1. Rearrange the sum as . The last fraction reduces to . Then it is clear that each grouping is of the form which is equal to 1 by the Pythagorean identity. So the whole sum is equal to **4**.

2. The first inequality bounds a circle centered at with a radius of sqrt(2). The region in the circle consists of an isosceles triangle with sides sqrt(2) and a sector that is 90 degrees. The area is then 1 + pi/2. The Gaussian integers in the set are those with coordinates 0, 1, or 2, except the 3 that lie below the line y=x, and hence the total number is 6. So .

3. The probability is the ratio of the areas, which will be the square of the ratios of the radii. Also, and , so . Using Heron’s formula for the area gives . Cancelling the “s” and then substituting gives (4\*4\*3\*2)/(5\*6\*7) = 16/35. The probability is the square of this ratio (since areas involve the radius squared), so the answer is **256/1225**.

4. The numerator can be factored as , so that the factor cancels with the denominator. The so . From the second limit, it is obvious that , so the last limit must equal 0. Then , from which we get . This means . The final limit can be evaluated by substituting in 1 for x, since the left and right hand limits will be equal since the function is continuous. So . (Note that it was not necessary to use the last limit to find what and are to arrive at the answer.)

5. Consider the sum .

Now consider the difference .

Then .

6. The value of A is found simply by the number of factors of 5 in N. This is the floor of N/5 = 4. The sum of the digits must be divisible by 9, so mod 9. So or 16. But and , which means the sum equals 16. From the divisibility rule by 11, it is evident that or 15. But the max of , if both are 9. However, , so the difference is less than 15. Hence, . Adding these two equations gives . Therefore, . (Although it is irrelevant to the question, the exact value in the question is N=20).

7. The period of sin(x) is , and so the inner argument will repeat every , making this the period.



Simple evaluation of f yields .

For this we would need . But then , or .We will show that whenever x is positive. increases at a decreasing rate, while increases at a constant rate. When , . On this interval, , and . So the linear graph will always lie above the curve, and there will be no more intersections other than 0. Since both of these graphs are symmetric about the origin, .

Note that we cannot just ignore the arcsin even though the sin’s will “cancel” them out. If we don’t have , then there are issues with the evaluation of the first arcsin. Precisely of the interval satisfies this condition. So . Then .

8. We know that , so is co-terminal with or , both of whose square of their cosine is .

Next, we get . We note that cosine cannot be zero here, or else the tangent in the original equation would not be defined. Using the Pythagorean identities, we get . We substitute , so the equation becomes

*.* We want all possible values of , or .

(since ) But the negative sign it outside of the range of sin.

Next, we have , in which case we have either or , both of which lead to .

The desired sum is then .

9. Since , to every point (x,y) which satisfies this inequality, the point symmetric about the origin does not. Similarly, for every point which does not satisfy this, the point symmetric about the origin does. So ½ of the points satisfy this.

Each of the trig functions is at most 1, so the sum is no more than 2, let alone 4. Hence B = 1.

The total area of the possible region is 4pi. And pi/4pi = 1/4= C.

The inequality is transformed into , whose complement region is . Therefore the desired region is , so the probability is

The final answer is .

10. We note that , or the upper half of a circle with center at and radius . Hence, the length of the range is from to , so A=.

, which is the upper half of an ellipse with center (1,). The minor axis (parallel to the x-axis) ranges from to . This interval is of length , which is less , the difference between . So B=.

Let so that . The area of the full ellipse is , and we need half of that, so . The ellipse also completely contains the semicircle, which has an area of . The area contained between them is hence .

Finally, .

11. First note that . The leading coefficient will be 1, so .Vertical asymptotes occur when the denominator is 0, so . The zeros of the function occur when this numerator is 0, except 3 is a removable discontinuity. So the only zero is

Hence, the answer is .

12. Note that is the binomial expansion , separating out the complex terms and re-writing the cosines in terms of sines. The even terms are on the left and the odd terms are on the right. The double angle formula for sin was used in the second summand to get the 102 and ½ factor in front. The magnitude of is hence 1. And the argument is 510, which is coterminal with 150. Hence , and so our sum is **6**. (DO NOT ACCEPT 18)

13. Expansion by minors or any other method will give -8.

The inverse may be found by the standard algorithm or by using the transposed adjacent matrix method. It is . The sum of the entries is ½

In the last part, the eigenvalues are found as solutions to the equation . So the sum of the squares is 6. Hence A+B+C = -8 + ½ + 6 = **-3/2** .

14. The angle between u and v is 60 degrees, so the angle between u and v in the triangle of desired area is 120 degrees. The area is then .

By the law of cosines, we have , and so .

Finally,