

For all questions, answer E. "NOTA" means that none of the above answers are correct.

1. Define two arithmetic sequences, a_n and b_m ($m, n > 0$), such that $a_{10} = 31$ with a common difference of 3, and $b_{15} = 35$ with a common difference of 2. How many natural numbers less than 1000 are elements of both a_n and b_m ?

- A. 83 B. 84 C. 165 D. 166 E. NOTA

2. Let T_n denote the n th triangular number. Compute the value of $\sum_{n=1}^{\infty} \frac{T_n}{2^{n-1}}$.

- A. 6 B. 8 C. 8.5 D. does not converge E. NOTA

3. Determine the value of $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{i}{n}\right)^n$.

- A. $\frac{1}{1-e}$ B. $\frac{e}{e-1}$ C. $\frac{1}{1+e}$ D. $\frac{1}{e-1}$ E. NOTA

4. Which of the following statements is false?

- A. The sequence of partial sums of the harmonic series diverges.
B. According to the ratio test, the harmonic series diverges.
C. The terms of the alternating harmonic series can be rearranged such that its sum equals 2014
D. If every term whose denominator contained the digit "7" were removed from the harmonic series, the new series will converge.
E. NOTA

5. Stone Cold Steve Austin drops a ball from an 80 foot tall building, and each time the ball bounces, it rebounds to $p\%$ of its previous height. If the ball travels a total of 320 feet, what is p ?

- A. 75 B. $66.\bar{6}$ C. 60 D. $33.\bar{3}$ E. NOTA

6. What is the value of $\sum_{i=0}^{11} \binom{11}{i} (1.3)^i (.7)^{11-i}$?

- A. $(.6)^{11}$ B. 1023 C. 1024 D. 2047 E. NOTA

7. For what value(s) of x does the series $\sum_{n=1}^{\infty} n^{-2x^3-x^2+5x-3}$ converge?

- A. $\left(-2, \frac{1}{2}\right) \cup (1, \infty)$ B. $\left(\frac{1}{2}, 1\right)$ C. $(-2, 1)$
 D. $(-\infty, -2) \cup \left(\frac{1}{2}, \infty\right)$ E. NOTA

8. The roots of the equation $8x^3 - px^2 + 78x - 27$ are consecutive terms in a geometric progression. Find the value of p .

- A. 52 B. -52 C. 36 D. -36 E. NOTA

9. Determine the value of $\sum_{n=3}^{2014} \ln\left(\frac{n-1}{n}\right)$.

- A. $\ln\left(\frac{1}{2014}\right)$ B. $\ln\left(\frac{1}{1007}\right)$ C. $\ln\left(\frac{1}{1006}\right)$ D. $\ln\left(\frac{2}{2015}\right)$ E. NOTA

10. Find the sum of the coefficients of the polynomial $f(x)$ such that $\sum_{k=1}^n f(k) = n^2 + 3n$.

- A. 3 B. 4 C. 5 D. 6 E. NOTA

11. The Rock is thinking of a number that is $\frac{1}{4}$ raised to some positive integer power, and CM Punk is thinking of a number that is $\frac{1}{7}$ raised to some positive integer power. Chris Jericho then takes the two numbers that were chosen, and multiplies them. What is the sum of all possible products that Chris can form?

- A. $\frac{1}{18}$ B. $\frac{1}{28}$ C. $\frac{1}{20}$ D. $\frac{1}{14}$ E. NOTA

12. Define $E_n = \frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \dots + \frac{1}{2n}$ and let $S_n = \frac{1}{(2n+2)E_n E_{n+1}}$. Determine the value of $\sum_{n=1}^{\infty} S_n$.

- A. $\frac{3}{2}$ B. $\frac{2}{3}$ C. $\frac{1}{2}$ D. 2 E. NOTA

13. Compute the value of $\sum_{n=0}^{\infty} \frac{2n^2 + 1}{n!}$.

- A. $3e$ B. $5e$ C. $2e^2 + e$ D. $4e^2 + e$ E. NOTA

14. What is the value of $\frac{\left(\frac{1+\sqrt{5}}{2}\right)^{10} - \left(\frac{1-\sqrt{5}}{2}\right)^{10}}{\sqrt{5}}$?

