

Note: For all questions, answer "(E) NOTA" means none of the above answers is correct.

1. Find the sum of all integers from 1 to 2013, **non-inclusive**.

(A) 2029091 (B) 2129791 (C) 2027089 (D) 2027093 (E) NOTA

2. Let p_n denote the n th positive prime number. For example, $p_1 = 2$, $p_2 = 3$, etc. Find the value of $\sum_{n=1}^{p_5} p_n$.

(A) 100 (B) 129 (C) 160 (D) 197 (E) NOTA

3. The first term of an arithmetic sequence is 1, and the fifth term is 60. Find the common difference of the sequence.

(A) 4 (B) 15 (C) $\frac{59}{4}$ (D) $\sqrt{2\sqrt{15}}$ (E) NOTA

4. Evaluate: $\sum_{l=1}^{10} l \sum_{k=1}^{10} k \sum_{j=1}^{10} j \sum_{i=1}^{10} i$

(A) 5764801 (B) 9150625 (C) 13845481 (D) 4477456 (E) NOTA

5. What is the sum of the first smallest twenty-five positive perfect squares?

(A) 4550 (B) 5525 (C) 6150 (D) 8725 (E) NOTA

6. Find the value of the fourth smallest Pentagonal Number.

(A) 20 (B) 22 (C) 25 (D) 50 (E) NOTA

7. Which of the following is equivalent to the fourth term of an arithmetic sequence whose first three terms, in order, are: 23_4 , 24_5 , and 25_6 ?

(A) 25_7 (B) 36_7 (C) 24_8 (D) 267_{10} (E) NOTA

8. Let $i = \sqrt{-1}$. Evaluate: $\prod_{n=1}^{100} i^n$

(A) i (B) -1 (C) $-i$ (D) 1 (E) NOTA

9. What is the sum of the first five terms of a geometric sequence with a first term of $3/4$ and a common ratio of $4/3$?

(A) $\frac{781}{108}$ (B) 87 (C) $\frac{718}{79}$ (D) $\frac{817}{27}$ (E) NOTA

10. Chris and Kadie make a pact to get in better shape. On the first day, Chris runs 1 mile, on the second day he runs 2.5 miles, on the third day he runs 4 miles, and so on, adding an additional 1.5 miles to the number of miles he runs each day with each passing day. Kadie runs 2 miles on the first day and takes the second day off. She runs 5 miles the third day, 8 miles the fifth day, and so on, taking every even-numbered day off. How many days will it take for Chris and Kadie to have run more than 1400 miles combined?
- (A) 24 (B) 25 (C) 30 (D) 40 (E) NOTA
11. Chris and three of his friends are playing *Battlefield 4*. Afterwards, Chris remarks that the total number of points the team had accrued was 30, and that, when the points each person earned are arranged in ascending order, it forms either an arithmetic or geometric sequence. Unfortunately, Chris can't remember exactly which type of sequence. What is the sum of all possible values of the number of points earned by the second worst player? (Note: Assume points are nonnegative integers. Also, we are measuring a player's skill in terms of points; the more points, the better the player.)
- (A) 22 (B) 15 (C) 16 (D) 23 (E) NOTA
12. Evaluate: $\sum_{k=1}^{\infty} \frac{k}{5^k}$
- (A) 0.80 (B) 0.3125 (C) 0.25 (D) 0.20 (E) NOTA
13. Two trains are 30 miles apart, approaching each other on a collision course! The trains are travelling at 10 miles per hour and 20 miles per hour, respectively. A fly travelling at 30 miles per hour begins at the front of the first train and flies to the front of the second train. Once it reaches the front of the second train, it instantaneously turns around and flies back to the nose of the first train, and so on. Assuming acceleration is instantaneous, how many miles will the fly travel before the trains collide?
- (A) 10 (B) 20 (C) 40 (D) 30 (E) NOTA
14. A sequence defined as $a_1 = 7$, $a_2 = 8$, $a_3 = 9$, and for $n > 3$, $a_n = a_{n-1} - a_{n-2} + a_{n-3}$. Find the units digit of a_{2013} .
- (A) 8 (B) 7 (C) 6 (D) 5 (E) NOTA
15. Evaluate: $\sum_{i=2}^5 \left(\prod_{n=2}^i \frac{i+n}{n} \right)$
- (A) 63 (B) 42 (C) 43 (D) 132 (E) NOTA

16. Consider the sequence 1, 1, 1, 2, 2, 1, 1, 2, 3, 3, 2, 1, 1, 2, 3, 4, 4, 3, 2, 1, 1, 2, 3, 4, 5, 5, 4, 3, 2, 1, ..., where the terms increase from 1 through n , inclusive, and then decrease from n to 1, inclusive, for $n = 1, 2, 3, \dots$. What is the 2013th term in this sequence?

