

*Please note that for all questions NOTA means None of the Above.
Good Luck and Have Fun!*

1. Evaluate d^2y/dx^2 for $y = t^2 + 3t$ and $x = t$ at $t=1$.
(A) 1
(B) 2
(C) 3
(D) 4
(A) NOTA
2. A 13ft ladder rests against a vertical wall. The bottom of the ladder is slipping away from the wall at a rate of 2ft/s. How fast, in ft/sec, is the top of the ladder sliding down the wall when the bottom of the ladder is 5ft from the wall?
(A) 5/6
(B) 6/5
(C) 5
(D) 6
(E) NOTA
3. Compute the area bounded by $y = x^2$ and $y = 4$.
(A) 8/3
(B) 16/3
(C) 32/3
(D) 16
(E) NOTA
4. Compute the volume of the solid obtained by rotating the region bounded by $y = 3 + 2x - x^2$ and $y = 0$ about the line $x = 3$.
(A) $\frac{128\pi}{3}$
(B) $\frac{256\pi}{3}$
(C) 64π
(D) 128π
(E) NOTA

5. If $f(x) = \lim_{t \rightarrow x} \frac{\sin t - \sin x}{t - x}$, compute $f'(\frac{\pi}{2})$.
- (A) -1
(B) -1/2
(C) 0
(D) 1/2
(E) NOTA
6. Evaluate the value of $\lim_{n \rightarrow \infty} \left(\frac{1}{\sqrt{n}\sqrt{n+1}} + \frac{1}{\sqrt{n}\sqrt{n+2}} + \dots + \frac{1}{\sqrt{n}\sqrt{n+n}} \right)$.
- (A) $\sqrt{2}$
(B) $2\sqrt{2}$
(C) $2 + 2\sqrt{2}$
(D) $2 - 2\sqrt{2}$
(E) NOTA
7. I have an inverted cone of radius 6 and height 8. Given that the water evaporates at a rate proportional to the area of the surface of the water with proportional constant k , find the rate at which the depth of the water is decreasing.
- (A) k^2
(B) $2k$
(C) k^3
(D) k
(E) NOTA
8. Given that $f(x)$ is a continuous function and that $x + \cos x = \int_{-42}^{x^2} f(t) dt$, find $f(169)$.
- (A) $(1 - \sin 13) / 26$
(B) $(1 + \sin 13) / 13$
(C) $(1 + \sin 13) / 26$
(D) $(1 - \sin 13) / 13$
(E) NOTA

9. The cost of producing x number of goods is defined by $C(x) = x^3$ for $x > 0$. Given that if you produce x number of goods, you can sell them for 15 dollars each, how many goods should you produce to maximize your profit?
- (A) 0
(B) 1
(C) 2
(D) 3
(E) NOTA
10. Evaluate $\lim_{x \rightarrow \infty} \sin\left(x + \frac{1}{x}\right)$
- (A) -1
(B) 0
(C) 1
(D) DNE
(E) NOTA
11. What is the average value of the function $f(x) = 4x - x^2$ from 0 to 4?
- (A) $\frac{4}{3}$
(B) $\frac{8}{3}$
(C) $\frac{12}{3}$
(D) $\frac{16}{3}$
(E) NOTA
12. Evaluate $\int_2^3 \frac{1}{x^7 - x} dx$.
- (A) $(1/6) \ln(6561/6656)$
(B) $(1/6) \ln(6656/6561)$
(C) $6 \ln(6656/6561)$
(D) $6 \ln(6561/6656)$
(E) NOTA
13. Use Newton's method to find the second approximation to the root of the equation $x^3 - 2x - 5$ with $x_1 = 2$.
- (A) 2.05
(B) 2.08
(C) 2.10
(D) 2.12
(E) NOTA

14. I write 2014 1's in a row with space between each subsequent 1 for a + or - sign. I then have each of my 2013 students randomly select an empty space and randomly choose to write either a + or - sign. Given that each space has an equal probability of having either a + or - sign after my activity with my students, how many possible ways are there for the final expression to evaluate to 120?
- (A) $\binom{2013}{947}$
(B) $\binom{2013}{1067}$
(C) $\binom{2014}{1067}$
(D) $\binom{2014}{120}$
(E) NOTA
15. I roll five fair six-sided die and sum the numbers that show up on each of the five dice. In how many ways could a sum of 20 be obtained? Hint: $(1 + x + x^2 + x^3 + x^4 + x^5)^5 = x^{25} + 5x^{24} + 15x^{23} + 35x^{22} + 70x^{21} + 126x^{20} + 205x^{19} + 305x^{18} + 420x^{17} + 540x^{16} + 651x^{15} + 735x^{14} + 780x^{13} + 780x^{12} + 735x^{11} + 651x^{10} + 540x^9 + 420x^8 + 305x^7 + 205x^6 + 126x^5 + 70x^4 + 35x^3 + 15x^2 + 5x + 1$.
- (A) 126
(B) 205
(C) 420
(D) 651
(E) NOTA
16. A tetrahedron has vertices at $(1,2,3)$, $(1,4,5)$, $(2,-1,4)$, and $(2,3,-1)$. What is the volume of this tetrahedron?
- (A) 54
(B) 18
(C) 3
(D) 9
(E) NOTA

For the next three questions: Consider $f(x) = x^4 + 5x^3 - 3x + 2$ which has non-zero roots $a_1, a_2, a_3,$ and a_4 .

