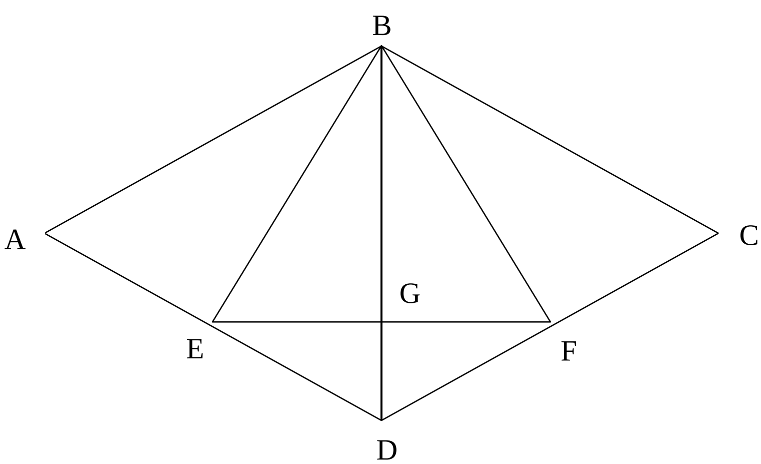
1. A triangle that contains one side that has the same length as the diameter of its circumscribing circle must be a right triangle, which cannot be acute, obtuse, or equilateral. **A**

2. $A = \frac{s^2\sqrt{3}}{4} = \frac{\frac{1}{2}^2\sqrt{3}}{4} = \frac{\sqrt{3}}{16}$. **D**

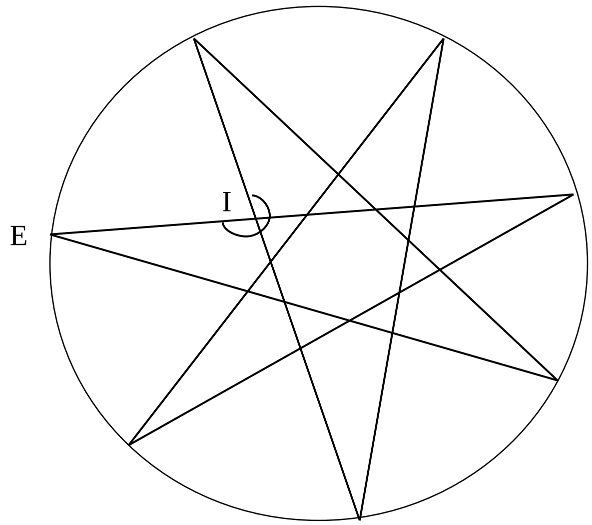
3. Radius of incenter, $r = 2 * \frac{\text{area of triangle}}{\text{perimeter}} = 2 * \frac{24}{24} = 2.$ $A = \pi r^2 = \pi 2^2 = 4\pi.$ **C**

4. The centroid is the point that corresponds to the center of gravity in a triangle. **B**

5. Triangle EBG is a 30-60-90 triangle with hypoteneuse 6, as stated. Using our knowledge of ratios of sides in 30-60-90 triangles, we know that $\overline{EG} = 3$ and $\overline{GB} = 3\sqrt{3}$. Triangle EGD is also a 30-60-90 triangle which shares a side with triangle EBG, $\overline{EG} = 3$. Using ratios, we find that $\overline{GD} = \sqrt{3}$. $\overline{BD} = 3\sqrt{3} + \sqrt{3} = 4\sqrt{3}$. The area of rhombus ABCD is twice the area of two equilaterial triangles with sides of length $4\sqrt{3}$. Area= $\frac{(4\sqrt{3})^2\sqrt{3}}{4} * 2 = 24\sqrt{3}$ **A**

