1. Each digit is independent of each other, so we have 10 \* 10 \* 10 \* 10 different combinations of digits. **D**
2. This is a permutation of 8 items with 3 “repetitions” = 8!/3! = 6270. **C**
3. Kevin has 4 choices of armor \* 3 choices of helmets \* (5 chose 2 choices of weapons) = 120. **E**
4. Because boxes can get lost, we basically have a sixth person that all the lost boxes can go to. Thus we have fifteen identical items that we’re trying to divide among six containers. This is the same thing as arranging fifteen identical items and five dividers. So we have twenty slots and we need to choose where the five dividers go (because once the dividers are fixed, the fifteen items are automatically distributed). This is 20 choose 5 = 20!/(15!5!). **C**
5. First count the number of possible committees: (6 chose 2 combinations of monks) \* (5 chose 3 combinations of knights) = 150. Next, count the number of invalid committees where the monk and knight serve together (5 chose 1 monks because the stubborn monk has been fixed on the committee) \* (4 chose 2 knights because the stubborn knight is fixed) = 30. Thus, the total number of committees without the stubborn knight and monk = 150 – 30 = 120. **A**
6. Karel needs to travel five steps in the positive x-direction (right) and seven steps in the positive y-direction (up) and he must make exactly this set of moves in order to get to (6, 8) because he cannot move backwards in any direction. Thus the problem reduces to choosing which of Karel’s twelve steps are going to be steps to the right: 12 chose 5. **B**
7. In order for the second number to be exclusively in between the first and third, all three numbers must be different: P(all numbers are different) = 20 \* 19 \* 18/(20^3) = (19 \* 18)/(20 \* 20). Now, we just need to compute the probability that these three numbers are also in the same order; there are three slots and we must fix the middle number to be the 2nd slot, so there are 2! valid permutations out of 3! possible ways to arrange these numbers, thus our answer is $\frac{19×18×2!}{20×20×3!}=\frac{57}{200}$. **B**
8. For a given expression f(x,y,z…), the sum of the coefficients is just f(1,1,1,1…) by definition. Thus the sum of the coefficients is just $(2×1+3×1)^{4}=5^{4}.$ **C**
9. First we pick the two ranks (e.g. Ace, King, Queen…, Two) of cards that will be our pairs: $\left(\begin{matrix}13\\2\end{matrix}\right)$ since the silver and gold cards can’t make pairs with any other cards. Now, among those two ranks, there are four cards that can form pairs: $\left(\begin{matrix}4\\2\end{matrix}\right)×\left(\begin{matrix}4\\2\end{matrix}\right)$ for each of the two pairs. Finally, we can pick any remaining card to be the fifth card: 46 since it can’t be any of the eight cards that might form a pair. There are $\left(\begin{matrix}54\\5\end{matrix}\right)$ to chose a group of five cards, so our final probability is: $\frac{\left(\begin{matrix}13\\2\end{matrix}\right)×\left(\begin{matrix}4\\2\end{matrix}\right)^{2}× 46}{\left(\begin{matrix}54\\5\end{matrix}\right)}$. **C**
10. First, we know that there are 6^4 total outcomes because each dice value is independent of the other three dice; thus, the denominator is 6^4. Now, we need to choose 2 values that we’re repeating so that’s (6 choose 2 possible pairs of values). Finally, we have four dice that we’re grouping into two groups of two, which is the multinomial coefficient: $\left(\begin{matrix}4\\2,2\end{matrix}\right)$. Thus, the probability is $\frac{\left(\begin{matrix}4\\2,2\end{matrix}\right)\left(\begin{matrix}6\\2\end{matrix}\right)}{6^{4}}$. **D**
11. The possible ways to get a sum of seven are rolling a {1,6}, {2,5}, or {3,4} and there are two permutations for each of these roll pairs, so we have 3 \* 2 desired outcomes and 6\*6 total outcomes; this is a 6/36 probability. **C**
12. **D.** $P\left(Vowel\right)=\frac{P\left(American AND Vowel\right)}{P\left(Vowel\right)}=\frac{P\left(American AND Vowel\right)}{P\left(American AND Vowel\right)+P\left(European AND Vowel\right)}=\frac{P\left(American\right)×P\left(American\right)}{P\left(American\right)×P\left(American\right)+P\left(European\right)×P\left(European\right)}=\frac{\frac{3}{5}×\frac{2}{5}}{\frac{3}{5}×\frac{2}{5}+\frac{2}{5}×\frac{3}{6}}=\frac{^{6}/\_{25}}{^{11}/\_{25}}=\frac{6}{11}$.
