1. Apollonius, **C**.

2. $4-2x^2-16x-2y^2+12y=6$
   $x^2+8x+y^2-6y=-1$
   $(x+4)^2+(y-3)^2=24$
   Center is (–4, 3), **B**.

3. $\sqrt{(3+7)^2+(5-6)^2}=\sqrt{101}$, **D**.

4. $16x^2-64x-9y^2+54y=161$
   $16\left(x^2-4x+4\right)-9\left(y^2-6y+9\right)=144$
   $16(x-2)^2-9(y-3)^2=144$
   $\frac{(x-2)^2}{9}-\frac{(y-3)^2}{16}=1$
   Asymptotes: $y=3\pm\frac{4}{3}(x-2)$
   Asymptote with positive slope is
   $y=\frac{4}{3}x+\frac{1}{3}$ or $4x-3y+1=0.$
   $\text{D}=\frac{\left|4(5)-3(3)+1\right|}{\sqrt{4^2+(-3)^2}}=\frac{12}{5}$, **B**.

5. $9x^2+54x-16y^2+64y-127=0$
   $9\left(x^2+6x+9\right)-16\left(y^2-4y+4\right)=144$
   $9(x+3)^2-16(y-2)^2=144$
   $\frac{(x+3)^2}{16}-\frac{(y-2)^2}{9}=1$
   $c=\sqrt{16+9}=5$, so the points given are the foci. By definition of a hyperbola, the difference between the focal radii is $2a$. $a^2=16$, so the difference is 8, **D**.

6. Nikitha lives on the directrix on the right side of the hyperbola. The equation for the directrix is $x=-3+\frac{16}{5}=\frac{1}{5}$, **A**.

7. $4x-5y=4$
   x-intercept: (1, 0); y-intercept: $\left(0,-\frac{4}{5}\right)$
   Diameter length: $\sqrt{1^2+\left(\frac{4}{5}\right)^2}=\sqrt{\frac{41}{25}}$
   Radius length: $\frac{1}{2}\sqrt{\frac{41}{25}}=\frac{\sqrt{41}}{10}$
   Area: $\frac{41}{100}\pi$, **A**.

8. $x^2+y^2+Ax+By+C=0$
   $\left(x+\frac{A}{2}\right)^2+\left(y+\frac{B}{2}\right)^2=\frac{-4C+A^2+B^2}{4}$.

   To be a point, the right side of the equation must equal 0, so $A^2+B^2=4C$, **C**.

9. Item I is a parabola, so its eccentricity is 1.
   Item II is an ellipse, so $0<e<1$.
   Item III is a hyperbola,
   $\frac{y^2}{4}-\frac{x^2}{9}=1.\ e=\frac{\sqrt{13}}{2}$.
   Item IV is a circle, so its eccentricity is 0.
   Item V is a equilateral hyperbola. These always have eccentricity $\sqrt{2}$. Answer: **A**.

10. By substitution, $y+y^2=1$, so $y=\frac{-1\pm\sqrt{5}}{2}$.
    Using the given information, $a=1$ and $b=5$, so $a+b=6$, **D**.

11. By substitution, $\left(\frac{7-3y}{2}\right)^2-3y^2=1$ which leads to $9y^2-12y^2-42y+49-4=0\rightarrow$ $y^2+14y-15=0$. The sum of the y-values is –14, **E**.

12. Place the center at the origin to get $\frac{x^2}{76^2}+\frac{y^2}{38^2}=1$. We are interested in finding the y-value at $x=19$. $\frac{19^2}{76^2}+\frac{y^2}{38^2}=1 \to y^2=\frac{38^2 \bullet 15}{16}$, so $y=\frac{19\sqrt{15}}{2}$, **B**.

13. The distance from the focus to the directrix is twice the distance than that between the focus and vertex. The answer is therefore **A**.

14. The inequality becomes $\frac{x^2}{3^2}+\frac{y^2}{2^2}\le 1$, so the area is $(3)(2)\pi=6\pi$, **D**.

15. Let the focus be located at $(0, a)$; therefore, the line is $y=\frac{1}{2}x+a$. The parabola can be written as $y=\frac{1}{4a}x^2$. We need to find the a-value at $x=2$: $\frac{1}{2}(2)+a=\frac{1}{4a}(2)^2$. This gives $1+a=\frac{1}{a} \to a^2+a-1=0 \to a=\frac{-1\pm\sqrt{5}}{2}$. The focal width is $4a$. Using only the positive value of a, we get $2\sqrt{5}-2$, **B**.

