Answers:

1. A
2. B
3. D
4. A
5. C
6. D
7. A
8. B
9. A
10. D
11. C
12. C
13. A
14. B
15. A
16. C
17. E
18. D
19. A
20. A
21. C
22. C
23. B
24. E
25. D
26. C
27. A
28. D
29. C
30. A

Solutions:

1. The common difference is $\frac{13}{36}-\frac{5}{18}=\frac{1}{12}$, so $a_{13}=\frac{5}{18}+\left(13-1\right)\cdot\frac{1}{12}=\frac{23}{18}$.

2. The common difference is 3, so $S_{13}=\frac{13}{2}\left(2\cdot 4+\left(13-1\right)\cdot 3\right)=286$.

3. If d is the common difference, $400=31+\left(53-8\right)d\Rightarrow d=\frac{41}{5}$. Therefore, $a_{51}=400-2\cdot\frac{41}{5}$ $=\frac{1918}{5}$.

4. The common ratio is $\frac{48\sqrt{7}}{64\sqrt{7}}=\frac{3}{4}$, so the sum is $S=\frac{64\sqrt{7}}{1-\frac{3}{4}}=256\sqrt{7}$.

5. $-243=-3r^{5-1}=-3r^{4}\Rightarrow r^{4}=81\Rightarrow r=-3$ (since $r<0$). Therefore, $a=9, b=-27,$ and $a+b=-18$.

6. Since the common difference is 6, $\sum_{j=6}^{45}\left(6j-20\right)=\frac{45-6+1}{2}\left(16+250\right)=5320$.

7. Since the common ratio is -2, $S_{15}=\frac{-3\left(1-\left(-2\right)^{15}\right)}{1-\left(-2\right)}=-32769$.

8. $\sum_{n=5}^{\infty}7\left(\frac{1}{8}\right)^{n-1}=\frac{\frac{7}{8^4}}{1-\frac{1}{8}}=\frac{7}{8^4}\cdot\frac{8}{7}=\frac{1}{8^3}=\frac{1}{512}$

9. Using simple trigonometry, the length of the side of an inscribed square will be $\frac{\sqrt{2}}{2}$ the length of the square in which it is inscribed. Therefore, the sequence of perimeters is

geometric with common ratio $\frac{\sqrt{2}}{2}$. The original square has perimeter 16, so the sum of all perimeters is $\frac{16}{1-\frac{\sqrt{2}}{2}} = 32+16\sqrt{2}$.

10. $a_2 = \frac{a_6}{r^4} = \frac{128x^{17}y^5}{\left(2x^3y^2\right)^4} = \frac{128x^{17}y^5}{16x^{12}y^8} = \frac{8x^5}{y^3}$

11. $44 = 17+\left(k-3-k\right)d = 17-3d \Rightarrow d = -9$, so $a_{k-5} = 17+\left(k-5-k\right)\left(-9\right) = 17+45 = 62$.

12. By the given information, $a_4 = 33$. Let $a_3 = x$. Then the sequence becomes $x-33$, 3, x, 33, 30, $30-x$, $-3-x$, $-33-x$, -63, …, so $a_9 = -63$.

13. $\frac{3x}{5x-8} = \frac{2x+2}{3x} \Rightarrow 10x^2-6x-16 = 9x^2 \Rightarrow 0 = x^2-6x-16 = \left(x-8\right)\left(x+2\right) \Rightarrow x = 8$ or $x = -2$, and both values generate geometric sequences (32,24,18 for $x=8$; –18,–6,–2 for $x=-2$). Therefore, the sum of the values is 6.

14. This is similar to a geometric sequence with common ratio -5, so $q=-5$. However, if you didn't realize that, just compute the first three terms of the sequence and solve a system of equations. The first three terms of the sequence are 14, -46, and 254, so $14 = pq+r$, $-46 = pq^2+r$, and $254 = pq^3+r$. Therefore, $14-r, -46-r, 254-r$ is geometric with common ratio q, so $\frac{-46-r}{14-r} = \frac{254-r}{-46-r} \Rightarrow r = 4$. This makes the common ratio $q=-5$, and finally $p=-2$. Therefore, $p+q-r = -2+\left(-5\right)-4 = -11$.

15. From the given information, Sen. Byrd initially took office in January 1959. Since he would have started a new term every 6 years, the years in which he took office form an arithmetic sequence with common difference 6. Therefore, $Y_n = 1959+6\left(n-1\right)$.

16. Sally will save $\sum_{n=1}^{30} n^2 = \frac{30\cdot 31\cdot\left(30+31\right)}{6} = 9455$ dollars.

17. $108 = 3+7\left(n-1\right) \Rightarrow n = 16$, so the sequence has 16 terms.