17. Find $S(x)$ such that $S(x)$ has roots $1/a_1, 1/a_2, 1/a_3,$ and $1/a_4$.

- (A) $2x^4 - 3x^3 + 5x + 1$
- (B) $2x^4 - 3x^3 + 5x^2 + 1$
- (C) $x^4 + x^3/5 - x/3 + 1/2$
- (D) $x^4/2 + x^3/3 - x/5 + 1/2$
- (E) NOTA

18. Find $H(x)$ such that $H(x)$ has roots $a_1/2, a_2/2, a_3/2,$ and $a_4/2$.

- (A) $x^4 + 5x^3/2 - 3x/4 + 1/4$
- (B) $x^4 + 5x^3/2 - 3x/4 + 1/8$
- (C) $x^4 + 5x^3/2 - 3x/8 + 1/16$
- (D) $x^4 + 5x^3/2 - 3x/8 + 1/8$
- (E) NOTA

19. Find $S_2(x)$ such that $S_2(x)$ has roots $-a_1, -a_2, -a_3,$ and $-a_4$.

- (A) $-x^4 - 5x^3 + 3x - 2$
- (B) $x^4 - 5x^3 + 3x + 2$
- (C) $-x^4 + 5x^3 + 3x - 2$
- (D) $x^4 + 5x^3 + 3x + 2$
- (E) NOTA

20. Given that f is differentiable at a , for $a > 0$, find the value of $\lim_{x \rightarrow a} \frac{f(x)\sqrt{x} - f(a)\sqrt{a}}{x - a}$
in terms of $f'(a)$.

- (A) $f'(a)\sqrt{a} + \frac{f'(a)}{2\sqrt{a}}$
- (B) $f'(a)\sqrt{a} + \frac{f(a)}{2\sqrt{a}}$
- (C) $f''(a)\sqrt{a} + \frac{f(a)}{2\sqrt{a}}$
- (D) $f'(a)\sqrt{a} - \frac{f(a)}{\sqrt{a}}$
- (E) NOTA

21. Consider $f(x,y,z) = x + 2y + 4z$. Find the number of solutions to $f(x,y,z) = 400$ such that x,y,z are non-negative integers.

- (A) 10201
- (B) 10000
- (C) 40000
- (D) 40401
- (E) NOTA

22. Consider $f(x) = \frac{4x + \sqrt{4x^2 - 1}}{\sqrt{2x+1} + \sqrt{2x-1}}$. Evaluate $\sum_{x=1}^{3333} f(x)$.

- (A) 999999/2
- (B) 500000
- (C) 999999
- (D) 1000000
- (E) NOTA

23. Consider a function $F(x)$ returns the smallest base such that x is a perfect square if such a base exists. If no such base exists, $F(x)$ returns -1. Evaluate $F(294)$. (Hint: $abc_x = ax^2 + bx + c$.)

- (A) -1
- (B) 13
- (C) 14
- (D) 12
- (E) NOTA

24. Let $f(x) = \begin{cases} x + 3 & x > 100 \\ f(3x) + x & x \leq 100 \end{cases}$. Compute $f(11)$.

- (A) 344
- (B) 348
- (C) 439
- (D) 443
- (E) NOTA

25. Consider $f(x/3) = 1 + x + x^2 + x^3 + \dots$. For what values of x , can $f(x)$ be evaluated?

- (A) $(-3,3)$
- (B) $(-1/3, 1/3)$
- (C) $(-1,1)$
- (D) $[-3,3]$
- (E) NOTA

26. Evaluate $\frac{d}{dx} \left(\frac{\sin^2 x}{1+\cot x} + \frac{\cos^2 x}{1+\tan x} \right)$.

- (A) $-\cos(2x)$
- (B) $-\cos x$
- (C) $\cos x$
- (D) $\sin(2x)$
- (E) NOTA

27. Let $P(x)$ represent the number of pandas alive in an alternate universe in year

x . If $P(x) = \left\lfloor \frac{18x+1}{x+1} \right\rfloor$, find $P(20140000)$.

- (A) 16
- (B) 2014
- (C) 18
- (D) 17
- (E) NOTA

28. Using differentials, calculate the linear approximation of $\sqrt{3.98}$ using

$f(x) = \sqrt{x+3}$ and $a = 1$.

- (A) 1.993
- (B) 1.994
- (C) 1.995
- (D) 1.996
- (E) NOTA

29. Assume that a perfectly round snowball melts in such a way that its volume decreases at a rate that is proportional to its surface area. If it takes three hours for the snowball to decrease to half of its original volume, how many hours does it take for the snowball to melt completely?

- (A) $\frac{\sqrt[3]{2}}{\sqrt[3]{2}-1}$
- (B) $\frac{3\sqrt[3]{2}}{\sqrt[3]{2}-1}$
- (C) $\frac{3\sqrt[2]{2}}{\sqrt[2]{2}-1}$
- (D) 3
- (E) NOTA

30. What is the 2014th derivative of $f(x) = x / (1 - x)$?

- (A) $-2014!(x-1)^{-2015}$
- (B) $2012!(x-1)^{-2012}$
- (C) $-2013!(x-1)^{-2013}$
- (D) $2014!(x-1)^{-2014}$
- (E) NOTA