16. This is a vertical hyperbola. Using the given slope and vertices, we get the actual values for a and b: $\frac{a}{b}=\frac{4}{3}=\frac{8}{6}$. Focal width is $\frac{2b^2}{a} \to \frac{2(36)}{8}=9$, **D**.

17. This is a semicircle of radius 4. (This can be seen by squaring each side and rearranging.) The area is $\frac{1}{2}\pi(4)^2=8\pi$, **C**.

18. The answer is **C**.

19. Using a rectangle 25 ft wide and 20 ft tall, the least eccentric will be the horizontal hyperbola. The asymptotes are $y=\pm\frac{20}{25}x \to y=\pm\frac{4}{5}x$, so $\frac{b}{a}=\frac{4}{5}$. This gives $a=\frac{5b}{4}$. The shorter transverse axis is 2 (given), and this is the conjugate axis for the hyperbola we need; therefore, $2b=2, b=1$, an $a=\frac{5(1)}{4}=\frac{5}{4} \to \frac{x^2}{\frac{25}{16}}-\frac{y^2}{1}=1$. When $x=5, \frac{25}{\frac{25}{16}}-y^2=1 \to 15=y^2$. Width is $2\sqrt{15}$, **B**.

20. $9x^2-y^2+4y=4$

$9x^2-\left(y^2-4y+4\right)=4-4$

$9x^2-(y-2)^2=0$

$3x=\pm(y-2)$

A pair of intersecting lines, **D**.

21. $9x^2+4y^2+54x-16y=-97$

$9\left(x^2+6x+9\right)+4\left(y^2-4y+4\right)=81+16-97$

$9(x+3)^2+4(y-2)^2=0$

This is only true for the point (−3, 2), **C**.

22. Using the distance formula for two points (left side) and the formula for distance between a point and a line (right side), we get:

$\sqrt{(x-4)^2+(y-0)^2}=\left(\frac{1}{2}\right)\frac{x-16}{\sqrt{1^2+0^2}}$

$x^2-8x+16+y^2=\frac{x^2-32x+256}{4}$

$3x^2+4y^2=192$, **A**.

23. $xy-2x+y-3=0$

$y(x+1)=2x+3$

$y=\frac{2x+3}{x+1}$ or $y=\frac{1}{x+1}+2$

This is a hyperbola with asymptotes $x=-1,\ y=2$, **B.**

24. $8x-3x^2$

Find the vertex then the corresponding value.

$x=\frac{-b}{2a}=\frac{-8}{2(-3)}=\frac{4}{3}$. $8\left(\frac{4}{3}\right)-3\left(\frac{16}{9}\right)=\frac{16}{3}$, **E.**

25. Like #24, use the discriminant. Luckily, the *a*-value is 1. We are only concerned with $b^2-4c\geq 0$.

c	6	5	4	3	2	1
b	5 6	5 6	4 5 6	4 5 6	3 4 5 6	2 3 4 5 6
# of possible b-values	2	2	3	3	4	5

There are 19 total possibilities, **B.**

26. By substitution, $x^2+4(mx+1)^2=1$.

This is a quadratic equation, so simplify and set the discriminant equal to 0. That way, there is only one solution.

$x^2+4m^2x^2+8mx+3=0.$

$\left(4m^2+1\right)x^2+(8m)x+3=0$

$64m^2-4\left(4m^2+1\right)(3)=0$

$16m^2-12=0$

$m^2=\frac{3}{4}$, **C.**

27. $a=5, c=4.\ a^2-b^2=c^2\rightarrow b=3.\quad 2b=6$, **A.**

28. $9y^2-144=16x$

$\frac{9}{16}y^2-9=x\rightarrow a=\frac{4}{9}$

Vertex: (–9, 0), Focus: $\left(-\frac{77}{9},0\right)$.

At $x=-\frac{77}{9},\ y=\pm\frac{8}{9}$. Using *x*-axis as line of symmetry:

Area: $2\left[\frac{1}{2}\left(\frac{4}{9}\right)\left(\frac{8}{9}\right)\right]=\frac{32}{81}$, **B.**

29. Extend the "10" chord and call the missing chord length *x*; then

$10x=(15)(15)=225$

$x=22.5\rightarrow r=(10+22.5)/2=16.25$, **E.**

30. Substituting each point into the equation:

$$\begin{cases}4+4-2a+2b+c=0\\16+4+4a+2b+c=0\\4+4+2a-2b+c=0\end{cases}\rightarrow\begin{cases}-2a+2b+c=-8\\4a+2b+c=-20\\2a-2b+c=-8\end{cases}$$

Adding the first and third rows, we get $c=-8$. Adding the second and third rows, we get $a=-2$. After substituting these values, we get $b=-2$.

$abc=(-2)(-2)(-8)=-32$, **A.**