- (A) 30 (B) 31 (C) 32 (D) 33 (E) NOTA

17. Name the type of series described by $\sum_{k=1}^n a_k$, where $a_k = \frac{1}{k}$.

- (A) Arithmetic (B) Geometric (C) Gradient (D) Harmonic (E) NOTA

18. The sum of seven positive numbers is 21. Find the smallest possible value of the arithmetic mean of the squares of these numbers.

- (A) 10 (B) 11 (C) 12 (D) 13 (E) NOTA

19. Let C equal the sum of the first 2013^{2013} smallest positive perfect cubes. Which of the following is equal to the number of positive integral factors of C ?

- (A) 504 (B) 506 (C) 508 (D) 510 (E) NOTA

20. A geometric sequence has a nonzero first term a and a common ratio equal to r . Which of the following is a necessary condition for the sum $S = \sum_{n=2013}^{\infty} a^2 r^n$ to equal a finite quantity?

- (A) $r \leq 1$ (B) $r = 0$ (C) $r > 1$ (D) $|r| < 1$ (E) NOTA

21. Find the sum of all values of x such that $x - 1, 2x$, and $5x + 3$ form a geometric sequence of positive real numbers.

- (A) 3 (B) 2 (C) 1 (D) 0 (E) NOTA

22. When $4.1\overline{36}$ is written as a fraction $\frac{m}{n}$, where m and n are relatively prime positive integers, what is the value of $m + n$?

- (A) 111 (B) 113 (C) 115 (D) 117 (E) NOTA

23. How many terms does the geometric sequence 6, 12, 24, ..., 768 have?

- (A) 8 (B) 11 (C) 32 (D) 54 (E) NOTA

24. Suppose the sum of the first n terms of a sequence $a_1, a_2, a_3 \dots$ is given by the formula $S_n = 4n(n - 1)$. Find the value of a_{2013} .
- (A) 16088 (B) 16090 (C) 16092 (D) 16094 (E) NOTA
25. The sum of an infinite geometric series is $2013 = (3)(11)(61)$. Each of the terms in this series is squared, resulting in a series with a sum of $66429 = (2013)(33)$. Find the common ratio of the original series.
- (A) $\frac{26}{27}$ (B) $\frac{28}{29}$ (C) $\frac{30}{31}$ (D) $\frac{32}{33}$ (E) NOTA
26. Solve for y if $\sum_{n=1}^6 (3ny + 2) = 3414$.
- (A) 54 (B) 55 (C) 56 (D) 57 (E) NOTA
27. What is the coefficient of a^2b^{11} when $(2a - b)^{13}$ is expanded and like-terms combined?
- (A) 26 (B) -11440 (C) 2941 (D) -312 (E) NOTA
28. Two distinct numbers a and b are selected at random from the first 100 terms of the Fibonacci Sequence, where any term is equally likely to be chosen. Recall that the Fibonacci Sequence is the sequence with the first and second terms both equal 1, and every term after the third is the sum of its previous two terms. Find the probability that $a + b$ is odd.
- (A) $\frac{17}{75}$ (B) $\frac{5}{8}$ (C) $\frac{67}{150}$ (D) $\frac{1}{2}$ (E) NOTA
29. The geometric sequence x_1, x_2, x_3, \dots is strictly increasing and has terms consisting of only integer powers of 2. If $\sum_{n=1}^{10} \log_2(x_n) = 500$ and $90 < \log_2(\sum_{n=1}^{10} x_n) < 100$, find the value of $\log_2(x_{20})$.
- (A) 185 (B) 190 (C) 195 (D) 200 (E) NOTA
30. The inhabitants of a faraway planet communicate using a language consisting of an alphabet of only four letters: A, B, C, and D. A valid word in this language consist of an even amount of As and an even amount of Bs, and any number of Cs and Ds. Some examples of valid words are AABBA, ABBBBBC, and AABBDCC. How many valid four-letter words are in this language?
- (A) 64 (B) 68 (C) 72 (D) 76 (E) NOTA