1. E.

 $tan\frac{θ}{2}=\frac{1-cosθ}{sinθ}$; squaring we get $\frac{1-2cosθ+cos^{2}θ}{sin^{2}θ}$.

 $1+tan^{2}\frac{θ}{2}=\frac{sin^{2}θ}{sin^{2}θ}+\frac{1-2cosθ+cos^{2}θ}{sin^{2}θ}=\frac{2-2cosθ}{sin^{2}θ}$

Cross multiply to get $rsin^{2}θ=2-2cosθ$.

Multiply both sides by r: $r^{2}sin^{2}θ=2r-2rcosθ$.

Convert to rectangular: $y^{2}=2\sqrt{x^{2}+y^{2}}-2x$

Simply: $y^{4}+4xy^{2}+4x^{2}=4x^{2}+4y^{2}$

 $∴y^{4}+4xy^{2}-4y^{2}=0$

1. C $r=\frac{a+b-c}{2}$ $∴ r=3$, and the center is at (3,3), making the

8

15

17

equation $(x-3)^{2}+(y-3)^{2}=9$, which simplifies to

$$x^{2}-6x+y^{2}-6y+9=0$$

1. B
The sum of the distances from the foci of an ellipse to a point on the ellipse is equal to 2a, in this case 2x3=6. The perimeter of the quadrilateral is twice this, so 12.
2. A

We can describe the tunnel as half of an ellipse with a vertical axis of 60 and a horizontal axis of 20, centered at the origin. The equation of such an ellipse would be $\frac{y^{2}}{900}+\frac{x^{2}}{100}=1$. We must find y when x = 5. $\frac{y^{2}}{900}+\frac{25}{100}=1, so \frac{y^{2}}{900}=\frac{3}{4} and y^{2}=675, making y=15\sqrt{3}$

1. C
$8cosθsinθ=4sin2θ$. This is a rose curve with a petal length of 4.
2. B
We must go up a total of 4 times and to the right a total of 7 times. The problem can be reduced to the number of words that can be formed from 11 letters, four of which are “a” and seven of which are “b.” This is equal to $\frac{11!}{7!4!}=330$
3. D
The volume of a torus is the same as that of a cylinder with a height equal to the internal circumference of the torus and a radius equal to the radius of the torus. The height of the cylinder is $πd=12π$. The radius of the cylinder is 1. The volume is therefore $π\left(1\right)\left(1\right)\left(12π\right)=12π^{2}$.
4. A
The side length of the first shaded square is $5\sqrt{2}$ (because it is the hypotenuse of the 45-45-90 triangle with a leg length of half the side of the largest square), so its area is 50. The side length of the next (unshaded) square is $\frac{5\sqrt{2}}{2}\*\sqrt{2}=5$. The side length of the next (shaded) square is then $\frac{5\sqrt{2}}{2}$, making its area $\frac{25}{2}.$ The ratio of the area of the inner shaded square to the largest shaded square is then $\frac{25}{2}:50=\frac{1}{4}$. To find the sum of the infinite geometric series: $\frac{a}{1-r}=\frac{50}{1-\frac{1}{4}}=\frac{200}{3}.$
5. E
$x^{2}-4x+y^{2}-6y-12=0$ defines a circle with radius 5 and center (2,3). When revolved around the line y=3, it forms a sphere with radius 5. The volume of the sphere is $\frac{4}{3}π\left(125\right)=\frac{500π}{3}$
6. B
The fourth roots of 16 are 2, -2, 2*i*, and -2*i*. When plotted on the Argand plane, these points form a square with a diagonal of 4 – the area is 8.
7. E
The slope from the point (1,3) to the point (2,2) is –1. Going up 1 and to the left 1 from the point (-1,-2) gives us (0,-3).
8. A
The absolute value of a complex number *z* is that number’s distance from the origin. Thus, this equation forms a circle with a radius of 3 that is centered at the origin.
9. C
This equation is for an ellipsoid. The formula for the volume of an ellipsoid in the form
 $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}+\frac{z^{2}}{c^{2}}=1 $is $\frac{4}{3}πabc$. Divide both sides by two to get it into standard form. The volume is $\frac{4}{3}\*π\*2\sqrt{2}\*3\sqrt{2}\*10=160π$.
10. C
Reflect the point (5,-4) around the line x=1, getting a new point of (-3,-4). Find the distance between (3,2) and (-3,-4) = $\sqrt{(-3-3)^{2}+(-4-2)^{2}}=\sqrt{36+36}=6\sqrt{2}.$
11. D
To put the points into Shoelace Theorem, they must be in an order than would make the non-overlapping shape. (2,3), (4,1), (5,0), (4,-1), (-2,0) will work.
|$\begin{matrix}2&4&5\\3&1&0\end{matrix}\begin{matrix} 4&-2&2\\ -1&0&3\end{matrix}$|. Taking the downward diagonals, we get (2-5-6)= -9. Taking the upward diagonals, we get (12+15+2)=19. Subtracting gives us -28; taking the absolute value and dividing by two gives the area: 14.
12. E
$e^{iθ}=cisθ=cosθ+isinθ$. The absolute value of $cisθ$ for any angle is equal to $\sqrt{cos^{2}θ+\left(-1\right)sin^{2}θ}=\sqrt{cos2θ}$. $\sqrt{cos\frac{π}{3}}=\frac{\sqrt{2}}{2}$
13. B
Taking the limit as x approaches $\infty $ and $-\infty $, we get horizontal asymptotes of y=0 and y=$\frac{5}{2}$
14. A
The general form of a polar conic is $r=\frac{eh}{1-ecosθ}$. In order to get the “1” on the denominator, we factor out the 5 and get $e=\frac{2}{5}$. The eccentricity is between 0 and 1, so the conic is an ellipse.
15. D
The reindeer can fly in an area equal to $\frac{7}{8}$ of a sphere. The radius of the sphere is 18 feet, which is 6 yards. Finding the volume: $\frac{7}{8}\*\frac{4}{3}\*π\*6^{3}=252π$.
16. C