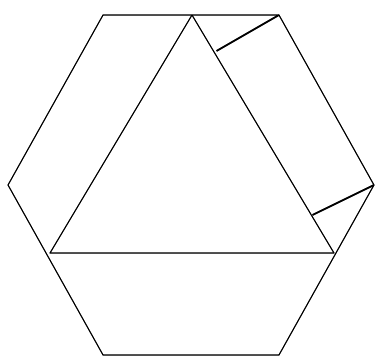


6. In a regular 7-pointed star, there are 7 exterior acute angles and 7 interior obtuse angles, labeled E and I respectively in the diagram below. Each exterior angle is equal to half of the subtended arc length in the circumscribed circle. $E = \frac{1}{2} * \left(\frac{360}{7}\right)° = \frac{180}{7}°$. I', the value of (360-I) is equal to half of the two subtended arc lengths along the circumscribed circle. $I' = \frac{1}{2} * \left(\frac{360}{7} + 2 * \frac{360}{7}\right)° = \frac{540}{7}°.$ $I = 360 - \frac{540}{7}° = \frac{1980}{7}°$. The sum is equal to $7 * \left(\frac{1980}{7}° + \frac{180}{7}°\right) = 2160°$. **C**

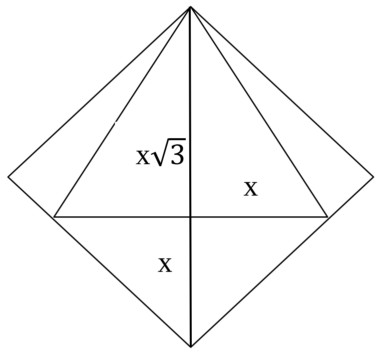


7. Solving the length of $\overline{AC}$ using angle bisector ratios, we find that $\overline{AC} = 16$, which exceeds the sum of the other two sides. This is not a possible triangle. **E**

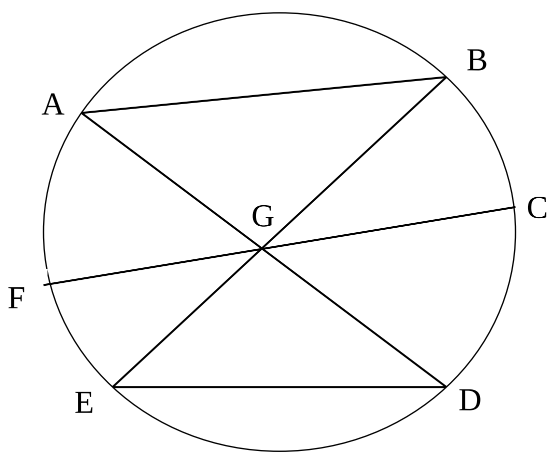
8. By drawing perpendicular lines from the sides of the hexagon to the triangle, as drawn below, we find that the side of the triangle is equal to the sum of 6 and two bases of 30-60-90 triangles with hypotenuse of length 3. Side of triangle $= 6 + \frac{3}{2} * 2 = 9$ **D**



9. As in the diagram below, if you label one half the base of the triangle x, we find that the diagonal of the triangle has length $x + x\sqrt{3}$. Setting this equal to $6\sqrt{2}$, we can solve for x. $6\sqrt{2} = x(1+\sqrt{3})$. $x = \frac{6\sqrt{2}(\sqrt{3}-1)}{2} = 3\sqrt{2}(\sqrt{3}-1) = 3\sqrt{6} - 3\sqrt{2}$. The side of the triangles $=2x = 6\sqrt{6} - 6\sqrt{2}$. **A**



10. Because $\angle ABE = 30°$, we know that $\widehat{AE} = 60°$. Also, since $\widehat{FE} = 30°$, we know that $\widehat{AF} = 30°$. Using the information that $\angle CGD = 25°$, we can solve $\widehat{CD} = 20°$. The sum of arcs $\widehat{AFE}$ and $\widehat{BCD}$ is 120º. Therefore, the remaining arcs sum to 240º, which is equal to twice the sum of $\angle AGB$. $\angle AGB = 120°$. **D**



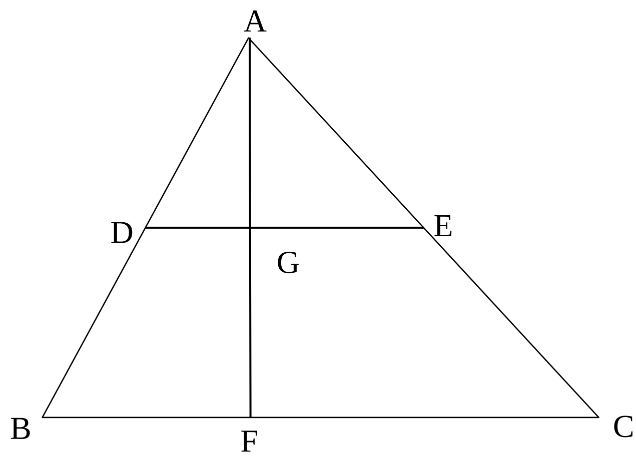
11. $y = -3x^2 + 6x + a = -3(x-1)^2 + 3 + a$. The height of the triangle is $(3+a)$ and the width is $2\sqrt{\frac{3+a}{3}}$. Thus, the area is $\frac{1}{2} * h * b = \frac{1}{2}(3+a)(2)\left(\sqrt{\frac{3+a}{3}}\right) = 81.$ $(3+a)^{\frac{3}{2}} = 3^{\frac{9}{2}}.$ $3 + a = 27.$ $a = 24.$ **C**

