The abbreviation NOTA denotes “none of these answers”. Figures are not necessarily to scale. Good luck!

1. Given a triangle that is inscribed within a circle whose diameter is equal to the length of a side of the triangle can be which of the following?
   A. scalene  B. acute  C. equilateral  D. obtuse  E. NOTA

2. What is the area of an equiangular triangle with sides of length $\frac{1}{2}$?
   A. $\frac{\sqrt{3}}{2}$  B. $\frac{\sqrt{3}}{4}$  C. $\frac{\sqrt{3}}{8}$  D. $\frac{\sqrt{3}}{16}$  E. NOTA

3. Given a triangle with area and perimeter 24, what is the area of the inscribed circle?
   A. $\frac{3\pi}{2}$  B. $2\pi$  C. $4\pi$  D. $\frac{8\pi}{3}$  E. NOTA

4. Which point corresponds to the center of gravity in a triangle?
   A. Orthocenter  B. Centroid  C. Incenter  D. Circumcenter  E. NOTA

5. Given an equilateral triangle with sides of length 6 inscribed within a rhombus with one angle 60, what is the area of the rhombus? Refer to the diagram to the right.
   A. $24\sqrt{3}$  B. $36\sqrt{3}$  C. $48\sqrt{3}$  D. $64\sqrt{3}$  E. NOTA

6. What is the sum of the interior angles of a regular 7-pointed star?
   A. $1260^\circ$  B. $1890^\circ$  C. $2160^\circ$  D. $2520^\circ$  E. NOTA

7. In triangle $ABC$, the angle bisector $AD$ bisects the base into fragments $BD = 2$ and $DC = 4$. If $\overline{AB} = 8$ and $\overline{AE}$ is a perpendicular bisector, what is the length of $\overline{DE}$?
   A. $3\sqrt{2}$  B. $2 - \sqrt{3}$  C. $1 + \frac{\sqrt{7}}{2}$  D. 2  E. NOTA

8. An equilateral triangle is inscribed within a regular hexagon with side lengths 6 as depicted below. What is the length of one side of the triangle? Refer to the diagram below.
   A. $6\sqrt{3}$  B. $8\sqrt{3}$  C. $6 + \frac{3\sqrt{3}}{2}$  D. 9  E. NOTA
9. An equilateral triangle is inscribed within a square with sides of length 6 as depicted below. What is the length of one side of the triangle? Refer to the diagram to the right.

A. $6(\sqrt{6} - \sqrt{2})$
B. $3(\sqrt{6} + \sqrt{2})$
C. $\frac{3}{2}(\sqrt{6} + 2\sqrt{3})$
D. $3(\sqrt{6} + \sqrt{3})$
E. NOTA

10. Given $\angle ABE = 30^\circ$, $\overline{BC} = 40^\circ$, $\angle CGD = 25^\circ$, $\overline{FE} = 30^\circ$, what is $\angle AGB$? Refer to the diagram to the right.

A. $85^\circ$
B. $90^\circ$
C. $110^\circ$
D. $120^\circ$
E. NOTA

11. An isosceles triangle has its base along the x-axis with it vertices consisting of the vertex and x-intercepts of a parabola with equation $y = -3x^2 + 6x + a$, where $a$ is a number. If the triangle has an area of 81, what is the value of $a$?

A. 15
B. $9 + 6\sqrt{3}$
C. 24
D. $12 + 6\sqrt{3}$
E. NOTA

12. What is the radius of the circumscribed circle around triangle with sides of length 6, 8, and 4?

A. $\frac{16\sqrt{3}}{5}$
B. $\frac{16\sqrt{15}}{15}$
C. $9\sqrt{2}$
D. $\frac{8\sqrt{3}}{3}$
E. NOTA

13. Which point is not contained within the Euler line?

A. Centroid
B. Circumcenter
C. Orthocenter
D. Incenter
E. NOTA

14. What is the diameter of the inscribed circle within a triangle with side lengths 4, 7, and 9?

A. $\frac{3\sqrt{5}}{5}$
B. $\frac{6\sqrt{5}}{5}$
C. $\frac{12\sqrt{5}}{5}$
D. $3\sqrt{7}$
E. NOTA

15. Given that $\overline{DE}$ is a midsegment of $\triangle ABC$, $\overline{EC} = 4$, $\angle B = 30^\circ$ and $\angle DEC = 135^\circ$, what is the length of $\overline{DE}$? Refer to the diagram to the right.

A. $2(\sqrt{2} + \sqrt{6})$
B. $2(\sqrt{2} - \sqrt{6})$
C. $2(3 - \sqrt{6})$
D. $3\sqrt{2} - \sqrt{6}$
E. NOTA
16. Given \( \angle BAC = 25^\circ, \angle IBA = 123^\circ, \angle GEC = 145^\circ, \angle GFJ = 125^\circ \), what is \( \angle BIH \) in degrees? Refer to diagram below.

A. 43  
B. 53  
C. 57  
D. 63  
E. NOTA

17. Triangle \( ABC \) is inscribed in a circle such that the circumcenter lies along \( BC \). Given \( \sin(B) = 0.75 \) and \( b = 9 \), what is the circumference of the circle?

