1. **B** d^2y/dx^2 = d/dt (dy/dx) / $(dx/dt)^2$. dy/dx = dy/dt / dx/dt = (2t + 3) / (1) ⇔ d/dt(dy/dx) = 2. Thus, the second derivative that we want is 2 / $(1)^2$ = 2.
2. **A** $x^2+y^2=169$ ⇔2x dx/dt + 2y dy/dt = 0 ⇔ dy/dt = -(x/y) dx/dt = -(5/12) * 2 = -5/6 ft/sec, so the ladder is sliding down at a rate of 5/6 ft/sec
3. ***C*** We want $\int_{-2}^{2} 4 - x^2 dx = 4x - \frac{x^3}{3} |_{-2}^{2} = \frac{32}{3}$
4. **A** The parabola has zeroes at 3 and -1. Hence, we must use shell method and compute $\int_{-1}^{3} 2\pi(3 - x)(3 + 2x - x^2)dx = \frac{128\pi}{3}$
5. **A** f(x) is just the derivative of sin, so f(x)=cos x, meaning that f'($\frac{\pi}{2}$)=-sin($\frac{\pi}{2}$)=-1
6. **D** We have $\lim_{n\to\infty}\left(\frac{1}{\sqrt{n}\sqrt{n+1}} + \frac{1}{\sqrt{n}\sqrt{n+2}} + \cdots + \frac{1}{\sqrt{n}\sqrt{n+n}}\right) = \lim_{n\to\infty}\left(\frac{\sqrt{n}}{\sqrt{n}\sqrt{n}\sqrt{n+1}} + \frac{\sqrt{n}}{\sqrt{n}\sqrt{n}\sqrt{n+2}} + \cdots + \frac{\sqrt{n}}{\sqrt{n}\sqrt{n}\sqrt{n+n}}\right) = \lim_{n\to\infty}\left(\frac{1}{n}\sum_1^n \sqrt{\frac{n}{n+i}}\right) = \lim_{n\to\infty}\left(\frac{1}{n}\sum_1^n \sqrt{\frac{1}{1+\frac{i}{n}}}\right) = \int_0^1 \sqrt{\frac{1}{1+x}}dx = -2\sqrt{1+x} |_0^1 = 2 - 2\sqrt{2}$
7. **D** dV/dt = $k\pi r^2$. And from similar figures, we have r/h=6/8=3/4 ⇔r=3h/4. Thus, dV/dt = $k\pi 9h^2/16$. And from V= $\pi r^2h/3 = 9\pi h^3/48$, we have dV/dt = $9\pi h^2/16$ * dh/dt. Thus, setting this equal to the previous expression, we have dh/dt=k.
8. **A** Differentiating, we get 1 – sinx = $2xf(x^2)$ ⇔ $f(x^2)$ = (1- sinx) / (2x) ⇔ f(169) = (1- sin13) / 26
9. ***C*** Profit = 15 * x – x^3 ⇔ 15-$3x^2$ = 0 ⇔ x=$\sqrt{5}$. Thus, testing x=2 and x=3, we see that x=2 maximizes the profit.
10. **D** x goes to infinity and 1/x goes to 0. However, since sin oscillates between -1 and 1, the limit does not exist.
11. **B** We want $\frac{1}{4}\int_0^4 (4x - x^2)dx = \frac{x^2}{2} - \frac{x^3}{12} |_0^4 = \frac{8}{3}$
12. **B** $\int \frac{1}{x^7-x}dx = \int \frac{x^5}{x^6(x^6-1)}dx = \frac{1}{6}\int \frac{du}{u(u+1)} = \frac{1}{6}\int \frac{1}{u} - \frac{1}{u+1}du = 1/6(\ln|x^6 - 1| - \ln|x^6|)$. Thus, we want (1/6) ln (6656/6561)
13. ***C*** $x_2 = x_1 - (x_1^3 - 2x_1-5) / (3x_1^2 - 2) = 2 - (2^3-2(2)-5)/(3(2)^2-2)=2.1$
14. **A** In order for the expression to evaluate to 120, we must have a +1's and b -1's. Thus, a+b=2014 and a-b=120. Thus, a=1067 and b=947. The first 1 in my string of 1's is automatically a +1 so I only needs 1066 more +1's. Since there are 2013 spaces to place either a + or -, there are ${}_{2013}C_{1066}$ or ${}_{2013}C_{947}$ ways to obtain an expression that evaluates to 120.
15. **D** Let x represent rolling a 1, x^2 represent rolling a 2, x^3 represent rolling a 3, x^4 represent rolling a 4, x^5 represent rolling a 5, x^6 represent rolling a 6. Hence, if we multiply the hint by x^5, we can let x^n represent rolling a sum of n. Hence, we must look at the coefficient of x^{15} in the given, which is 651.