12. The radius of the circumscribed circle is as follows: $r = \frac{(6)(8)(4)}{4\sqrt{9*3*1*5}} = \frac{16\sqrt{15}}{15}$. **B**

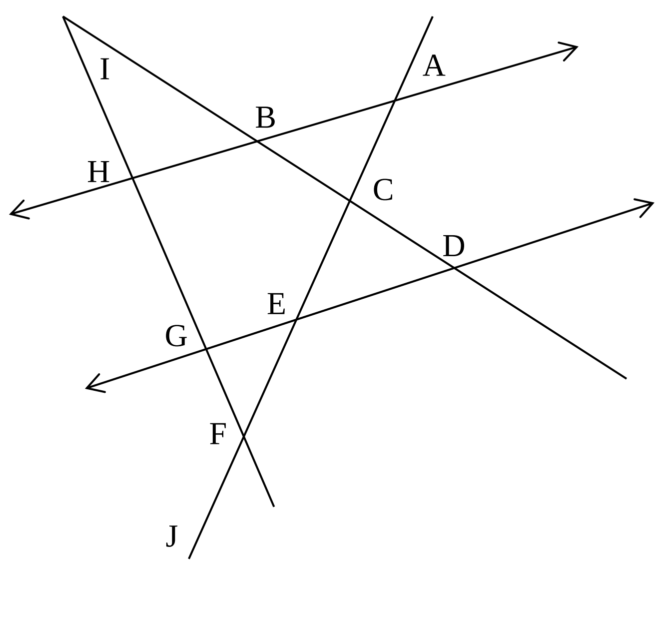
13. The Euler line contains the centroid, circumcenter, and orthocenter. **D**

14. The radius of the inscribed circle in a triangle is $\frac{2(\text{area of triangle})}{\text{perimeter}}$. Using Heron's formula, we can solve for the area of the triangle. $A = \sqrt{10*3*6*1} = 6\sqrt{5}$. Thus, the radius of the inscribed circle is $2 * \frac{6\sqrt{5}}{20} = \frac{3\sqrt{5}}{5}$ and the diameter is $\frac{6\sqrt{5}}{5}$. **B**

15. Since segment DE is a midsegment, we know that the length of $\overline{AE} = 4$. Drawing a perpendicular line from point A to point F along $\overline{BC}$(such that segment DE intersects segment AF at point G), we find that triangle AFB and AGD are 30-60-90 triangles and triangle AFC and AGE are 45-45-90 triangles. Given segment AE has length 4, we know segment GE and AG has length $2\sqrt{2}$. Given segment AG has length $2\sqrt{2}$, we know that segment DG has length $2\sqrt{6}$.
$2\sqrt{2} + 2\sqrt{6}$ **A**



16. $\angle ABC$ is supplementary to $\angle IBA$, therefore, $\angle ABC = 57°$. Within triangle ABC, using the information known about the other two angles, we can solve $\angle ACB = (180 - 57 - 25)° = 98°$. Using the property of vertical angles, we know $\angle DCE = 98°$ as well. $\angle GEC$ is supplementary to $\angle CED$, therefore $\angle CED = 35°$. Within triangle CED, based on the other two angles, we can solve $\angle CDE = 47°$. $\angle GFE$ is supplementary to $\angle GFJ$, therefore $\angle GFE = 55°$. Within triangle GEF, based on the other two angles, we can solve $\angle EGF = 90°$. $\angle IGE$ is supplementary to $\angle EGF$, therefore $\angle IGE = 90°$. Within triangle IDG, based on the other two angles, we can solve $\angle BIH = (180 - 47 - 90)° = 43°$ **A**