A. \( 9\pi \)  
B. \( 6\sqrt{7}\pi \)  
C. \( 8\sqrt{2}\pi \)  
D. \( 12\pi \)  
E. NOTA

18. Quadrilateral \( ABCD \) has sides of length \( AB = 7, BC = 20, CD = 15, \) and \( DA = 24 \), what is the length of \( BD \)?

A. \( 12\sqrt{5} \)  
B. \( 17\sqrt{2} \)  
C. 21  
D. 25  
E. NOTA

19. What is the sum of all of the altitudes in a triangle with sides of length 6, 10, and 12?

A. \( \frac{28\sqrt{14}}{5} \)  
B. \( \frac{56\sqrt{14}}{5} \)  
C. \( \frac{51\sqrt{7}}{10} \)  
D. \( \frac{49}{6}\sqrt{3} \)  
E. NOTA

20. Given triangle \( ABC \) with \( \sin(C) = \frac{7}{8}, AC = 3 \) and \( AB = 5 \), what is the length of \( BC \)?

A. 5  
B. \( \frac{35}{8} \)  
C. 7  
D. \( 3\sqrt{7} \)  
E. NOTA

21. An isosceles right triangle and a right triangle with one angle 30° share the same hypotenuse of length 8. What is the perimeter of the quadrilateral created from the combined triangles?

A. \( 8 + 4\sqrt{3} + 4\sqrt{2} \)  
B. 16  
C. \( 4 + 4\sqrt{3} + 8\sqrt{2} \)  
D. \( 20 + 4\sqrt{3} + 8\sqrt{2} \)  
E. NOTA

22. Given isosceles triangle \( ABC \) with vertex \( C \) with \( AC = 12 \) and base of length 14, what is not a possible length of a segment drawn from \( C \) to \( AB \)?

A. \( 5 + \pi + 2e \)  
B. \( \sqrt{119} \)  
C. \( e^2 \)  
D. 12  
E. NOTA
23. Refer to the figure below to answer the question, where all vertices are labeled. Given \( \frac{AH}{HG} : \frac{GF}{} = 5:1:4 \), \( \overline{DI} = \overline{IJ} = \overline{IH}, \overline{CI} \) is parallel to \( \overline{BJ} \), and the area of \( \triangle ADF \) is 12, what is the area of triangle \( \triangle CJI \)? Refer to the diagram to the right.
   A. \( \frac{1}{2} \)
   B. \( \frac{2}{3} \)
   C. \( \frac{4}{3} \)
   D. 1
   E. NOTA

24. A regular hexagon is inscribed within a circle of radius 4. What is the ratio of the area of the regular triangle inscribed in the region between the hexagon and circle to the area of the regular hexagon? Refer to the diagram to the right.
   A. \( \frac{7-4\sqrt{3}}{16} \)
   B. \( \frac{7-4\sqrt{3}}{18} \)
   C. \( \frac{7-4\sqrt{3}}{24} \)
   D. \( \frac{7-4\sqrt{3}}{32} \)
   E. NOTA

25. Triangle \( DAC \) is similar to triangle \( ABC \) where \( \angle DAC = \angle ABC = 90^\circ \). Given \( \angle DCB = 60^\circ \) and \( \overline{DC} = 12 \), what is the length of the distance between the circumcenters of the triangles? Refer to the diagram to the right.
   A. \( \sqrt{3} \)
   B. 3
   C. \( 3\sqrt{3} \)
   D. 4
   E. NOTA

26. Given a triangle with coordinates \((4, 2), (3, -1), \) and \((7, 1)\), what is the equation of the circle circumscribing the triangle?
   A. \((x - 6)^2 + (y - 1)^2 = 9\)
   B. \(\left(x - \frac{9}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 = \frac{25}{2}\)
   C. \((x - 3)^2 + (y + 1)^2 = 7\)
   D. \((x - 5)^2 + y^2 = 5\)
   E. NOTA
Refer to the diagram below and the following information for questions 27-29. All vertices are labeled in the figure.

Pentagon $ABCDE$ contains square $GEDC$. $\overline{BG} = \overline{BC} = 4\sqrt{6}$, $\overline{ED} = 8\sqrt{2}$, and $\overline{AG} = 4\sqrt{2}$.

27. How many triangles are pictured? Refer to the diagram and information above.
A. 19  B. 20  C. 21  D. 23  E. NOTA

28. At least how many scalene triangles are present? Refer to the diagram and information above.
A. 9  B. 10  C. 11  D. 12  E. NOTA

29. What is the area of pentagon $ABCDE$? Refer to the diagram and information above.
A. $144 + 56\sqrt{2}$  B. $72 + 64\sqrt{2}$  C. $144 + 64\sqrt{2}$  D. $72 + 48\sqrt{2}$  E. NOTA

30. Given that points $E, F$ and $D$ bisect $\overline{BC}, \overline{CA}$ and $\overline{AB}$ respectively, $\overline{BG}$ is one-third the length of $\overline{CE}$, $\overline{AF} = 6$, $\overline{DC} = 9$, and $\overline{BI} = 8$, what is the area of $\triangle BGI$? Refer to the diagram below.
A. $\frac{7\sqrt{3}}{2}$
B. $3\sqrt{3}$
C. $\frac{8\sqrt{2}}{3}$
D. $4\sqrt{2}$
E. NOTA