

For all questions, answer choice "E) NOTA" means none of the above answers are correct.

1. Solve: $\begin{bmatrix} 1 & 2 \\ -1 & -3 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ -2 & 1 \end{bmatrix}$.

A) $\begin{bmatrix} 0 & 3 \\ 2 & -4 \end{bmatrix}$

B) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

C) $\begin{bmatrix} 1 & 3 \\ -10 & 4 \end{bmatrix}$

D) $\begin{bmatrix} 0 & 1 \\ -2 & 4 \end{bmatrix}$

E) NOTA

2. Find an equation of the plane P in \mathbb{R}^3 that passes through (1,2,3) and is normal to $\langle 2, -4, 3 \rangle$. Use the form $Ax + By + Cz = k$, where A, B, C are relatively prime integers with $A > 0$. Find k.

A) 1

B) 3

C) 2

D) -3

E) NOTA

3. Generalize the previous problem to n dimensions. A hyperplane lies in \mathbb{R}^n which passes through $P = (2, 2, 2, \dots)$ and is normal to $u = \langle 1, \frac{1}{2!}, \frac{1}{3!}, \frac{1}{4!}, \frac{1}{5!}, \dots \rangle$. Use the form $A_1x_1 + A_2x_2 + A_3x_3 + \dots = k$, where $A_1 = 1$. Find $\ln(k)$.

A) $2\ln(e-1)$ B) $2 - \ln(2)$ C) $\ln(e-1) + \ln(2)$ D) $\ln(2)$

E) NOTA

4. Let the complex number $z = a + bi$, where a and b are real, positive numbers. Let \bar{z} be the complex conjugate of z. Plotting the two on the Argand plane, what is the smaller angle between z and \bar{z} ?

A) $\tan^{-1}\left(\frac{b}{a}\right)$

B) $2 \tan^{-1}\left(\frac{b}{a}\right)$

C) $2 \cos^{-1}\left(\frac{b}{a}\right)$

D) $\sin^{-1}\left(\frac{b}{a}\right)$

E) NOTA

5. $v = \langle 1, -2, 5 \rangle$ is a linear combination of $u_1 = \langle 1, 1, 1 \rangle$, $u_2 = \langle 1, 2, 3 \rangle$, $u_3 = \langle 2, -1, 1 \rangle$, which can be written in the form $au_1 + bu_2 + cu_3 = v$. Find $\frac{a}{b \cdot c}$.

A) 1

B) 2

C) 0

D) -1

E) NOTA

6. In the figure, u_p is the projection of the vector u onto v. Note that the magnitude of u_p is determined by θ , the angle between the vectors u and v, but its direction is determined by v. If $u = \langle 1, -3, 4 \rangle$ and $v = \langle 3, 4, 7 \rangle$, find the projection of u onto v.

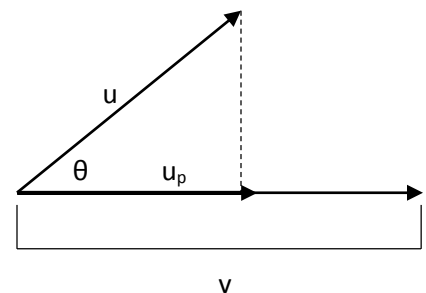
A) $\left\langle \frac{57}{74}, \frac{38}{37}, \frac{133}{74} \right\rangle$

B) $\left\langle \frac{19}{74}, \frac{18}{74}, \frac{19}{74} \right\rangle$

C) $\left\langle \frac{19}{74}, \frac{18}{74}, \frac{19}{74} \right\rangle$

D) $\left\langle \frac{4}{74}, -\frac{12}{74}, \frac{28}{74} \right\rangle$

E) NOTA



7. Which of the following is NOT true about traces? (Assume A, B are 2x2 matrices).

- A) $\text{tr}(A) + \text{tr}(B) = \text{tr}(A+B)$ B) $\text{tr}(A^T) = \text{tr}(A)$ C) $\text{tr}(9A) = 9\text{tr}(A)$
 D) $\text{tr}(AB) = \text{tr}(BA)$ E) NOTA

8. Let A be a real, 2x2, rotation matrix. The determinant of A is always equal to what?

- A) 1 B) -1 C) 0 D) 2 E) NOTA

9. Find the determinant of $M = \begin{bmatrix} 5 & 4 & 2 & 1 \\ 2 & 3 & 1 & -2 \\ -5 & -2 & -3 & 9 \\ 1 & -2 & -1 & 4 \end{bmatrix}$.

- A) 1 B) -38 C) 103 D) 0 E) NOTA

10. u_1, u_2, u_3 uniquely determine a tetrahedron. What is the volume of the tetrahedron if $u_1 = \langle 1, 1, 0 \rangle$, $u_2 = \langle 1, 1, 1 \rangle$, and $u_3 = \langle 0, 2, 3 \rangle$?

- A) 1/3 B) 2 C) 1/2 D) 4 E) NOTA

11. Which of the following sets contain vectors that determine a unique plane?

- I. $\langle 0, 0, 1 \rangle, \langle 1, 0, 0 \rangle$
 II. $\langle 3, 4, -1 \rangle, \langle 8, 0, 16 \rangle$
 III. $\langle -2, 5, 3 \rangle, \langle 4, -10, -6 \rangle$
 IV. $\langle 0, 0, 0 \rangle, \langle 1, 5, 3 \rangle$

- A) I only B) II, III, IV C) III, IV D) I, II E) NOTA

12. Find the rank of the following matrix: $\begin{bmatrix} 1 & 2 & 0 & -1 \\ 2 & 6 & -3 & -3 \\ 3 & 10 & -6 & -5 \end{bmatrix}$.

