Alpha Bowl State 2014 solutions

**1. Answer: 20**

**A = 9**: Powers of 3 cycle through 3, 9, 7, 1, … Note leaves a remainder of 2 so 9 is the units digit.

**B = 501**: Note that 10 = (2)(5) so the number of zeros at the end of the expansion relies on the number of factors of 10. Since there are less factors of 5 than 2, we find that there are such factors and thus 501 zeros.

**C = 512**: Noting , we see which has (8)(8)(8) = 512 factors.

We see that **A – B + C**  = **9 – 501 + 512 = **

**2. Answer: 12**

**A = 16**: Using the Shoelace Theorem, the area is 

**B = 1**: Let . Then we require: . Solving this system, we achieve  for  and thus .

**C = 4/5**: First using the Distance Formula, , , .

By the Law of Cosines, , hence . Using a 3-4-5 triangle, we have .

The desired quantity is then .

We see that 

**3. Answer: -10**

**A = 5:** 

We require  AND 

From the first inequality,. From the second, 

The intersection of the two sets is .  
The integers contained in this set are: -2, -1, 1, 2, 3 accounting for 5 total integers.

**B = **: Factor as . The only vertical asymptote occurs at  as there is a hole at *x* = -1. The horizontal asymptote occurs at  as the degrees or numerator and denominator are equal. Thus the desired sum is .

**C = -5:** We require 

Factor as  to see that , but  is not in the domain, so 

We see that **ABC = (5) (-5) = **

**4. Answer: 154**

**A = 272**: .

Thus, the terms are 6, 14, 22 with *d* = 8. Thus  and the sum 

**B = ** : .

The product of the values of *k* is .

**C = -256**: Note  has *x* = 0 as one of the roots and must have odd symmetry. Thus . Testing , we find the three given values to evaluate to 0, -16, and 16. Thus,  and .

We see that 

**5. Answer: -2**

**A =**  **2** : 



Hence,  but  is extraneous hence *x*  = 2 only.

**B = 4** : We require 

Using Change of Base, 

**C = -8** :By the Binomial Expansion Theorem, this can be rewritten as 

Using the identity , we have 

We see that **A + B + C** = 2 + 4 – 8 = ****

**6. Answer: 54**

**A =** : Using the sine rule for area, the area is 

**B = 14**: The radius of the circumcircle is  .

First use Law of Cosines to find the third side: 

Now 

**C = **: Use the Law of Sines: 

Thus, 

**7. Answer = 27**

**A =** : Start by completing the square on *x* and *y* in to achieve. This shows the center is (2, 1). The points (5, 1) and (2, 3) are the two endpoints in the first quadrant with a slope of .

**B =** : For the equation , we see that *a* = 3 and *b*= 2 and thus the asymptote’s slopes are  whose product is .

**C = 18** : Complete the square to write in standard form: . From this the vertex is at (2, 1) with focus at (-1, 1) so the triangle has height 3. When *x* = -1, we haveor  so the base of the triangle measures 12 units. The area of the triangle is then  square units.

We see that  = 

**8. Answer = 2**

**A = 4π**:  thus  which occurs at . The sum of these values is 

**B =** **2π :**  whose sum is 

**C =**: Squaring,  thus  and . Sine and cosine must be the same sign, thus 

**D = 6π2/5** : The old period is ****and the new period is ****.

Hence we have  = 

**9. Answer = **

**A = ** : The probability desired is ****

**B = **: The probability is an infinite series: ****

Thus, 

**10. Answer = 13**

**A = 8**: Let *d* bet the distance between Y and Z. Then 2*d* is the distance between X and Y and 6*d* is the distance between W and X. Thus, 9*d* = 72 and *d* = 8 miles.

**B = 27**: Let *a, b, c* be the current ages of Ann, Bob, and Carol, respectively. We have *b* = *c* + 3, and *a* = *b* + *c* = 2*c* + 3. Nine years ago, .

Thus,  and Ann’s current age is 2(12)+3 = 27.

**C = 48**: Let *d* be the distance traveled one way. Then  represents the total time for the round trip and 2*d* is the total distance traveled. Hence his average speed is 

We see that **C – B – A**  = 48 – 27 – 8 = 

**11. Answer = 17**

**A = 5**: When plotted, these points with the origin form a right triangle with hypotenuse 5. One could also use the Law of Cosines.

**B = 16**: Note  so 

**C = -4**: Eliminating *t*, we find that  whose minimum value is *y* = -4.

We see that **A + B + C** = 5 + 16 – 4 = .

**12. Answer =** 

**A = 20**: There arediagonals.

**B = 135**: Each interior angle measures degrees

**C = **: One way is to break up the octagon into 4 congruent right triangles, 4 congruent rectangles, and a central square. Respectively, these areas are ,  and  totaling .

We see that **A + B + C** = 20 + 135 +  = 

**13. Answer = -29**

**A =** : There are  arrangements.

**B =** : The are possible schedules.

**C = ** ways

We see that **A – B + C** = 20 – 84 + 35 = 

**14. Answer = 81**

First, let *h* be the height of the cone. Then .

Thus the cone has radius 6 units, height 8 units, and slant height 10 units.

**A = **: We require , thus  units

**B = 216**: The radius of the sector is equal to 10 units, the slant height. Letting  be the central angle in radians, we have , the circumference of the base. Thus 

We see that  = 