17. Because the circumcenter lies along $\overline{BC}$, we know that $\overline{BC}$ is the diameter of the circle as well as the hypotenuse of the right triangle ABC with right angle at A. Because A is a right angle, we know $\sin A = 1$. Using law of Sines, we can solve for a, the length of the diameter of the circle. $\frac{\sin B}{b} = \frac{\sin A}{a}$. $\frac{0.75}{9} = \frac{1}{a}$. $a = 12$. The circumference of the circle is $2\pi r = \pi d = 12\pi$. **D**

18. Quadrilateral $ABCD$ is composed of two right triangles that share the same hypotenuse, diagonal $\overline{BD}$, of length 25. **D**

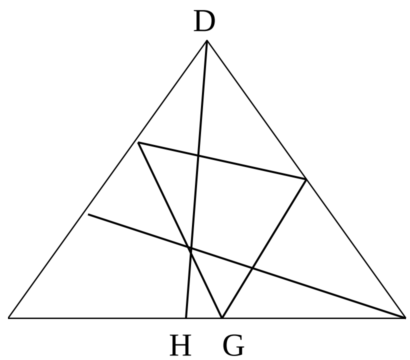
19. Using Heron's formula, we can solve for the area of the triangle.
$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{14(8)(4)(2)} = 8\sqrt{14}$. The altitude to each side can be found by $\text{altitude (aka. height)} = 2 * \frac{\text{Area}}{\text{base}}$. Therefore, the three altitudes have lengths $\frac{8}{3}\sqrt{14}, \frac{8}{5}\sqrt{14}, \frac{4}{3}\sqrt{14}$ and the sum is $\frac{28\sqrt{14}}{5}$. **A**

20. Given two sides the sine of an angle that is not the interior angle, we do not have enough information to solve for $\overline{BC}$. **E**

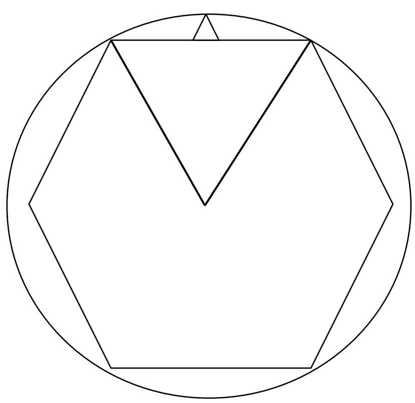
21. A right isosceles triangle with hypotenuse 8 has two legs of length $4\sqrt{2}$. A 30-60-90 triangle with hypotenuse 8 has legs of length 4 and $4\sqrt{3}$. Thus, the perimeter of the quadrilateral whose sides are composed of the legs of the triangles is $4 + 4\sqrt{3} + 8\sqrt{2}$. **C**

22. The shortest distance from the base $\overline{AB}$ to vertex C is the altitude which is length of the leg of a right triangle with hypotenuse of length of 12 and other leg of length 7. Using Pythagorean Theorem, we can solve the shortest distance possible from vertex C to $\overline{AB}$ to be $\sqrt{12^2 - 7^2} = \sqrt{119}$. The maximum length from vertex C to $\overline{AB}$ is along the edge of the triangle, which has legs of length 12. e^2 is the only value outside of the range. **C**

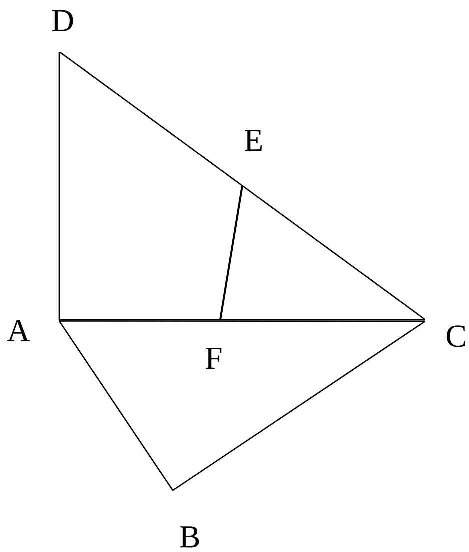
23. Looking at the ratios of the sides, we see that $\overline{DH}$ is a median as it intersects $\overline{AF}$ at its midpoint. Point J is the centroid of the triangle since it intercepts the median $\overline{DH}$ two-thirds of the way from the vertex D. Since $\overline{BF}$ goes through point J, we know that $\overline{BF}$ is also a median. Because the medians of a triangle separate the triangle into 6 triangles of equal area, we know that triangle BDJ is a sixth of the area of triangle ADF. Area of triangle $BDJ = 2$. Since $\overline{DI}:\overline{IJ} = 1:1$ and $\overline{CI}$ is parallel to $\overline{BJ}$, we know that $\overline{DC}:\overline{CB} = 1:1$ and therefore, triangle DCJ is ½ the area of triangle BDJ. Area of triangle $DCJ = 1$. Within triangle DCJ, $\overline{DI}:\overline{IJ} = 1:1$ and therefore, triangle CIJ is ½ the area of triangle DCJ. Area of $CIJ = \frac{1}{2}$. **A**



