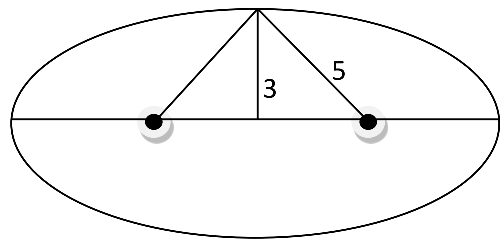
Precalculus Hustle Solutions

1. $= \frac{\sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right)+\sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right)}{\cos\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi}{6}\right)+\sin\left(\frac{\pi}{2}\right)\sin\left(\frac{\pi}{6}\right)} = \frac{\frac{\sqrt{3}}{2}\times\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\times\frac{1}{2}}{0+\frac{1}{2}} = 2\left(\frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{4}\right) = \frac{\sqrt{6}+\sqrt{2}}{2}$
2. $= \frac{2\tan\left(\frac{\pi}{8}\right)}{1+\tan^2\frac{\pi}{8}} = \frac{2\sin\left(\frac{\pi}{8}\right)}{\cos\left(\frac{\pi}{8}\right)} \times \cos^2\frac{\pi}{8} = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$
3. $= \left(\frac{e^{\frac{i\pi}{6}}+e^{-\frac{i\pi}{6}}}{2}\right)\left(\frac{e^{\frac{i\pi}{6}}-e^{-\frac{i\pi}{6}}}{2}\right) = \cos\left(\frac{\pi}{6}\right) i\sin\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{4}i$
4. $= \cos\left(\frac{18\pi}{4}\right) + i\sin\left(\frac{18\pi}{4}\right) = \cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right) = i$
5. $c = \sqrt{a^2+b^2} = \sqrt{64+36} = 10$. We know the y coordinate of the foci must be 2, and the x coordinates are ± 10 units away from the center. So $(a,b) = (4-10,2) = (-6,2)$ and $(c,d) = (4+10,2) = (14,2)$-> LCM=42
6. Jack traces an ellipse. A=5, b=3, so the area is 15π



7. Use law of cosines: $c^2 = a^2+b^2-2ab\cos C = 36+100-120(-.5) = 196 \rightarrow c = 14$
8. Third roots can be described as an equilateral triangle inscribed in a circle with radius 1. The side length is therefore $\sqrt{3}$
9. Complete the square to get: (x+1)^2 + (y-3)^2 = 5. With a radius of $\sqrt{5}$, this makes the conic a circle.
10. $2^{12}(\cos\left(12 * \frac{2\pi}{3}\right) + i\sin\left(12 * \frac{2\pi}{3}\right) = 4096$
11. Only the x^9 term matters for both the numerator and denominator. So the fraction reduces to $\frac{\binom{10}{8}x^9}{1!x^9} = 45$
12. $= \frac{(x+1)^2(x+2)(x-7)}{(x+1)^2(x+4)} \rightarrow \frac{(x+2)(x-7)}{(x+4)}$ → x = -4 is an asymptote + the slant asymptote → 2 asymptotes.
13. $-5x^2+12x-6 = 0 \rightarrow x = \frac{6\pm\sqrt{6}}{5}$ →Larger: $\frac{6+\sqrt{6}}{5}$
14. $\begin{pmatrix} i & j & k \\ 1 & 4 & -4 \\ -2 & 3 & 2 \end{pmatrix} = (20,6,11) \rightarrow mag = \sqrt{557}$
15. There are $\frac{6!}{2!2!} = 180$ ways to uniquely arrange the blocks, one of which spells coffee. $\frac{1}{180}$
16. $= \frac{10-24}{14} = -\frac{14}{14} = -1$
17. Normally the period of this function is $\frac{2\pi}{2} = \pi$. However, the power flips the negative values, so the period is halved: $\frac{\pi}{2}$
18. $a_n = a_{n-1} + 6a_{n+1} \rightarrow x^2 - x - 6 = 0 = (x-3)(x+2)$, so $a_n = c_1(3)^n + c_2(-2)^n$. Using the initial conditions, we can solve for the coefficients to find they are 1 and -1. Thus, the average of x and y is 2.5
19. $= \frac{-5+i}{-2+10i} = \frac{-5+i}{-2(1-5i)}\frac{1+5i}{1+5i} = \left(\frac{(-2)(5+12i)}{-2(26)}\right) = \frac{5+12i}{26}$

20. $= \frac{\left(\frac{5}{2}\right)\left(\frac{1}{4}\right)}{1+\left(\frac{1}{4}\right)\cos(\theta)} \to e = 1/4$
21. The term is $\binom{7}{5}(x)^5(3y)^2 = 189x^5y^2 \to 189$
22. Putting them in the correct order: (5,3),(3,-2),(-2,-2),(-3,1),(5,3) the area is 51/2.
23. Solving the system, X=6, y=-8. $\frac{y}{x} = -4/3$
24. $= \frac{\tan(45)+\tan(30)}{1-\tan(45)\tan(30)} = \frac{\left(1+\frac{1}{\sqrt{3}}\right)}{1-\frac{1}{\sqrt{3}}} = 2 + \sqrt{3}$
25. 0 is a root, so the product is necessarily 0.