13. **A.** There are 3! ways that Nick, Alex, and Duke can be arranged in the first three slots and 7! ways the rest of the line can be arranged behind them, so this is 3!7!/10! = 3!/(8\*9\*10) = 1/120.
14. **E.** $P\left(Richard Correct AND Diane Incorrect\right)=P\left(Richard Correct\right)=\frac{n-2}{n-1}×\frac{1}{n}=\frac{n-2}{n^{2}-n}$. $P\left(Richard Correct XOR Diane Correct\right)=2×\frac{n-2}{n^{2}-n}=\frac{2n-4}{n^{2}-n}$.
15. To make the shorter piece exactly 1/3 the size of the longer piece, we need to divide the pipe into segments of length x and 3x; in other words, we need to divide the pipe such that we cut off 1/4 of the total pipe as the shorter piece – any cut that is less than 1/4 the length of the pipe will yield a smaller piece that is less than 1/3 the length of the longer piece. But note that we can cut this piece from either of the pipe, so we have a 2 \* ¼ probability = ½. **D**
16. Using Bayes Rule: P(Speak German | Speak Chinese) = P(Speak German and Speak Chinese)/P(Speak Chinese) -> .3 \* .16 /.2 = .24 -> **D**
17. By the Pigeonhole Principle, we are guaranteed that at least one dartboard will have 10 or more darts on it (88/9 = 9 remainder 7); however, if we’re trying to guarantee a maximum, then we can only assume that the dartboard will have 10 darts (because the darts could be distributed 9,9,10,10,10,10,10,10,10). Given that the dartboard has 10 darts, we simply use the Pigeonhole Principle to guarantee that at least 4 of the darts will have the same color (because there are 3 colors/“buckets” that the darts can be, so 10/3 = 3 remainder 1). Thus we can guarantee the maximum number of same colored darts on one dartboard is 4. Note that while it would be possible for us to guarantee larger numbers given certain probability distributions (e.g. Sandy only has green darts), we don’t know any information about how often Sandy hits certain boards and what the color distribution of the darts are. Therefore, in the “worst case” scenario described above, where everything is uniform, there will be no more than 4 darts of the same color on one board. **B**
18. The sum of the probability of the disc landing in one of the three buckets plus the probability that the disc lands in no bucket must equal one since these are all the outcomes 1 – 0.3 – 0.5 – 0.1 = 0.1. **A**
19. The probability that Dieterich wins is 1 \* 7/15 = 7/15 (the first pawn can be any color and the next one must be one of the remaining 7). His probability of losing is just the complement. By definition of expectation, E[winnings] = Winning\_Value \* P(Win) + Losing\_Value \* P(Lose) = 2 \* 7/15 -1 \* 8/15 = 6/15 = $0.40. **B**
20. Every positive integer can be written as x = 9n + r. $x^{2}=81n^{2}+18nr+r^{2}\rightarrow \frac{x^{2}}{9}=9n^{2}+2nr+\frac{r^{2}}{9}$, so the remainder has 9 options depending on whether r = 0 -> 8 (since x = 9n + r and r < 9). If r = {0, 3, 6} then the remainder is 0. If r = {1, 8}, then (r^2)/9 yields a remainder of 1. If r = {2, 7}, then (r^2)/9 yields a remainder of 4. Finally, if r = {4, 5}, then (r^2)/9 yields a remainder of 7. Thus the probability is 2/9. **C**
21. Since we’re guaranteed that at least one program is a virus, then the total sample space is {at least one program is a virus} -> P(at least one program is a virus) = 1 – P(neither program is a virus) = 1 – (1/5)^2 = 24/25. P(both programs are viruses) = (4/5)^2 = 16/25. Thus the conditional probability is (16/25)/(24/25) = 16/24 = 2/3. **E**
