Mu Alpha Theta 2014 National Convention

- 1. **B** The total number of choices is the product of: ${}_{10}C_2 \cdot 4 \cdot {}_6C_2 = 2,700$.
- 2. C First determine the rate of the upstream and downstream travel, using (distance)/(time). This gives us the rate downstream = 9 mph, and the rate upstream = 3 mph. Defining two variables, b = boat's speed in still water, and c = rate of the current, we can represent the upstream and downstream rates by the equations, respectively: b c = 3 and b + c = 9. Solving, we get c = 3.

3. A Let *m* = Jamey's age and *f* = Jeffy's age. As described, we get the equation $m - f = \frac{m + f}{2}$, which

can be transformed to $\frac{m}{f} = \frac{3}{1}$, so the desired ratio is **3:1.**

- 4. **B** Our two equations are $x^2 + y^2 = 1753$ and $x^2 y^2 = 295$. Adding them gives us $2x^2 = 2048$, which leads us to x = 32. Substituting and solving we get y = 27. The product $32 \cdot 27 = 864$.
- 5. **B** I like to set up this type of problems using each person's unit rate of work, in other words, what fraction of the job does each person complete in one hour. Each person's rate is then multiplied by the number of hours he or she works; these products represent what fraction of the job each person completes. The sum of all of these fractions of the job equals 1, representing the entire job complete.

Using this method, our equation is:
$$\frac{1}{2}\left(\frac{1}{2}\right) + \frac{1}{2}\left(\frac{1}{3}\right) + \frac{1}{2}\left(\frac{1}{d}\right) = 1$$
. Solving gives us $d = \frac{6}{7}$

- 6. A First we can determine the amount of time that Kim walks by dividing distance by rate. That gives us $1\frac{1}{2}$ hours. Since Woolfey is running continuously during this time and his rate is 7mph, we can find his total distance by simply multiplying his rate and his time: (7 mph)(1.5 h) = **10.5 miles.**
- 7. **B** Let x = the number of increases of 20 cents to the original ticket price. We can use the information in the problem to set up the expression (3 + .25x)(400 20x) to represent the total amount of money taken in for tickets as the price and number of attendees change with the ticket price change described. We want to find the value of x that results in the maximum value of this expression. First multiply out and combine terms; that gives us $-5x^2 + 40x + 1200$; then simplify to $x^2 8x 240$. The max occurs at x = -b/2a which is x = 4, so the increase in ticket price is \$1.00 and the ideal price is \$4.00.
- 8. B Let x = the amount removed. Our equation is 10(.5) x(.5) = 10(.4). Solving we get x = 2.
- 9. D Pages 1 99 involve 10 numbers with 5 in the ones place and 10 numbers with 5 in the tens place. That's 20 5's. Then for pages 100 899, for each interval of 100 pages there are likewise the same number of 5's in the ones and tens places, so that is 20(8) or 160 5's. But for the interval from 500-599 there are also 100 5's in the hundreds place. From 900 913 only one 5 occurs. The total is 281.
- 10. **D** Let s = average score and h = hours studying. This is a direct proportion: s = kh². Substituting the given values we solve and get k = 12. To find the score that corresponds to 2 hours 45 minutes, we use $s = 12\left(\frac{11}{4}\right)^2$, which becomes $s = 12\left(\frac{121}{16}\right)$, which easily simplifies to s = 363/4 or about **91**.
- 11. A Let the two numbers be 9x and 13x. Set up the equation $\frac{9x-5}{13x-11} = \frac{5}{7}$ and solve. We get x = 10, so the original larger number was **130**.