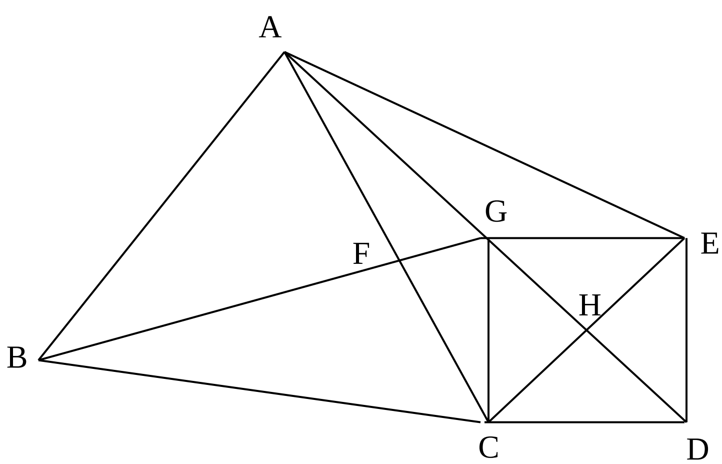
24. The regular hexagon can be split up into 6 regular triangles, such as drawn below. The altitude of the inscribed smaller triangle can be evaluated by subtracting the altitude of the big triangle from the radius of the circle to give $4 - 2\sqrt{3}$. Using ratios of 30-60-90 triangles, we can solve the length of the side of the inscribed triangle to be $\frac{2(4-2\sqrt{3})\sqrt{3}}{3} = \frac{8\sqrt{3}-12}{3}$. The area of the small equilateral triangle is $\frac{\left(\frac{8\sqrt{3}-12}{3}\right)^2\sqrt{3}}{4}$. The area of the regular hexagon is $\frac{6(4)^2\sqrt{3}}{4}$. The ratios of the area of the triangle to the hexagon is as follows: $\frac{\left(\frac{\left(\frac{8\sqrt{3}-12}{3}\right)^2\sqrt{3}}{4}\right)}{\frac{6(4)^2\sqrt{3}}{4}} = \frac{(7-4\sqrt{3})}{18}$. **B**



25.Since the triangles are both right triangles, we know the hypotenuse coincides with the diameter of the circumscribed triangle. Thus, the circumcenter is the midpoint of the hypotenuse. Given this information, we can solve for the lengths $\overline{EC} = 6$ and $\overline{FC} = 3\sqrt{3}$. We also know $\angle ECF = 30°$ using the property of similar triangles and given the fact that $\angle ECF = \angle ACB$ (otherwise, the angles would sum to 90 degrees). Using the Law of Cosines, we can solve for $\overline{EF} = \sqrt{6^2 + (3\sqrt{3})^2 - 2(6)(3\sqrt{3})\cos 30°} = \sqrt{36 + 27 - 54} = \sqrt{9} = 3$. It is also possible to simply recognize that triangle EFC is proportional to triangle EAC. **B**



26. The equation of a circle is $x^2 + y^2 + ax + bx + c = 0$, where a, b, and c are real numbers. Thus, we can plug in the three pairs and solve for a, b, and c. Solving, we get $a = -10,\ b = 0,$ $c = 20,\ x^2 + y^2 - 10x + 20 = 0$. Completing the square, we get $(x-5)^2 + (y)^2 = 5$. **D**

