BC Calculus Alpha Test - MAO National Convention 2014

Note that "NOTA" means none of the above.

1. The tangent line to the function g at $x = -1$ passes through $(3, 5)$. If $g(-1) = -3$, find $g'(-1)$.

A. 4 B. –2 C. 2 D. $\frac{1}{2}$ E. NOTA

2. Determine $\lim_{x\to-\infty} \frac{7+4x-6x^2}{\sqrt{4x^4+2x-1}}$

A. 0 B. $-\frac{3}{2}$ C. -3 D. 3 E. NOTA

3. A curve is described by the parametric equations $x(t) = e^{2t} + 2\sin t$ and $y(t) = \tan t + 3$. Determine the equation of the tangent line to the curve at the point when $t = 0$.

A. $x + 4y = 13$ B. $4x - y = -1$ C. $x - 4y = -11$
D. $x = 3$ E. NOTA

4. Find $\frac{dy}{dx}$, given $4x^3 - 2xy^2 + \ln(xy) = 2$.

A. $\frac{dy}{dx} = \frac{12x^3y-2xy^3+y}{4x^2y^2-x}$ B. $\frac{dy}{dx} = \frac{12x^3y+y}{4xy^2-x}$ C. $\frac{dy}{dx} = \frac{12x^3y-2xy^3}{4x^2y^2-1}$

D. $\frac{dy}{dx} = \frac{12x^3y+y}{2xy^3-x}$ E. NOTA

5. What is the exact value of $\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}(\pi \cdot \ln 2)^k$?

A. $\frac{2}{\pi}$ B. $\frac{1}{\pi^2}$ C. $\frac{1}{2^\pi}$ D. 2^π E. NOTA

6. Consider the three integrals:

I: $\int_1^\infty \frac{3}{9x^2-6x+2}\,dx$ II. $\int_0^\infty \frac{x+7}{x^2-x-6}\,dx$ III. $\int_1^2 \frac{2}{\sqrt[3]{x-1}}\,dx$

Which of these integrals converge?

A. I only B. II only C. III only D. I and III only E. NOTA

7. Consider the function $f(x) = \int_{\ln x}^{\sqrt{x}} \frac{\tan^{-1}\sqrt{3}t}{\sqrt{1+t^2}}\,dt$. Determine $f'(1)$.

A. $\frac{\pi}{3} - 1$ B. $\frac{\pi\sqrt{2}}{12}$ C. $\frac{\pi\sqrt{2}}{24}$ D. $\frac{\pi\sqrt{2}}{12} - 1$ E. NOTA

8. Find the average value of $f(x) = xe^{-2x}$ on the interval $[0, \ln 2]$.

A. $\frac{\ln 2}{16}$ B. $\frac{\ln 4+1}{16\ln 2}$ C. $\frac{3-\ln 4}{16}$ D. $\frac{3-\ln 4}{16\ln 2}$ E. NOTA

9. Let $h(x) = \frac{f(x^3)}{g(2x)}$ for functions *f* and *g* below. Find $h'(-2)$.

x	-8	-7	-6	-5	-4	-3	-2	-1	0
$f(x)$	3	2	-3	-5	-9	-1	DNE	0	3
$f'(x)$	-1	-.5	-4	-2	-4	3	DNE	5	2
$g(x)$	0	2	5	3	2	-1	4	8	6
$g'(x)$	DNE	4	3	-3	-2	-2	6	4	0

A. 36 B. 9 C. –3 D. 27 E. NOTA

10. Determine $\lim_{x\to 0} \frac{\tan x - x}{x^3}$

A. 1 B. $\frac{1}{3}$ C. $-\frac{1}{3}$ D. $\frac{1}{2}$ E. NOTA

11. Find the arc length for the curve $y = \ln(\sec x)$ for $0 \le x \le \frac{\pi}{4}$.

A. $\ln(\sqrt{2}+1)$ B. $\ln(\frac{\sqrt{2}}{2}+1)$ C. $\sqrt{2}$ D. $\ln(\sqrt{2})+1$ E. NOTA

12. Suppose you approximate ln (1.5) using the series $\ln(1+x) = \sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^n}{n}$. Use Lagrange's Error Bound to bound the error of the first four terms in the Taylor approximation.

A. $\frac{1}{1024}$ B. $\frac{1}{324}$ C. $\frac{1}{160}$ D. $\frac{1}{24}$ E. NOTA

13. Consider a spring whose force, *F*, as a function of displacement, *x*, in centimeters, is given by $F(x) = \sin x + 4x$. What is the work required to displace the spring π centimeters from equilibrium?