- A. 18 B. 34 C. 55 D. 89 E. NOTA

15. What is the sum of the last three digits of 49^{60} ?

- A. 1 B. 7 C. 12 D. 15 E. NOTA

16. Determine the interval of convergence for $\sum_{n=1}^{\infty} \frac{(-1)^n n}{3^n} (x-2)^n$.

- A. $(0,4)$ B. $(-3,7)$ C. $(-1,5)$ D. $(-5,5)$ E. NOTA

17. Which of the following series diverges?

- A. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^{2.2} - 5n + 3}}$ B. $\sum_{n=0}^{\infty} \frac{(-1)^n}{n^3 - 1}$ C. $\sum_{k=1}^{\infty} \left(\frac{2k}{5k-2}\right)^k$
 D. $\sum_{n=0}^{\infty} \frac{n+2}{2n+57}$ E. NOTA

18. What is the Taylor Series for $f(x) = x^5 e^{-2x^3}$ centered about $x = 0$?

- A. $\sum_{n=0}^{\infty} \frac{(-2)^n x^{3n+5}}{n!}$ B. $\sum_{n=0}^{\infty} \frac{x^{-2n^3+5}}{n!}$ C. $\sum_{n=0}^{\infty} \frac{x^{-10n^3}}{n!}$ D. $\sum_{n=1}^{\infty} \frac{-2x^{n^3+5}}{(n-1)!}$
 E. NOTA

19. Evaluate $\sum_{n=1}^{\infty} \frac{1}{n3^n}$

- A. $\ln(3)$ B. $\ln\left(\frac{3}{2}\right)$ C. $\frac{1}{2}$ D. $\frac{1}{2}\ln(3)$
 E. NOTA

20. Define a recursive sequence by $a_n = 6a_{n-2} - a_{n-1}$ such that $a_0 = 1$ and $a_1 = 3$. The explicit formula for a_n is of the form $a_n = \lambda_1 a^n + \lambda_2 b^n$, where $a > b$. Find $a + b + \lambda_1 + \lambda_2$.

- A. 3 B. 2 C. 1 D. 0 E. NOTA

21. What is the generating function for the set of positive perfect squares?

- A. $\frac{1+x}{(1-x)^3}$ B. $\frac{x(1+x)}{(1-x)^3}$ C. $\frac{x}{(1-x)^2}$ D. $\frac{1}{(1-x)^2}$
E. NOTA

22. Determine the value of $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{\sqrt{n^2 + 4k^2}}$.

- A. $\frac{\pi}{8}$ B. $\frac{\pi}{4}$ C. $\frac{1}{2} \ln(2 + \sqrt{5})$ D. $\ln(2 + \sqrt{5})$
E. NOTA

23. Find the first three nonzero terms of the Taylor polynomial for $\ln(x)$ centered at $x = 4$.

- A. $\ln(4) + \frac{x-4}{4} - \frac{1}{16}(x-4)^2$
B. $\ln(4) + \frac{x-4}{4} + \frac{1}{16}(x-4)^2$
C. $\ln(4) + \frac{x-4}{4} - \frac{1}{32}(x-4)^2$
D. $\ln(4) + \frac{x-4}{4} + \frac{1}{32}(x-4)^2$
E. NOTA

24. Find the sum of all solutions on the interval $[0, 2\pi]$: $\sum_{n=0}^{\infty} (-1)^n \tan^{2n}(\theta) = \sin^2(\theta) - 3\sin(\theta) + 2$

- A. $\frac{9\pi}{2}$ B. $\frac{8\pi}{3}$ C. $\frac{3\pi}{2}$ D. 3π E. NOTA

25. Evaluate the following nested sum: $\sum_{i=1}^{12} \sum_{j=1}^i (i+j)$

- A. 144 B. 1014 C. 1024 D. 1728
E. NOTA

26. Evaluate $\binom{11}{1} + 2\binom{11}{2} + 3\binom{11}{3} + \dots + 11\binom{11}{11}$
- A. 12287 B. 12282 C. 11269 D. 11264 E. NOTA
27. Which of the following is true regarding $\sum_{n=1}^{\infty} \frac{(-1)^n \sin(n)}{n!}$?
- A. Absolutely Convergent B. Conditionally Convergent
C. Divergent D. Not enough information E. NOTA
28. What is $\prod_{n=1}^{89} \cot(n^\circ)$?
- A. $-\frac{1}{2}$ B. 0 C. $\frac{1}{4}$ D. $\frac{1}{2}$ E. NOTA
29. Let a_n and b_n be sequences of positive real numbers such that $a_n < b_n$ for all n and $\lim_{n \rightarrow \infty} b_n = 0$. Which of the following must be true?
- A. $\sum_{n=1}^{\infty} a_n$ diverges B. $\sum_{n=1}^{\infty} a_n$ converges C. a_n diverges
D. $\lim_{n \rightarrow \infty} a_n < \lim_{n \rightarrow \infty} b_n$ E. NOTA
30. What is the value of $\sum_{n=0}^{\infty} \frac{3^n (-1)^n}{(2n)!}$?
- A. $\sin(3)$ B. e^{-3} C. $\cos(\sqrt{3})$ D. $\sin(\sqrt{3})$ E. NOTA