National Convention 2014 Alpha Ciphering solutions

Q#0- $\cos^2 x+\sin^2 x \quad \cos^2 x-\sin^2 x \;=\cos 2x \rightarrow \frac{2\pi}{b}=\pi$

Q#1- $\begin{vmatrix} 3 & 4 & -2 & 1 \\ 5 & 1 & 0 & -1 \\ 3 & 1 & 0 & 4 \\ 4 & -1 & 0 & 5 \end{vmatrix} = -2\begin{vmatrix} 5 & 1 & -1 \\ 3 & 1 & 4 \\ 4 & -1 & -5 \end{vmatrix} = -2\begin{vmatrix} 9 & 0 & -6 \\ 7 & 0 & -1 \\ 4 & -1 & -5 \end{vmatrix} = -2\begin{vmatrix} 9 & -6 \\ 7 & -1 \end{vmatrix} = -66$

Q#2- $a_1+a_1r=90 \qquad a_1\ 1+r\ =90$

$a_1r^5+a_1r^6=\frac{-10}{27} \qquad a_1r^5\ 1+r\ =\frac{-10}{27} \qquad r^5=\frac{-10}{27}\bullet\frac{1}{90}=\frac{-1}{3^5}\rightarrow r=\frac{-1}{3}$

$$S_7=\frac{135\left(1-\frac{-1}{3^7}\right)}{1-\frac{-1}{3}}=\frac{3}{4}\bullet 135\left(1+\frac{1}{2187}\right)=\frac{3\ 135}{4}\bullet\frac{2188}{2187}=\frac{2735}{27}\ or\ 101\frac{8}{27}$$

Q#3 $x-y=\ \sqrt{x}-\sqrt{y}\ \ \sqrt{x}+\sqrt{y}\ =12\rightarrow 8\ \sqrt{x}-\sqrt{y}\ =12\rightarrow\ \sqrt{x}-\sqrt{y}\ =\frac{3}{2}$

$$2\sqrt{x}=\frac{19}{2}\rightarrow\sqrt{x}=\frac{19}{4}\rightarrow x=\frac{361}{16}$$

Q#4-I would draw a picture of a 6 by 6 and a 1 by 6 to get a visual. Two or more means two of the same and one different or all 3 the same. There will be 216 total possible outcomes. There are 6 ways to get all the same. In the 6 by 6 there are 30 squares that produce different numbers and they can be matched up with 2 in the 1 by 6. Also in the 6 by 6 there are 6 squares with doubles and they can be matched up with 5 of the squares in the 1 by 6.

$$\frac{6+6\ 5\ +30\ 2}{216}=\frac{4}{9}$$

Q#5- Draw picture!! Extend the larger radius out 5 units at the tangent point and then create a rectangle to the center of the smaller circle. This creates a right triangle where the distance from the centers is the hypotenuse, one side is now 11 and the other side is equivalent to the common internal tangent we are trying to solve for. $16^2-11^2=x^2=256-121=135\rightarrow x=\sqrt{135}=3\sqrt{15}$

Q#6- Find all the numbers divisible by 4 between 224 and 692. Find all the numbers divisible by 7 between 224 and 693. Add these together and then subtract out all the numbers between 224 and 672 that are divisible by 28. They have been double counted.

$4\ \ 56 \leftrightarrow 4\ \ 173 \rightarrow 173-56+1=118$

$7\ \ 32 \leftrightarrow 7\ \ 99 \rightarrow 99-32+1=68$

$28\ \ 8 \leftrightarrow 28\ \ 24 \rightarrow 24-8+1=17$

$118+68-17=169$

Q#7- Make perfect squares under the radicals and this is an easy question.

$$\frac{\sqrt{\sqrt{7}+\sqrt{11}^{\,2}}}{\sqrt{\sqrt{14}+\sqrt{6}^{\,2}}}=\frac{\sqrt{7}+\sqrt{11}}{\sqrt{6}+\sqrt{14}}\bullet\frac{\sqrt{6}-\sqrt{14}}{\sqrt{6}-\sqrt{14}}=\frac{\sqrt{42}-\sqrt{98}+\sqrt{66}-\sqrt{154}}{-8}\rightarrow 42+66+154=262$$

Q#8-Complete some squares and transform into: $x-6^{\,2}+\ y+4^{\,2}=45$. Find the slope of the line through tangent point and center then take negative reciprocal to obtain the slope of our line. Negative 2 is the slope then negate and flip and you get one-half. Plug in the point (3,2) and you get your answer of: X - 2Y = -1

Q#9-sinXcosY+cosXsinY. Draw two triangles in the proper quadrants and obtain the pieces.

$$\frac{2\sqrt{5}}{5}\bullet\frac{3}{\sqrt{10}}+\frac{-\sqrt{5}}{5}\bullet\frac{-1}{\sqrt{10}}=\frac{7\sqrt{5}}{5\sqrt{10}}=\frac{7\sqrt{50}}{50}=\frac{7\sqrt{2}}{10}$$

Q#10-Use tangent formula:

$$Tan\ \ A+B\ =\frac{TanA+TanB}{1-TanA\bullet TanB}=Tan\left(\frac{8\pi}{12}+\frac{9\pi}{12}\right)=Tan\left(\frac{2\pi}{3}+\frac{3\pi}{4}\right)=\frac{\tan\frac{2\pi}{3}+\tan\frac{3\pi}{4}}{1-\tan\frac{2\pi}{3}\bullet\tan\frac{3\pi}{4}}$$

$$\frac{-\sqrt{3}-1}{1-\sqrt{3}}\bullet\frac{1+\sqrt{3}}{1+\sqrt{3}}=\frac{-2\sqrt{3}-4}{-2}=2+\sqrt{3}$$

Answers:

0. π

1. -66

2. $101\frac{8}{27} or \frac{2735}{27}$

3. $\frac{361}{16}$

4. $\frac{4}{9}$

5. $3\sqrt{15}$

6.169

7. 262

8. X -2Y = -1

9. $\frac{7\sqrt{2}}{10}$

10. $2+\sqrt{3}$