16. **C** We will use (1,2,3) as our "origin" for the three vectors that define the given tetrahedron. Hence, we have (0,2,2), (1,-3,1), and (1,1,-4). Hence, the volume of the tetrahedron is $\frac{\begin{vmatrix} 0 & 2 & 2 \\ 1 & -3 & 1 \\ 1 & 1 & -4 \end{vmatrix}}{6} = 3.$
17. **A** $S(x) = x^4 - (1/a_1+1/a_2+1/a_3+1/a_4)x^3 + (1/(a_1a_2) + 1/(a_1a_3) + 1/(a_1a_4) + 1/(a_2a_3) + 1/(a_2a_4) + 1/(a_3a_4))x^2 - (1/(a_1a_2a_3) + 1/(a_1a_2a_4) + 1/(a_1a_3a_4) + 1/(a_2a_3a_4))x + 1/(a_1a_2a_3a_4)$. Multiplying through by $a_1a_2a_3a_4$, we have $a_1a_2a_3a_4x^4 - (a_1a_2a_3 + a_1a_2a_4 + a_1a_3a_4 + a_2a_3a_4)x^3 + (a_1a_2 + a_1a_3 + a_1a_4 + a_2a_3 + a_2a_4 + a_3a_4)x^2 - (a_1 + a_2 + a_3 + a_4)x + 1$. Furthermore, from $f(x) = 3x^4 + 5x^3 - 3x + 1 = x^4 - (a_1 + a_2 + a_3 + a_4)x^3 + (a_1a_2 + a_1a_3 + a_1a_4 + a_2a_3 + a_2a_4 + a_3a_4)x^2 - (a_1a_2a_3 + a_1a_2a_4 + a_1a_3a_4 + a_2a_3a_4)x + a_1a_2a_3a_4$. Thus, $S(x) = 2x^4 - 3x^3 + 0x^2 + 5x + 1$.
18. **D** Based on the expansions written out in the previous solution, $H(x) = x^4 - (1/2)(a_1 + a_2 + a_3 + a_4)x^3 + (1/4)(a_1a_2 + a_1a_3 + a_1a_4 + a_2a_3 + a_2a_4 + a_3a_4)x^2 - (1/8)(a_1a_2a_3 + a_1a_2a_4 + a_1a_3a_4 + a_2a_3a_4)x + (1/16)a_1a_2a_3a_4$. So, $H(x) = x^4 + 5x^3/2 - 3x/8 + 1/8$
19. **B** $f(x) = (x-a_1)(x-a_2)(x-a_3)(x-a_4)$. Substituting $-x$ in f(x), we get $f(-x) = (x+a_1)(x+a_2)(x+a_3)(x+a_4) = S_2(x) = x^4 - 5x^3 + 3x + 2$
20. **B** Let the limit be equal to g′(x), the derivative of g(x). Hence, when we rationalize the denominator we see the formal definition of the derivative of g(x), where $g(x) = f(x)(\sqrt{x})$, leading to $g'(a) = f'(a)\sqrt{a} + \frac{f(a)}{2\sqrt{a}}$
21. **A** There are 101 choices for z (0 to 100) and among those 101 choices, the average of 4z is 200. Furthermore, the average of x+2y is thus 200. Thus, for x+2y=200, there are also 101 choices for y (0 to 100). Hence, there are 101 choices for z and based on the choice of z, there are 101 choices of y, resulting in only one choice for x. Hence, there are 101*101 = 10201 total solutions to f(x,y,z).
22. **A** Let $a = \sqrt{2x+1}$ and $b = \sqrt{2x-1}$. Hence, $f(x) = (a^2 + ab + b^2)/(a + b) = (a-b)(a^2+ab+b^2)/((a-b)(a+b)) = (a^3 - b^3)/(a^2 - b^2) = (\sqrt{(2x+1)^3} - \sqrt{(2x-1)^3})/2$. Thus, the series that is asked is just $(1/2)((\sqrt{(2(1)+1)^3} - \sqrt{(2(1)-1)^3}) + (\sqrt{(2(2)+1)^3} - \sqrt{(2(2)-1)^3}) + (\sqrt{(2(3)+1)^3} - \sqrt{(2(3)-1)^3}) + \ldots(\sqrt{(3(3333)+1)^3} - \sqrt{(3(3333)-1)^3}) = \left(\frac{1}{2}\right)\left(\sqrt{(10000)^3} - \sqrt{(1)^3}\right) = 999999/2$
23. **D** From the hint, $294_a = 2a^2+9a+4 = (2a+1)(a+4)$. Since both 2a+1 and a+4 will neither be both odd nor both even and since 9 is a digit, the base must be greater than or equal to 10, 2a+1 and a+4 must both be perfect squares. Hence, we must test integers starting with 10. We soon realize that a=12.
24. **D** f(11) = f(33) + 11 = f(99) + 33 + 11 = f(297) + 99 + 33 + 11 = 300 + 99 + 33 + 11 = 443.