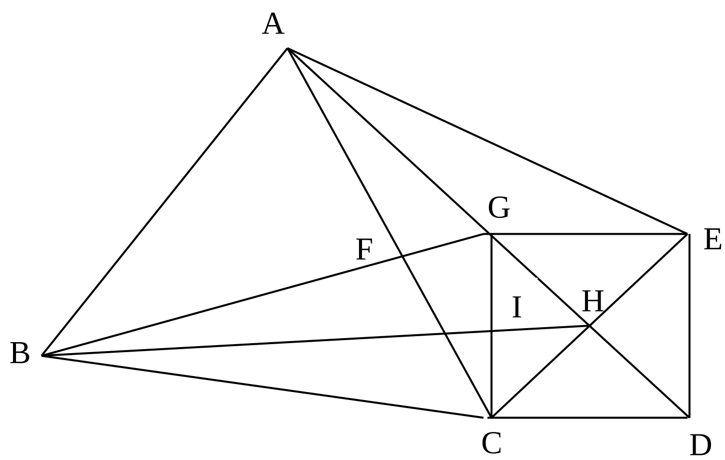


27. Triangles with the following vertices are pictured: GEH, HED, CHD, GHC, FGC, AFG, AGE, ABF, BFC, BGC, BAG, GEC, CED, GED, GCD, ACE, AHE, ACH, AGC, ACD, AED, ABC. 22 **E**

28. These 10 triangles with the following vertices are isosceles: BGC, ACE, GEC, CED, GED, GCD, GEH, HED, CHD, GHC. Triangle AEC is an isosceles triangle with vertex A as the altitude drawn from point A perpendicularly bisects the base $\overline{CE}$. There are no equilateral triangles. Thus, there are 22-10=12 scalene triangles. **D**

29. Draw an altitude down from B within isosceles triangle BGC to center H of the square and call the intersection of $\overline{GC}$ and $\overline{BH}$ I. Because I bisects $\overline{GC}$, $\overline{GI} = \frac{1}{2}\left(8\sqrt{2}\right) = 4\sqrt{2}$. In triangle BIG, $\angle BIG$ is a right angle since $\overline{BI}$ is the altitude from the vertex of an isosceles triangle. Given

that $\overline{BG} = 4\sqrt{6}$ and $\overline{GI} = 4\sqrt{2}$, we can use the Pythagorean theorem to solve $\overline{BI} = 8$ and thus, $\overline{BH} = 8 + 4\sqrt{2}$. $\overline{GH}$ is equal to the length of half the diagonal of square $GEDC$. $\overline{GH} = 8$ and thus, $\overline{AH} = 8 + 4\sqrt{2} = \overline{BH}$. Using the formula Area $= \frac{1}{2}ab\sin C$, where C is the angle between sides a and b, we can determine the area of pentagon $ABCDE$. Pentagon $ABCDE$ is composed of triangles $AHE,\ BHA,\ BHC,$ and CED. Area $= \frac{1}{2}(8 + 4\sqrt{2})(8)\sin(90°) + \frac{1}{2}\left(8 + 4\sqrt{2}\right)\left(8 + 4\sqrt{2}\right)\sin(45°) + \frac{1}{2}\left(8 + 4\sqrt{2}\right)(8)\sin(45°) + \frac{1}{2}\left(8\sqrt{2}\right)^2 = 32 + 16\sqrt{2} + 24\sqrt{2} + 32 + 16\sqrt{2} + 16 + 64 = 144 + 56\sqrt{2}$. **A**



30. $\overline{FC} = 6$ since $\overline{AF} = 6$. Point *I* is the centroid of the triangle and is two-thirds the distance from each vertex. Since $\overline{BI} = 8$ and $\overline{DC} = 9$, $\overline{IF} = 4$ and $\overline{IC} = 6$. Triangle IFC has sides of length 6, 6, and 4. Using Herons's formula, Area $= \sqrt{8 * 2 * 2 * 4} = 8\sqrt{2}$. The medians divide a triangle into 6 triangles of equal area. Thus, the area of triangle *IFC* is equal to the areal of triangle *BIE*. Since $\overline{BG}$ is one third the length of $\overline{BE}$, we know that the area of triangle *BIG* is one-third the area of triangle *BIE*. Area $= \frac{8\sqrt{2}}{3}$. **C**