- 12. C Let d = number of dimes, Q = number of quarters, and n = number of nickels. The equations are d = 2Q, 2d = n + 2, and 5n + 10d + 25Q = 185. We can express Q and n in terms of d and quickly get an equation in one variable: 5(2d 2) + 10d + 25(d/2) = 185. Solving we get d = 6, so Q = 3, n = 8. After she spent all her nickels, she has **\$1.35** left.
- 13. **B** Since the area of the square is 16 square inches, we know the side of the square is 4 inches. The width of the velvet is three times the side, so it must be **12** inches.
- 14. A Let p = total points earned so far, and let n = number of tests taken so far. We can see that her new test grades and resulting averages are $\frac{p+98}{n} = 83$ and $\frac{p+66}{n} = 79$. Cross multiply and get two linear equations: p+98 = 83n and p+66 = 79n. This solves to n = 8.
- 15. E To get 4 pieces, it takes 3 cuts, so each cut must take 4 minutes. To get 10 pieces, it takes 9 cuts. Total time needed is (9)(4) = 36 minutes.
- 16. C The elapsed time is 40 minutes, which is 2/3 of a full revolution or one circumference. Given the radius is 6, the circumference is 12π . Multiplying $(2/3)(12\pi)$ we get 8π inches.
- 17. A First, consider the rate at which each hand moves in terms of degrees per minute. It takes the minute hand 60 minutes to go 360 degrees, so it moves at a rate of 6 degrees per minute. The hour hand takes 60 minutes to move $1/12^{\text{th}}$ of a full revolution, so its rate is $\frac{1}{2}$ degree per minute. Now consider the "starting position" of each hand at 8:00. The minute hand is at the 12, which could be assigned the measure 0 degrees, and the hour hand is at the 8, which would then correspond to a measure of 240 degrees. Let x = the number of minutes elapsed since 8:00. For the two hands to coincide, their positions must be equal. The equation representing this is $6x = 240 + \frac{1}{2}x$. Solving we get x = 480/11 or 43 7/11. Rounded value: **8:44 am.**
- 18. **D** Using a Venn diagram we find that a total of 56 schools attended at least one of the theme parks, so the number of schools that stayed at the Doubletree was **11**.
- 19. C Let x = the side of the square base of the box. That box has volume $3x^2$ which = 972. Solving for x we get x = 18. The original side is then 18 + 3 + 3 = 24 inches.
- 20. A The surface area of the original cube is 6(36) = 216 sq in. The cut out area on two opposite faces of the cube are both circles of radius 1, so the total area removed is 2π . The new surface area which is created is the lateral area of the cylindrical hole; it equals $6(2\pi) = 12\pi$. The total surface area from these parts is $216 + 10\pi$.
- 21. **C** Use the distance formula: $\sqrt{a^2 + b^2 + c^2} = 10$ to represent the length of the diagonal. This becomes $a^2 + b^2 + c^2 = 100$. Use 2ab + 2bc + 2ac = 224 for the total surface area. Adding and factoring gives us $(a + b + c)^2 = 324$, so a + b + c = 18, and the sum of all the edges is 4(18) = 72.
- 22. C Position the arch so that the vertex is at (0, 10). It will have an *x*-intercept at (8, 0). We want to find the *y* coordinate for the point (6, *y*). Using the general form for the equation of a parabola $y k = a(x h)^2$, we can substitute (0, 10) for (*h*, *k*) and (8, 0) for (*x*, *y*). Solving for *a*, we find the value $a = -\frac{5}{32}$. So the equation of this parabola is $y 10 = -\frac{5}{32}(x 0)^2$. To answer this question,

all we have to do is substitute 6 for x and solve for y. Doing so we find that $y = \frac{35}{8}$ or $4\frac{3}{8}$.

- 23. A Let the two parallel sides be x, and the remaining side be 130 2x. The area is 1200 = x(130 2x). Expanding and simplifying gives us $x^2 - 65x + 1050 = 0$, which factors into (x - 30)(x - 35) = 0. So the two possible sets of dimensions are 30 by 70 and 35 by 60. The correct difference is **25**.
- 24. **C** The volume of the sphere is $(4/3)(6)^3\pi$, or 288 π . Let *s* be the side length of the cube; then the volume of the cube will be $6s^2$ and it must equal the volume of the original sphere, which was 288 π . Solve the equation $6s^2 = 288\pi$ and we get $s = 4\sqrt{3\pi}$.
- 25. **B** Treat the 3 vowels as a single entity. Arranging the 4 consonants and the vowel block has 5! permutations. Now consider the 3 vowels; among themselves they can be arranged 3! times. The total number of arrangements possible as described in the question is (5!)(3!) or **720**.
- 26. **C** We want to position this semi-ellipse so that its center is at the origin and its major axis is on the *x*-axis. Based on the information given, we know a = 15 and b = 10. The equation of the ellipse is $\frac{x^2}{225} + \frac{y^2}{100} = 1$ To find the height of the arch at a point 6 ft from the center, we simply substitute 6 for *x* and solve for *y*. A good approach is to multiply the equation by the LCM of the denominators, which is 900. Doing so, we get $4x^2 + 9y^2 = 900$. Substituting 6 for *x* gives us $144x^2 + 9y^2 = 900$. We can divide all terms by 9 to solve more easily; the new equation is $16 + y^2 = 100$. This gives leads us to $y^2 = 84$. Because the semi-ellipse is above the *x*-axis, we want the positive value, $2\sqrt{21}$.
- 27. **B** The strategy here is to first find the point of intersection of each pair of lines that correspond to the inequalities. These are the vertices of the feasible region. We will test the coordinates of each point in the profit function and find the point that yields the highest profit and what that profit is. The intersection point of x + y = 3 and x 3y = -3 is (3/2, 3/2). The point of intersection of x + y = 3 and 2x y = 6 is (3, 0). The point of intersection of x 3y = -3 and 2x y = 6 is (21/5, 12/5). The respective values of the profit function are 1.5, 9, and 7.8. So the maximum profit is **9**.
- 28. A There are 31 possible cases for a family of 5 to have at least one girl (in other words, $2^5 1$ (all boys)). We want the probability of having 2 girls and 3 boys, which would occur in (5!)/(3!)(2!) ways, or 10 ways. The probability asked for is $\frac{10}{31}$.
- 29. A Based on the formula and information provided, we have $I = 10^{4.5}$ which can be rewritten $I = 10^4 \cdot 10^{0.5}$. This is the same as $I = 10,000(\sqrt{10})$, which is close to 10.000(3), so **30,000**.
- 30. **D** First, the three consecutive integers whose sum is 72: Set up and solve x + x + 2 + x + 4 = 72; 3x = 66, x = 22. So the diameters are 22, 24, and 26. The radii are then 11, 12, and 13. The total surface area of the three spheres is $4\pi(11^2 + 12^2 + 13^2) = 4\pi(121 + 144 + 169) = 1736\pi$.