22. Zeroes at the end of a number a generated by factors of ten (in other words a pair of 2 and 5). With factorials, clearly there will always be more 2’s than 5’s as factors (because for finite numbers there are more even numbers than there are multiples of five); thus once we get six 5’s as factors, we will have a number with at least 6 zeroes. If we count up from 1, we see that 5, 10, 15, and 20 yield four factors of 5 and that 25 yields 2 factors of 5; so from 25 to 50, our number will have at least six zeroes at the end. Thus we have 26/50 numbers -> 13/25. **A**
23. Ari needs to pick 8 wrong turtles and then the right turtle: 49/50 \* 48/49 \* … \* 1/42 = 1/50 (by telescoping). **D**
24. Let B be the expected number of heads before Albert chooses a coin and A be the expected number of heads at the very end. Then E[A] = ½ \* E[B] + ½ \* E[B + 1] because the coin that Albert picks has a 50-50 chance of being either heads or tails. If it’s heads, then we just get E[A] = E[B], but if it’s tails, then E[A] = E[B + 1]. Thus, E[A] = E[B] + ½ by linearity of expectation and E[B] = 4 because we have ½ \* 8 = 4 coins that should land as head. So the answer is 4 + ½ = 4.5. **B**
25. The number of photos HongVan takes is: $\left(\begin{matrix}10\\0\end{matrix}\right)+\left(\begin{matrix}10\\1\end{matrix}\right)+\left(\begin{matrix}10\\2\end{matrix}\right)+…+\left(\begin{matrix}10\\10\end{matrix}\right)$. This is just the sum of the entries in Row 10 of Pascal’s triangle which is 2^10 = 1024 (a nice way to algebraically see this is to consider (1+1)^10 and expand using binomial expansion). **E**
26. First consider all the numbers with exactly four repeating digits; there are only 9 (because numbers can’t start with 0). Now, count the numbers that have exactly 3 repeating digits; either the most significant digit is one of the repeating digits or it’s not: (1) MS digit is one of the three repeating = $9×\left(\begin{matrix}3\\2\end{matrix}\right)×9=243$; the first 9 comes from the 9 digits that can be the MS digit, the (3 chose 2) comes from the two digits that will repeat the first, and the final 9 comes from the 9 other digits that the non-repeating digit can be. (2) MS digit is not one of the repeating three = $9×9×1×1=81$; the MS digit is non-repeating (9 options) and the remaining three digits are all the same (no permutations of 3 identical digits – also 9 options because they can’t be the first digit). Thus the total number is 9 + 243 + 81 = 333. **C**
27. If the Heat win in exactly five games, then they must win on the very last game and lose one of the first four: $\left(\begin{matrix}4\\1\end{matrix}\right)×\left(\frac{2}{5}\right)^{1}×\left(\frac{3}{5}\right)^{3}×\frac{3}{5}=\frac{648}{5^{5}}$. **C**
28. This is Pascal’s Identity: $\left(\begin{matrix}n\\r\end{matrix}\right)+\left(\begin{matrix}n\\r+1\end{matrix}\right)=\left(\begin{matrix}n+1\\r+1\end{matrix}\right)\rightarrow \left(\begin{matrix}20\\10\end{matrix}\right)+\left(\begin{matrix}20\\11\end{matrix}\right)=\left(\begin{matrix}21\\11\end{matrix}\right)$. **B**
29. This is another variation on dividing marbles into bags/balls into urns problem. You have three containers (x, y, and z) that you’re filling with 15 sticks (because x + y + z = 15). Since each x, y, z are positive integers, they all must have at least one stick in each of them, so you’re really only sorting 12 items into 3 containers, or choosing where to place two dividers in between twelve items: $\left(\begin{matrix}14\\2\end{matrix}\right)$. **A**
30. This is a twist on the classic Monte Carlo problem. The original door that Niels picked has a 1/5 probability of housing the Holy Grail because all five doors had an equally likely chance. However, there is a 4/5 probability that one of the four other doors contains the Holy Grail, so by switching Niels has a 4/5 probability that the remaining door contains the Holy Grail. **D**