

Trigonometry Alpha Test - MAO National Convention 2014

Note that "NOTA" means none of the above.

1. Which of the following angles is coterminal with 123456° ?

- A. 24° B. 336° C. 343° D. -123366° E. NOTA

2. In what quadrant does the terminal side of θ lie when drawn in standard position if $\sec \theta > 0$ and $\tan \theta < 0$?

- A. I B. II C. III D. IV E. NOTA

3. How many solutions on $[0, 2\pi)$ are there to the equation $2\sin\left(4x - \frac{\pi}{6}\right) - 1 = 0$?

- A. 1 B. 2 C. 4 D. 8 E. NOTA

4. The sides of a triangle are in the ratio of 3: 5: 6. Determine the cosine of the largest angle.

- A. $-\frac{1}{15}$ B. $\frac{1}{15}$ C. $\frac{13}{15}$ D. $-\frac{1}{225}$ E. NOTA

5. Which of the following is not an identity?

- A. $\frac{\sin x}{1+\cos x} = \frac{1-\cos x}{\sin x}$ B. $\frac{\cot x}{\csc x} = \cos x$ C. $\sin^4 x - \cos^4 x = -\cos 2x$
D. $1 - \tan^2 x = \sec^2 x$ E. NOTA

6. Suppose $\sin x = \frac{3}{5}$ and $\cos y = -\frac{5}{13}$ where x and y both terminate in Quadrant II. Determine $\sin(x - y)$.

- A. $-\frac{36}{56}$ B. $\frac{63}{65}$ C. $\frac{33}{65}$ D. $-\frac{63}{65}$ E. NOTA

7. An angle $0 < x < 2\pi$ is chosen at random and with equal likelihood. What is the probability that $|\sec x| < \sqrt{2}$?

- A. 50% B. 75% C. 33.3% D. 25% E. NOTA

8. Compute the exact value of $\sin(\cos^{-1}(\tan(\sin^{-1}(\frac{\sqrt{2}}{2}))))$.

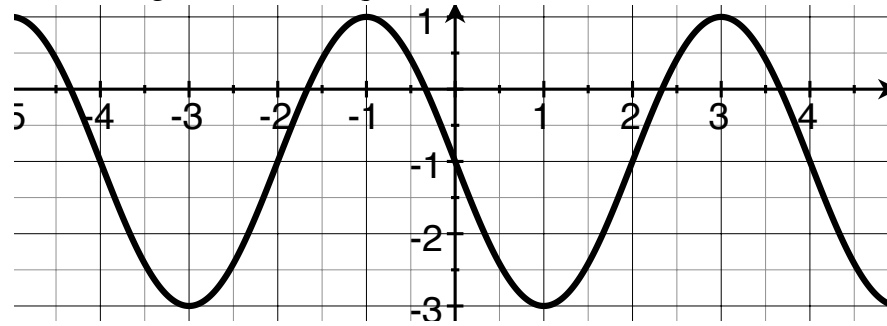
- A. Undefined B. 1 C. -1 D. $\frac{\sqrt{2}}{2}$ E. NOTA

9. On an analog clock the minute hand measures 8 cm. Determine the linear speed of the tip of the hour hand at the instant the time is 10:28 am.

- A. $\frac{4\pi}{99} \frac{cm}{min}$ B. $\frac{2\pi}{15} \frac{cm}{min}$ C. $\frac{4\pi}{15} \frac{cm}{min}$ D. $\frac{8\pi}{15} \frac{cm}{min}$ E. NOTA

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10. Which of the following could be the equation of the sinusoid shown below?



- A. $y = 2 \sin(4x) + 1$
- B. $y = 2 \sin\left(\frac{\pi}{4}x\right) - 1$
- C. $y = -2 \sin\left(\frac{\pi}{2}x\right) - 1$
- D. $y = 2 \cos\left(\frac{\pi}{2}(x - 1)\right) - 1$
- E. NOTA

11. What is the maximum possible area of a triangle two of whose sides measures 20 and 14?

- A. $140\sqrt{3}$
- B. $140\sqrt{2}$
- C. 140
- D. $140\sqrt{5}$
- E. NOTA

12. Which of the following is an eighth root of 16?

- A. $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$
- B. $\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$
- C. $1 - i$
- D. $-2 - 2i$
- E. NOTA

13. What is the sum of the solutions on $[0, 2\pi]$ to the equation $\cos 2\theta = -\cos \theta$?

- A. π
- B. 2π
- C. 3π
- D. $\frac{5\pi}{3}$
- E. NOTA

14. In triangle ABC , $\angle A = 30^\circ$ and $b = 16$. Which of the following could be the length, a , so that two possible triangles could exist?

- A. 8
- B. 10
- C. 7
- D. 17
- E. NOTA

15. Two campers are on opposite sides of a tree, where at the top, they see a large hawk 100 feet off the ground. The tree makes a right angle with the ground. The campers' angles of elevation to see the hawk are 45 and 60 degrees. Determine the approximate distance in feet that the two campers are apart along the ground.

- A. 73 feet
- B. 273 feet
- C. 158 feet
- D. 42 feet
- E. NOTA

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16. The domain of the function $f(x) = \sqrt{\frac{\pi}{2} - \cos^{-1}(x - \frac{1}{2})}$ can be written as the closed interval $[a, b]$ for real numbers a and b . Determine the value of $a + b$.