25.**C** f(x/3) is simply a geometric series with first term 1 and common ratio x, so |x| must be less than 1, so the domain of f(x/3) is (-1,1).

26.**A** $\left(\frac{\sin^2 x}{1+\cot x}+\frac{\cos^2 x}{1+\tan x}\right)=\left(\frac{\sin^3 x}{\sin x+\cos x}+\frac{\cos^3 x}{sinx+\cos x}\right)=\frac{(\sin x+\cos x)(\sin^2 x-sinx\ cosx+\cos^2 x)}{sinx+cosx}=1-sinxcosx$. Thus, the derivative is simply -cos2x

27.**D** From y=(18x+1)/(x+1), it is clear that the horizontal asymptote is at y=18 meaning that as x grows, the y value will approach 18. However, since P(x) takes the floor function of y, 18 will never be reached for large values of x. Thus, P(20140000) = 17.

28.**C** $f'(x)=0.5\ (x+3)^{-0.5}$, so L(x)=f(1) + f'(1) (x-1) = 7/4 + x/4. Hence, for x=0.98, we have 7/4 + 0.98/4 = 1.995.

29.**B** We have $\frac{dV}{dt}=4\pi r^2\frac{dr}{dt}$ and since $\frac{dV}{dt}=k\ 4\pi r^2$, we have $k=\frac{dr}{dt}\Leftrightarrow$r=kt+C. And since when t=0, r=original radius, C= r_0. Thus using the information about t=3, we have $\frac{4}{3}\pi(3k+r_0)^3=\frac{1}{2}\frac{4}{3}\pi r_0^3\Leftrightarrow 3k+r_0=\frac{1}{\sqrt[3]{2}}r_0\Leftrightarrow k=\frac{1}{3}r_0(\frac{1}{\sqrt[3]{2}}-1)$. Hence, we have r=0 when $\frac{1}{3}r_0\left(\frac{1}{\sqrt[3]{2}}-1\right)t+r_0=0$ when $t=\frac{3\sqrt[3]{2}}{\sqrt[3]{2}-1}$.

30.**A** By trying to take a few of the derivatives, we find that $d^n/dx^n = n!\ (-1)^{n+1} / (x-1)^{n+1}$ so the 2014th derivative is $-2014!(x-1)^{-2015}$