A. $2+2\pi^2$ B. $4+2\pi^2$ C. $4+4\pi^2$ D. $2\pi^2-4$ E. NOTA

14. What is the interval of convergence for the series $\sum_{n=1}^{\infty}\frac{(n+1)}{3^n n^2}(2x+1)^n$?

A. $-2 \le x \le 1$ B. $-2 \le x < 1$ C. $-1 < x \le 2$ D. $-\frac{2}{3} \le x < \frac{2}{3}$ E. NOTA

15. Which integral represents the area of the inner loop of $r = 2 + 4\cos\theta$?

A. $\frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}}(2+4\cos\theta)^2\,d\theta$ B. $\frac{1}{2}\int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}}(2+4\cos\theta)\,d\theta$ C. $\frac{1}{2}\int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}}(2+4\cos\theta)^2\,d\theta$

D. $\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}}\theta(2+4\cos\theta)^2\,d\theta$ E. NOTA

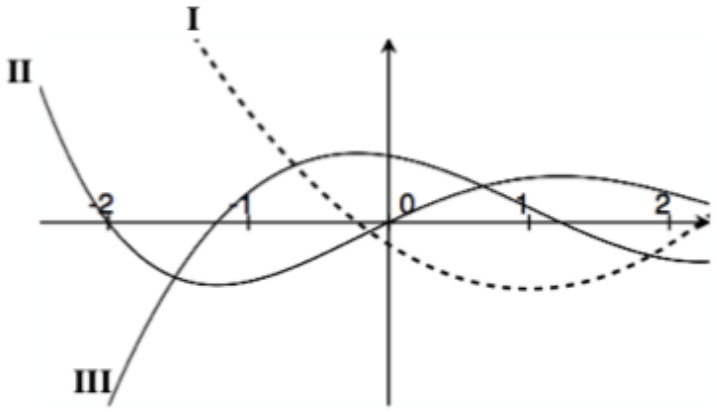
16. Which of the following series converges conditionally?

A. $\sum_{n=1}^{\infty} \frac{\sin n}{n^4}$ B. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ C. $\sum_{n=1}^{\infty} \cos(n+3)$
D. $\sum_{n=1}^{\infty} \left(\frac{5n+2n^3}{8n^3-1}\right)^n$ E. NOTA

17. If $\frac{dy}{dx} = \frac{y}{\sqrt{1-x^2}}$ and $y(1) = 1$, determine $y(0)$.

A.1 B. $\ln\frac{\pi}{2}$ C. $1+\frac{\pi}{2}$ D. $e^{-\frac{\pi}{2}}$ E. NOTA

18. Below are three graphs labeled I, II, and III. Of these three graphs, which of the following correctly identities f, f', and f''?



	f	f'	f''
A.	I	II	III
B.	II	III	I
C.	II	I	III
D.	III	II	I
E.	NOTA		

19. If f is differentiable at $x = a$, which of the following could be false?

A. f is continuous at $x = a$. B. $\lim_{x\to a} f(x)$ exists. C. $f''(a)$ exists.
D. $\lim_{x\to a^+} \frac{f(x)-f(a)}{x-a}$ exists. E. NOTA

20. Determine the derivative of $f(x) = (\sin 2x)^{-x}$

A. $f'(x) = -(\sin 2x)^{-x}(\ln(\sin 2x) + 2x\sec x)$
B. $f'(x) = -(\sin 2x)^{-x}(\ln(\sin 2x) + 2x\cot x)$
C. $f'(x) = -(\sin 2x)^{-x}(\ln(\sin 2x) + 2x\sec 2x)$
D. $f'(x) = -(\ln(\sin 2x) + 2x\sec 2x)$
E. NOTA

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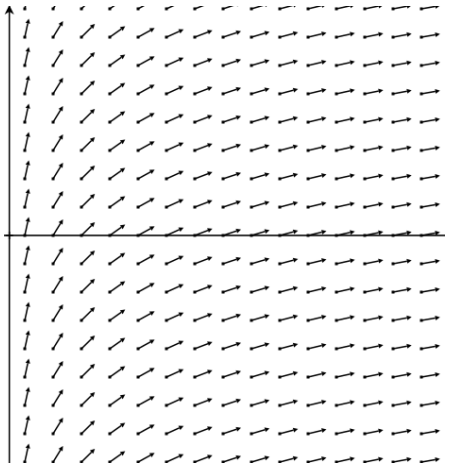
21. Consider the slope field shown at right for a certain differential equation. Which of the following could be a specific solution to this differential equation?



