Geometry Hustle Solutions

MAO National Convention 2014

- 1. The figure created is a sphere, so the volume is $\frac{4}{3}\pi(2^3) = 32\pi/3$.
- 2. This is a right triangle with legs 11 and 60, so the area is 0.5(11)(60)=330.
- 3. Let the smallest sphere have radius x. Thus, the cube that it is inscribed in has a side length of 2x. Thus, the sphere that the cube is inscribed in has a radius of the diagonal of a cube with side 2x, or $x\sqrt{3}$. Hence, the cube that the sphere is inscribed in has a side length of $2x\sqrt{3}$. Thus the ratio that we want is $2x\sqrt{3}/x = 2\sqrt{3}$.
- 4. Since the circle is circumscribed, the radius of the circle is ABC/4Area where ABC represents the product of the side lengths. Hence, we have (5)(12)(13)/(4(0.5)(5)(12))=6.5.
- 5. The diameter is $\sqrt{100^2 + 2^2 4^2 5^2} = 9\sqrt{123}$ so radius is $9\sqrt{123}/2$.
- 6. No.
- 7. (5/360)(100 π)=25 π /18.
- 8. Since the bases differ by 2, the height of the trapezoid is $\sqrt{7^2 1^2} = 4\sqrt{3}$. Thus, the area of the trapezoid is (0.5)($4\sqrt{3}$)(11+9)= $40\sqrt{3}$.
- 9. $(x+1)^2 + (x+8)^2 = (x+9)^2$, so $x^2 16 = 0$, meaning that x = +/-4. Since x+1 has to be positive, x=4.
- 10. The figure is a square with diagonal 2014*2, so the area is (2014*2)²*0.5=8112392.
- 11. Yes. Place two quarters touching side by side on top of another quarter such that half of each quarter is touching the bottom quarter. Then with the last two quarters make a tent and place it such that the base of each quarter rests on the bottom quarter.
- 12. (1/3)(Area of Base)(Height)=(1/3)($100\sqrt{3}/4$)(10)=250 $\sqrt{3}/3$.
- 13. The radius is Area/Semi-perimeter. Hence, the radius is 0.5(5)(12)/((5+12+13)/2)=2.
- 14. Volume = s^3 and lateral surface area = $4*s^2$, so the ratio is $s^3/(4s^2)=s/4$.
- 15. Number of diagonals can be found by (n)(n-3)/2. Hence, we have (21)(18)/2=189.
- 16. The largest possible perimeter would require the square with side length 5to be opposite the square with side length 4 with the square with side length1 between the two. This yields a square with perimeter 36.
- 17. The apothem of a square is half of the side length, so the area is $8^2 = 64$.
- 18.10(10)(2)(1) = 200.
- 19. There are (2)(2)*6 cubes with one side painted (this is a two by two square on each face and there are (2)(2)(2) cubes with no paint on them. Hence, there are 24+8=32 cubes with fewer than one side with red paint on them.
- 20.(4/3) $\pi r^3 = s^3$ so r/s = $\sqrt[3]{(4\pi)}$, so the ratio of the surface area of the sphere to the surface area of the cube is $4 \pi r^2/6s^2 = (2 \pi/3)(\sqrt[3]{3/(4\pi)})^2 = \sqrt[3]{\pi/6} = \sqrt[3]{36\pi/6}$.
- 21. V-E+F=2, so V-30+20=2. Hence, V=12.

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- 22. If the height is 10 and the diameter is 10, the slant height is $5\sqrt{5}$. The lateral surface area is $\pi rl = \pi(5)(5\sqrt{5}) = 25\pi\sqrt{5}$.
- $23.\sqrt{(2012-1)^2 + (2013-2)^2 + (2014-3)^2} = 2011\sqrt{3}.$
- $24.(4/3)(50)^3 \pi = 500000\pi/3.$
- 25.A dodecahedron has 12 faces made up of all pentagons, so there are 12*5/2 = 30 edges. And by Euler's polyhedron formula, V-30+12=2, so there are 20 vertices. Hence, V+E+F=20+30+12=62.