A

B

4

R

r

 $R^{2}-r^{2}=16$. The area of an annulus is

 $π\left(R^{2}-r^{2}\right)=16π$.

1. B
This will be the inverse function. Switching x and y, we get $x=3y+9$. Solving for y gives $y=\frac{1}{3}x-3$.
2. D
Completing the square gives $3(y-1)^{2}-4\left(x+3\right)^{2}=72$, which gives $\frac{(y-1)^{2}}{24}-\frac{\left(x+3\right)^{2}}{18}=1$. The latus rectum is equal to $\frac{2b^{2}}{a}$, which gives us $\frac{2(18)}{2\sqrt{6}}=3\sqrt{6}$
3. E
The two circles have equations of $(x+3)^{2}+\left(y-1\right)^{2}=25$ and $\left(x-5\right)^{2}+\left(y-4\right)^{2}=81$. The distance between the centers is $\sqrt{\left(-3-5\right)^{2}+\left(1-4\right)^{2}}=\sqrt{73}$. Subtracting the two radii, we get $\sqrt{73}-14$
4. D
This will make a frustrum with base radii of 8 and 4 and a height of 4. Using similar right triangles, we can find the height of the removed cone to be 4. The volume of the frustrum is then $\frac{1}{3}π\left(8\*8\*8-4\*4\*4\right)=\frac{448π}{3}$
5. B
For two vectors to be perpendicular, their dot product must equal zero. For choice B, 3\*1+ -1\*9 + 6\*1 = 0.
6. E
If you draw an altitude to the longest side of the triangle, you form a 45-45-90 triangle and a 30-60-90 triangle. Therefore, $AB=x+x\sqrt{3}$, making $x=\frac{6}{1+\sqrt{3}}$. The area of the triangle is then $\frac{1}{2}\*\frac{6}{1+\sqrt{3}}\*6=\frac{18}{1+\sqrt{3}}$. The area of a circumscribed triangle is **r\*s**, so we can set$ \frac{18}{1+\sqrt{3}}=r\*\left(6+\frac{6\sqrt{2}}{1+\sqrt{3}}+\frac{6\sqrt{3}}{1+\sqrt{3}}\right)∴r=\frac{3}{2\sqrt{3}+\sqrt{2}+1}$

1. C
$$\left(cosh\frac{π}{2}+sinh\frac{π}{2}\right)\left(sinh\frac{π}{2}-cosh\frac{π}{2}\right)=sinh^{2}\frac{π}{2}-cosh^{2}\frac{π}{2}$$

$=-\left(cosh^{2}\frac{π}{2}-sinh^{2}\frac{π}{2}\right). $ For any angle, $cosh^{2}\frac{π}{2}-sinh^{2}\frac{π}{2}=1$. $∴$ the answer is -1.

1. E

$$distance= \frac{\left|\left(4\right)\left(1\right)-\left(4\right)\left(0\right)+\left(2\right)\left(2\right)-5\right|}{\sqrt{4^{2}+4^{2}+2^{2}}}=\frac{3}{6}=\frac{1}{2}$$

1. C
There are two intersection points corresponding to $=0$ , r=0 and r=2. There are also intersection points at $θ=2π/3$, $r=-\sqrt{3}$ and $θ=4π/3$, $r=-\sqrt{3}$.

The first two solutions can be found by observing the graphs of the two functions; the second two can be found by setting the two equations equal and solving for $θ$

1. B

$$x=sintcost=\frac{1}{2}sin2t$$

$$y=cos^{2}t-sin^{2}t=cos2t$$

$$∴4x^{2}+y^{2}=1$$