A. $y = \frac{1}{x}$ B. $y = \ln x$ C. $y = \ln \frac{1}{x}$

D. $y = \frac{1}{x^2}$ E. NOTA

22. Consider the region bounded by the curves $y = \sqrt{x}, y = 0, x = 0,$ and $x = 1$. Determine the volume when this region is rotated about the *y*-axis.

A. $\frac{4\pi}{5}$ B. $\frac{2\pi}{5}$ C. $\frac{6\pi}{5}$ D. $\frac{2}{5}$ E. NOTA

23. How many inflection points are there for $f(x) = \frac{2}{5}x^5 - \frac{1}{3}x^4 + 2x + 1$ on [-5, 5]?

A. 4 B. 3 C. 2 D. 1 E. NOTA

24. The volume of a spherical balloon is decreasing at a rate of 4 cubic cm per minute. What is the rate of change of the radius the instant the balloon has a circumference of 2 centimeters?

A. 4 cm per minute B. $-\pi$ cm per minute C. $-\frac{4}{\pi}$ cm per minute

D. $-\frac{1}{4\pi}$ cm per minute E. NOTA

25. Consider the following three statements about $f(x) = \begin{cases} \frac{x^2-9}{x-3}, & \text{if } x \neq 3 \\ 6, & \text{if } x = 3 \end{cases}$.

I. *f* has a limit at $x = 3$.
II. *f* is continuous at $x = 3$.
III. *f* is differentiable at $x = 3$.

Which of the statements above are true about *f*?

A. I only B. II only C. I and III only D. I, II, and III E. NOTA

26. Which of the following is the power series for $\frac{x}{1+x^2}$?

A. $x - x^3 + x^5 - x^7 + \cdots$ B. $1 - x^2 + x^4 - x^6 + \cdots$ C. $x + x^3 + x^5 + x^7 + \cdots$
D. $x^3 + x^4 + x^5 + x^6 + \cdots$ E. NOTA

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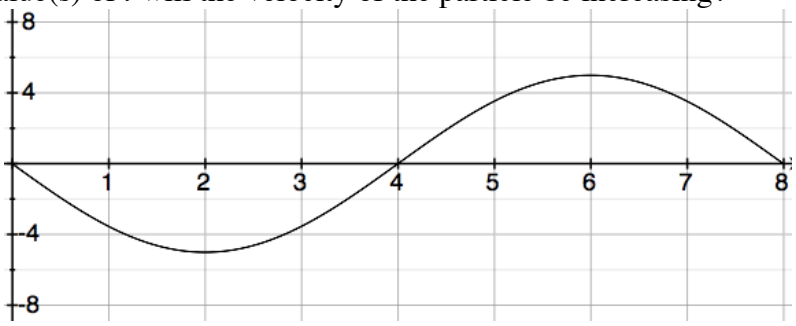
27. Consider the following graph showing a particle's position $x(t)$ at time t for $0 \le t \le 6$. For what value(s) of t will the velocity of the particle be increasing?



A. $0 < t < 2$ B. $0 < t < 4$ C. $4 < t < 6$ D. $2 < t < 6$ E. NOTA

28. A continuous function f is defined on [2, 10] and has values shown in the table below. Using the subintervals [2, 3], [3,7], and [7, 10], approximate $\int_2^{10} f(x)dx$ using a right Riemann Sum.

x	2	3	7	10
$f(x)$	8	12	14	4

A. 64 B. 80 C. 48 D. 89 E. NOTA

29. Determine $\int \sin^3 x \cdot \cos^2 x \, dx$. Note that C denotes some real constant.

A. $\frac{\cos^5 x}{5} - \frac{\cos^3 x}{3} + C$ B. $\frac{\cos^3 x}{3} - \frac{\cos^5 x}{5} + C$ C. $\frac{\cos^4 x}{4} - \frac{\cos^2 x}{2} + C$

D. $\frac{\sin^5 x}{5} - \frac{\sin^3 x}{3} + C$ E. NOTA

30. Suppose f is continuous and differentiable on [2, 12]. It is given that $f(2) = 8$ and $|f'(x)| \le 5$ for all x in [2, 12]. What is the largest possible value for $f(12)$?

A. 46 B. 58 C. 60 D. 64 E. NOTA