- A) 1 B) 2 C) 3 D) 0 E) NOTA

13. Find the characteristic polynomial of $M = \begin{bmatrix} 7 & -1 \\ 6 & 2 \end{bmatrix}$.

- A) $p(t) = t^2 - 9t + 14$ B) $p(t) = t^2 - 9t + 20$ C) $p(t) = t^2 + 9t + 20$ D) $p(t) = t^2 - 9t - 14$ E) NOTA

14. Find the distinct eigenvalues of $M = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$.

- A) 3 B) 4, 5, 2 C) 3, 5 D) 1, -1 E) NOTA

15. Given the system of equations, find xyz.

$$\begin{aligned} x + 2y + 3z &= 1 \\ x + 3y + 6z &= 3 \\ 2x + 6y + 13z &= 5 \end{aligned}$$

- A) 27 B) 15 C) -30 D) 30 E) NOTA

16. Find the trace of the inverse of the matrix $M = \begin{bmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{bmatrix}$.

- A) 8 B) -12 C) -1/12 D) 10 E) NOTA

17. Which vector is not orthogonal to 2 others in the following:

$$\begin{aligned} u_1 &= \langle 1, 1, 1 \rangle \\ u_2 &= \langle 1, -3, 2 \rangle \\ u_3 &= \langle 5, -1, -4 \rangle \\ u_4 &= \langle 3, -2, 1 \rangle \end{aligned}$$

- A) u_1 B) u_2 C) u_3 D) u_4 E) NOTA

18. Solve for x, assuming the system has 1 unique solution for (x, y):

$$\begin{aligned} \sqrt{a}x - \sqrt{b}y &= a^{5/2} \\ \sqrt{b}x + \sqrt{a}y &= b^{5/2} \end{aligned}$$

- A) $a^3 + b^3$ B) $a^2 - ab + b^2$ C) $a + b$ D) \sqrt{ab} E) NOTA

19. Find A^n if n is a positive integer and $A = \begin{bmatrix} 1 & 1/2 \\ 2 & 1 \end{bmatrix}$.

- A) $2^{n-2}A$ B) 2^nA C) $2^{n-2}A^2$ D) 2^nA^2 E) NOTA

20. Vishal plays a math game with Fred. Fred randomly creates a 3x3 matrix of non-negative integers less than 10. If the matrix is symmetric, Vishal wins a dog. What is the probability that Vishal will fairly win the game?

- A) $1/10^6$ B) $1/10000$ C) $1/1000$ D) $1/100$ E) NOTA

21. Which of the following is perpendicular to $4i - 5j + 2z$?

- A) $3i + 4j + 4z$ B) $3i - 4j + 4z$
 C) $-4i + 5 + 2z$ D) $5i - 4j - 2z$ E) NOTA

22. What is the trace of an $n \times n$ identity matrix?

- A) $n-1$ B) $n+1$ C) n^2 D) n E) NOTA

23. Which of the following describes the matrix $M = \begin{bmatrix} 0 & 2 & -4 \\ -2 & 0 & 1 \\ 4 & -1 & 0 \end{bmatrix}$.

- A) Symmetric B) Skew-symmetric C) Invertible D) Upper Triangular E) NOTA

24. Given that $A = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -4 \\ 1 & -2 \end{bmatrix}$, find $|\det(B^{-1}A^{-1})|$.

- A) $-1/4$ B) $1/4$ C) 10 D) -10 E) NOTA

25. Find the rotation angle θ when the rotation matrix Q is given by $Q = \begin{bmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$.

- A) 210° B) 180° C) 120° D) 30° E) NOTA

26. Given two vectors a and b , $\|a \times b\| = \sqrt{3}(a \cdot b)$. Given that the smaller angle between them is θ degrees, what is the sum of the digits of θ .

- A) 0 B) 3 C) 6 D) 9 E) NOTA

27. $\begin{bmatrix} 1 & -1 \\ 2 & 5 \\ -3 & 1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 5 & 0 \\ 2 & -3 & 2 \\ -1 & 4 & 0 \end{bmatrix} = ?$

- A) $\begin{bmatrix} -1 & 8 & -2 \\ 12 & -5 & 10 \\ -1 & -18 & 2 \\ 2 & 10 & 0 \end{bmatrix}$ B) $\begin{bmatrix} -1 & 4 & -2 \\ 0 & 3 & 1 \end{bmatrix}$ C) $\begin{bmatrix} -1 & 8 & 2 \\ 1 & 5 & -10 \\ 0 & 4 & 0 \end{bmatrix}$ D) $\begin{bmatrix} 1 & -2 & 6 \\ 2 & -5 & 7 \\ 0 & 0 & 1 \\ 1 & 4 & -4 \end{bmatrix}$ E) NOTA

28. Which of the following are eigenvectors of the matrix $A = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$?

A) $\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

B) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

C) $\begin{bmatrix} 1/3 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1/3 \end{bmatrix}$

D) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

E) NOTA

29. For what value of k is the matrix given by $M = \begin{bmatrix} 2 & 2 & -1 \\ -4 & -7 & -4 \\ 0 & 5 & k - 8 \end{bmatrix}$ singular?

A) 10

B) 18

C) 0

D) -2

E) NOTA

30. $\langle 3, 5, -7 \rangle \cdot \langle -4, 3, 2 \rangle = ?$

A) 13

B) 4

C) -11

D) 3

E) NOTA