1. Which of the following is a solution to the equation: ?
2. 0 B) 11 C) 12 D) 13 E) NOTA
3. David is on the planet Y2E2. On Y2E2, if it snows in the morning, then sparkling bunnies will appear everywhere in the afternoon and if it’s cloudy in the afternoon, then Hilary will swim. David wakes up one afternoon and observes there are no sparkling bunnies jumping anywhere and that it’s not cloudy. Which answer most completely describes what can David conclude?
4. It did not snow that morning
5. Hilary will not swim
6. It did not snow that morning and Hilary will not swim
7. David cannot conclude anything
8. NOTA

The next two questions deal with formal logic and use the following four predicates:

|  |  |
| --- | --- |
| Fastest(x) = x is fastest of x’s type | Magician(x) = x is a magician |
| Owns(x,y) = x owns y | Dragon(x) = x is a dragon |

1. Which of the following answer choices is equivalent to the propositional formula:

“

”

1. There is a magician who owns all the dragons.
2. There are only two dragons and a magician owns both of them.
3. There is a magician who owns all but two of the dragons.
4. There is a magician who owns two dragons.
5. NOTA
6. Which of the following answers choices is an equivalent propositional formula for the following statement: “No magician owns the fastest dragon”?
8. NOTA
9. Consider the island of Gnarnia where there are exactly two types of people: oracles and necromancers. Statements made by oracles are always true and statements made by necromancers are always false. Let X and Y be two different people on this island. Which of the following can you conclude if: X says, “Both X and Y are oracles” and Y says, “X is a necromancer”?

A) X is an oracle and Y is a necromancer B) X is an oracle and Y is an oracle

C) X is a necromancer and Y is a necromancer D) X is a necromancer and Y is an oracle

E) NOTA

1. How many positive, integral factors does the number 7920 have?
2. 8 B) 15 C) 30 D) 60 E) NOTA
3. Let be the decimal representation of the hexadecimal number . Compute the sum of the digits of as a decimal number.
4. 11 B) 12 C) 13 D) 14 E) NOTA
5. What is the units digit of?
6. 1 B) 3 C) 7 D) 9 E) NOTA
7. Which of the following answer choices is not always true?
8. Two integers are relatively prime if and only if no prime number is a factor of both of them.
9. If an integer *n* has a factor that is greater, it must have a factor that is strictly less than .
10. If an integer *n* is divisible by integers *a* and *b*, then *n* is also divisible by *ab*.
11. If *n* is an integer greater than 1, then *n* can be expressed as , where each is a distinct prime number and each is a natural number.
12. NOTA
13. What is the smallest positive integer with exactly 10 positive factors?
14. 80 B) 162 C) 512 D) 1024 E) NOTA
15. Let A and B be two sets with cardinalities and . If , what is the value of ?
16. 11 B) 14 C) 17 D) 31 E) NOTA
17. Which of the following statements are always true? For this question, the capital letters A, B, C represent ***finite sets***.
    1. Let X Y denote that set X is an element of set Y. If A B and B C, then A C.
    2. (A – B) U B = A.
    3. Let p(X) denote the power-set of set X. If p(A) = p(B), then A = B.
18. I B) II, III C) I, II D) I, II, III E) NOTA
19. Let A equal the set and B equal the set. Which of the following is the Cartesian product of set A and set B?
20. NOTA
21. From slowest growing to fastest growing, what is the correct ordering of the following four functions?

I. II. III. IV.

1. I, II, III, IV B) I, II, IV, III C) II, I, III, IV D) II, I, IV, III E) NOTA
2. Let f(*n*), for a natural number *n*, be the function defined as

. What is the value of f(5)?

1. 39 B) 59 C) 79 D) 83 E) NOTA
2. How many of the statements about functions are true?
   1. A function can be both even and odd
   2. The composition of two injective functions is injective.
   3. The composition of two surjective functions is surjective.
3. 0 B) 1 C) 2 D) 3 E) NOTA
4. A three digit binary number is randomly generated by selecting each of the three digits to be either a 0 or a 1 with uniformly equal probability. What is the probability that the two least significant digits are both 0?
5. 1/8 B) 1/4 C) 1/3 D) 1/2 E) NOTA
6. Ari randomly rearranges the Q, W, E, R, T, and Y keys on his keyboard. What is the probability that exactly one key remains in its original position?