- A. 1 B. -1 C. 1.5 D. 2 E. NOTA

17. Compute the value of $\sec \frac{8\pi}{3} \csc \frac{7\pi}{2} + \tan \frac{11\pi}{6} \cos \frac{5\pi}{6}$.

- A. $\frac{3}{2}$ B. $\frac{5}{2}$ C. $-\frac{1}{2}$ D. $\frac{4+2\sqrt{3}}{2}$ E. NOTA

18. For an angle x in radians whose terminal side lies in Quadrant IV, which trigonometric expression is equivalent to $\tan(x - \frac{\pi}{2}) \cdot \frac{\sec^2 x - \tan^2 x}{\csc(\pi - x)}$?

- A. $\sec(x)$ B. $-\csc(x)$ C. $\cos(x)$ D. $-\cos(x)$ E. NOTA

19. Find the product of the amplitude and the period for the graph of $y = 7 - \frac{\pi}{8} \cos(\frac{\pi}{2}x - 3)$.

- A. $\frac{\pi}{2}$ B. $\frac{2}{\pi}$ C. $\frac{\pi^2}{16}$ D. $\frac{\pi}{4}$ E. NOTA

20. For $0 \leq x \leq \pi$, find the sum of solutions that satisfy $\cos(4x) + \cos(2x) = 0$.

- A. $\frac{3\pi}{2}$ B. $\frac{\pi}{2}$ C. $\frac{2\pi}{3}$ D. $\frac{5\pi}{6}$ E. NOTA

21. In $\triangle ABC$, the measure of angle B is twice that of angle A . If side length b is 1.5 times that of side length a , find the exact value of $\cos A$.

- A. $\frac{\sqrt{3}}{4}$ B. $\frac{2}{3}$ C. $\frac{3}{4}$ D. $\frac{3}{8}$ E. NOTA

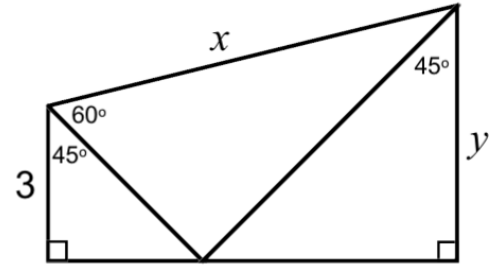
22. Chord \overline{AB} of length $9\sqrt{3}$ is drawn for the circle centered at point O , whose circumference is 18π . What is the exact area of the region outside of $\triangle AOB$ but inside the sector subtended by \overline{AB} ?

- A. $\frac{27(4\pi-3\sqrt{3})}{4}$ square units B. $\frac{9(4\pi-6\sqrt{3})}{4}$ square units C. $\frac{27(2\pi-3\sqrt{3})}{4}$ square units
 D. $\frac{27(4\pi-6\sqrt{2})}{2}$ square units E. NOTA

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23. In the diagram at right, find $\frac{x}{y}$.

- A. $\frac{4\sqrt{3}}{3}$ B. $\frac{3\sqrt{6}}{2}$ C. $\frac{2\sqrt{6}}{3}$
 D. $\frac{2\sqrt{3}}{6}$ E. NOTA

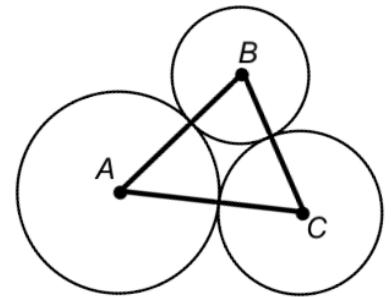


24. How many real solutions are there to the equation $\sin x = \frac{x}{100\pi}$?

- A. 99 B. 101 C. 199 D. 201 E. NOTA

25. Three externally tangent circles centered at A , B , and C have areas of 36π , 49π , and 64π . Find the exact area of $\triangle ABC$.

- A. 42 B. 84 C. 100
 D. $84\sqrt{2}$ E. NOTA



26. What is the maximum value of $\sin^6 x + 3\sin^4 x \cdot \cos^2 x + 3\sin^2 x \cdot \cos^4 x + \cos^6 x$?

- A. 4 B. 1 C. $\sqrt{2}$ D. 1.5 E. NOTA

27. Consider the set $S = \{x \mid x > 0 \text{ and } \sin 3x = \cos 7x\}$. If the elements of S are written in increasing order, what is the value of the third element?

- A. $\frac{\pi}{4}$ B. $\frac{3\pi}{8}$ C. $\frac{\pi}{20}$ D. $\frac{7\pi}{8}$ E. NOTA

28. Let w be an acute angle and define the following three values $x = \sin w$, $y = \tan w$, and $z = w$. Which choice lists the values of x , y , and z in *decreasing* order?

- A. y, z, x B. y, x, z C. x, z, y D. z, y, x E. NOTA

29. Find the exact value of $\tan 22^\circ + \tan 23^\circ + \tan 22^\circ \tan 23^\circ$.

- A. 1 B. $\sqrt{3}$ C. 2 D. -1 E. NOTA

30. Evaluate the sum $\cos \frac{\pi}{1000} + \cos \frac{2\pi}{1000} + \cos \frac{3\pi}{1000} + \dots + \cos \frac{1999\pi}{1000}$.

- A. 0 B. 1 C. -1 D. $\frac{\sqrt{2}}{2}$ E. NOTA