18. The sequence is geometric with common ratio 5, so $a_n = a_1(5)^{n-1}$, so we need only find the first term. $a_1 = \frac{a_5}{r^4} = \frac{5000}{5^4} = 8$, making the formula $a_n = 8(5)^{n-1}$.

19. After rationalizing, $\frac{1}{1+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{7}} + \ldots + \frac{1}{\sqrt{2n-1}+\sqrt{2n+1}} = \frac{\sqrt{3}-1}{2} + \frac{\sqrt{5}-\sqrt{3}}{2}$ $+\frac{\sqrt{7}-\sqrt{5}}{2} + \ldots + \frac{\sqrt{2n+1}-\sqrt{2n-1}}{2}$, and all terms cancel except the -1 in the first term and the $\sqrt{2n+1}$ in the last term. Therefore, $100 = \frac{\sqrt{2n+1}-1}{2} \Rightarrow n = 20200$.

20. A sequence of the form $a_n = Ba_{n-1} + Ca_{n-2}$ can be written explicitly as $a_n = Dt_1^n + Et_2^n$, where t_1 and t_2 satisfy the equation $t^2 - Bt - C = 0$. Therefore, $0 = t^2 - 3t + 2 = (t-2)(t-1)$, so $a_n = D(2)^n + E(1)^n = D(2)^n + E$. Plugging the first two points in yields $D = 1$ and $E = -1$. Therefore, $a_n = (2)^n - 1$, and $a_{15} = 2^{15} - 1 = 32767$.

21. $a_5 = a_1 r^{5-1} = (5+2x)(3)^4 = 405 + 162x$

22. $837 = \frac{n}{2}(5+57) \Rightarrow n = 27$, so $57 = 5 + (27-1)d \Rightarrow d = 2$.

23. Let *x* be the remainder when the first term is divided by 5, then $5-x$ is the remainder when the second term is divided by 5. This pattern will alternate until the remainder when the 101st term is divided by 5 will be *x* again. So we need the 51 smallest numbers with remainder *x* and the 50 smallest numbers with remainder $5-x$. Therefore, make $x=1$, and the 51 smallest numbers are 1, 6, 11, …, 251, while the 50 smallest numbers are 4, 9, 14, …, 249. The sum of all of these numbers is $5 + 15 + 25 + \ldots + 495 + 251 = \frac{50}{2}(5+495) + 251 = 12751$.

24. $5 = 12005r^{5-1} \Rightarrow r^4 = \frac{1}{2401} \Rightarrow r = \pm\frac{1}{7}$, but to make the sum as large as possible, we choose $r = \frac{1}{7}$. Therefore, the largest possible sum is $\frac{12005}{1-\frac{1}{7}} = \frac{84035}{6}$.

25. If we add any sequence of four terms consisting of positive, negative, negative, positive terms, we get $n^2-(n+1)^2-(n+2)^2+(n+3)^2=(n-(n+1))(n+(n+1))+(n+3-(n+2))$ $(n+3+(n+2))=-2n-1+2n+5=4$. So the sum of the first four terms is 4, as are the next four, the next four, etc. Therefore, since 2016 is a multiple of 4, the sum is 2016.

26. Series I and II are arithmetic while series III is geometric.

I) $\sum_{i=0}^{6}\left(\frac{5}{3}i+9\right)=\frac{7}{2}(9+19)=98$

II) $\sum_{i=0}^{6}\left(19-\frac{5}{3}i\right)=\frac{7}{2}(19+9)=98$

III) $\sum_{i=0}^{6}2^i=\frac{1-2(64)}{1-2}=127$

So series III has the greatest sum.

27. The 16th smallest prime number is 53, so the first term of the sequence is 54. Therefore, $\frac{54}{1-r}=36\Rightarrow r=-\frac{1}{2}$. Therefore, $a_5=54\left(-\frac{1}{2}\right)^4=\frac{54}{16}=\frac{27}{8}$.

28. $\sum_{n=10}^{23}n^3=\sum_{n=1}^{23}n^3-\sum_{n=1}^{9}n^3=\left(\frac{23\cdot 24}{2}\right)^2-\left(\frac{9\cdot 10}{2}\right)^2=74151$

29. $\sum_{n=1}^{45}\log_2\left(1+\frac{1}{n+2}\right)=\sum_{n=1}^{45}\log_2\left(\frac{n+3}{n+2}\right)=\log_2\left(\frac{4}{3}\cdot\frac{5}{4}\cdot\frac{6}{5}\cdot\ldots\cdot\frac{48}{47}\right)=\log_2\left(\frac{48}{3}\right)=\log_2 16=4$

30. $a_r=a_s+(r-s)d\Rightarrow s=r+(r-s)d\Rightarrow s-r=(r-s)d\Rightarrow d=-1$ (since $r-s\neq 0$). Therefore, $a_{r+s}=a_r+sd=s+s(-1)=0$.