1. **(A)** The sum of the geometric series is equal to



2. **(C)** Taking the common logarithm of both sides: .



3. **(B)** By the power of a point theorem,. So is a diameter since it is the perpendicular bisector of a chord. So the diameter is 26, which makes the radius 13.



4. **(B)** The maximum value of both functions occur at the same point. Both and are maximized when And thus, the maximum value of the function is



5. **(A)** Taking the tenth root gives. This gives.



6. **(A)** The three triangles constructed by connecting the centroid to the vertices form three congruent triangles. So the ratio of the area of one of them to the area of the whole triangle is 1/3.

7. **(C)** The composite function is so the domain of is all real numbers that are also in the domain of. So the answer includes all values except.



8. **(Thrown Out at Convention) (A)** By the rational root theorem, the only possible integer roots are . Since there are exactly six different roots, this must be all of them. So.



9. **(E)** Replace with and determine if the function is changed. All of them produce the same functions, so they are all even.



10. **(D)** Let and. Then . Hence, . So either or 1. For , we need . So we have to find the number of values such that. We are given . By the Fundamental theorem of algebra this equation has distinct solutions. So we have a total of solutions.



11. **(C)** There are three different fractions that it decomposes into. Multiply both sides by the denominator to find:. By correlating various powers of on both sides of the equation, we find that .



12. **(E)** We eliminate the parameter by squaring both sides and using the identity . Solving for and and then substituting gives. This equation is a hyperbola with center (2,2), which is the point where the asymptotes will intersect.



13. **(B)** This occurs at , so there are two solutions every , which means 2014 solutions total.



14. **(D)** If we raise each to the sixth power, we see that



15. **(C)** A surjection is a function such that for every there exists some such that . First we count that the number of well-defined functions from the first set (call *A*) to the second set (call *B*). Each of the 4 elements in *A* has three choices in *B* to be mapped to. So there are total functions. We must now subtract out the number of functions that do not have one of the elements of *B* in its range. There are 3 elements to choose and ways to assign the rest of the elements in the range, for a total of functions. However, we have over-subtracted the functions that have only one element in their range. There are 3 of these functions. So the total number of surjections is .



16. **(D)** We find the vectors that bound this triangle. Subtracting the first two points from the last gives and. The area of the triangle is.



17. **(C)** Substitute 6 for the repeating expression to get.



18. **(B)** The circle touches the vertices of the ellipse at a distance from the origin, making that the radius and the area .

19. **(D)** Expand the matrices to get the equations necessary. Let for simplicity. Also, . Then . Combining the results gives . The desired expression is.



20. **(A)** Converting to sines and cosines gives . But, so multiply through by 2 to be able to write the product as



Then we can use the angle addition/subtraction formulas to get

.



21. **(A)** We find the term where the powers of cancel. The power of the term must be twice as much as the power of the term. Each term of the sequence is given by so . This gives. Trying values for gives us .



22. **(D)** Clearly *b* will divide , but since *a* and *b* are relatively prime, *b* will never divide any number of *a*’s. So *b* will never divide the sum.

23. **(C)** Writing out the first few terms gives that . For this to be the square of an integer, say , then we have , which implies



. By the wording of the question, it can be assumed that there is a unique , so all we must do is find some particular value. If we suppose that the two given factors are equal, then take their difference to see that . This value will suffice and is the desired value for . Substituting into the initial conditions shows that



24. **(A)** We make the following algebraic manipulations:



Subtract and divide by 3 to get



Note that



Subtracting the previous two equations gives.



25. **(C)** Clearly the argument of arctan approaches infinity as *x* grows without bound. The principal angle whose tangent grows without bound is .

26. **(C)** The first is the graph of a straight line and the second is the graph of a circle, so they can intersect at most twice.

27. **(C)** For simplification, we use the “BAC-CAB rule,” which states that . Applying this rule and then the appropriate permutation for the second term give since the dot product is commutative. Again using this rule, we get this is equal to . The cross product is anti-commutative, but the result is unchanged if we commute the two vectors twice. So this is equal to .



28. **(E)** Looking at the product expanded out, we have the first few terms look like: . If we were to expand this Euler-style product by taking one term from each sum, using the fundamental theorem of arithmetic, and ordering the terms appropriately, we would see that this product is equal to . We know that the harmonic series diverges, and hence so does the product.



29. **(B)** Knowing whether the sum is even is equivalent to knowing if the number of odd integers is even. If we had statement (1), we would know that among thethe number of powers of two is some positive multiple of ; however, we have no information of the distribution of the odd primes. Given statement (2), we know that any power of an odd is odd. So the number of odd powers would odd, and therefore the number of odd numbers would be odd. So Benji could answer “no” to the given question. Hence, statement (2) alone can and will provide an answer.



30. **(A)** The rank of a matrix is the maximal number of linearly independent rows. If *k* is anything but 2, then the fourth row is not a multiple of the third row, which would make the rank equal to 4. So *k* must be 2.