A) 1/5 B) 3/5 C) 3/24 D) 5/24 E) NOTA

1. Five sets of twins enter an empty room. The ten individuals are then randomly grouped into five new groups of two. What is the probability that the five new groups are the five twin sets?
2. B) C) D) E) NOTA
3. The Oak Creek Market sells eight different types of Fury Stones. Michela is making one giant earring that requires three Fury Stones of the same type. If she randomly selects Fury Stones from the Oak Creek Market to buy, what is the minimum number of Fury Stones she needs to buy in order to guarantee that she can make her giant earring?

A) 4 B) 8 C) 11 D) 24 E) NOTA

1. Big-O notation is used to describe a family of functions based on their limiting behavior. Formally, we say that f(x) = O(g(x)) if there exist two constants c and k such that for all . Given the function, which of the following is NOT a valid g(x) such that f(x) = O(g(x))?
2. B) C) D) E) NOTA
3. Using Big-Theta notation, what is the runtime complexity of the following algorithm?

Note: Big-Theta notation is similar to Big-O notation except that (g(n)) = f(n) means that g(n) belongs to the family of functions which is the *tightest* upper bound on f(n).

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| For int i = 1 to n {  int j = 1;  While(j < n) { j \*= 2; }  } |

1. B) C) D) E) NOTA
2. Compute the determinant of the matrix A = .
3. 91 B) 55 C) -55 D) -91 E) NOTA
4. Let T be a linear operator on complex, finite-dimensional vector space V (i.e. T is a linear map from V to V), dim(V) equal the dimension of a vector space V, and null(T) equal the null space of a linear map T. Given that dim(null(T)) = m, for 1 < m < dim(V), which of the following is true?
5. T is an invertible linear map
6. T is an injective linear map
7. A basis for the image of T will have less than dim(V) vectors in it
8. A basis for V will have less than dim(V) vectors in it
9. NOTA
10. Let T be a linear operator on and A be the matrix of T with respect to the standard basis { , , }. What is the value of T () if the matrix A = ?
11. 10 B) 20 C) D) E) NOTA
12. Given A, B, and C are finite sets and that , which of the following must ALWAYS be TRUE?
13. B) C) D) One of the sets is empty E) NOTA
14. Grant is on the planet of CL3. The planet CL3 is exactly like Earth, with one exception: CL3 has VeniBoxes which are like the boxes on Earth, but if you can lift an arbitrary VeniBox, then you must have the strength to lift any other VeniBox with arbitrary content. Grant cannot lift an anvil and writes the following proof to conclude he can’t lift a VeniBox:

|  |
| --- |
| * If Grant could lift a VeniBox, then he could lift an anvil.   + This is because we could put an anvil in a VeniBox and he could lift the VeniBox * Since Grant cannot lift an anvil, he cannot lift a VeniBox. |

Which of the following best describes this proof?

1. The proof is correct by induction
2. The proof is incorrect because it incorrectly applies the inductive hypothesis.
3. The proof is correct by reduction.
4. The proof is incorrect because it should be reducing anvil lifting to VeniBox lifting
5. NOTA
6. Kevin runs the following algorithm on a connected weighted undirected graph.

|  |
| --- |
| * Create “Set E” that contains all edges from the graph and “Set N” that contains all nodes in the graph * Remove a random node from Set N and add it to a special set called “Set X” * While Set N is not empty, loop over each edge in Set E and do:   + Remove an edge from Set E with minimum weight that connects a node in Set X with a node in Set N and add this edge to Set X.   + From Set N, remove the node that was just connected by the newly added edge and add this removed node to Set X. * Output the graph determined by the nodes and edges in Set X. |

What graph structure term best describes the output of this algorithm?

1. Minimal Graph B) Minimum Spanning Tree C) Shortest Path Circuit

D) Shortest Directed Acyclic Graph E) NOTA

1. Let G be an undirected graph containing 15 nodes. If two nodes can have at most one (undirected) edge between them, what is the maximum number of undirected edges the graph can have?
2. 14 B) 30 C) 90 D) 210 E) NOTA
3. [The](http://www.artofproblemsolving.com/Resources/Files/NCTM08talk.pdf) island of Crittenden consists of 11 towns, each of which is connected to at least seven other towns by roads. If all towns must be directly connected to the same number of towns, what is the minimum number of roads per town?
4. 7 B) 8 C) 9 D) 10 E) NOTA