Attend the 35th Annual Convention in Honolulu, Hawaii

Mu Alpha Theta Newsletter

Spring 2005

(405) 325-4489 Fax: (405) 325-7184 matheta@ou.edu www.mualphatheta.org

MIAMI SPRINGS SENIOR HIGH SCHOOL WINS RUBIN AWARD

The Miami Springs Senior High School Mu Alpha Theta won the first annual Diane Rubin Award for community service. For the past five years, the chapter has been sending letters and cards of encouragement to several members of Mu Alpha Theta that had graduated and were serving in the military. The students then expanded the project to include other graduates and not just members. In 2004, their chapter of Mu Alpha Theta hosted a visit by three of these graduates to come speak to students at the school about their experiences in the military. One of the speakers had been in the military for nine years and was in the ambush in Iraq alongside Jessica Lynch. Another student serves in a medical unit in Germany. Besides this project, the students decorated doors at the Fair Havens Nursing Home during the winter holidays and donate Thanksgiving baskets of food to needy families. We congratulate Miami Springs' students for their contributions to their community. The school won two free registrations to the Hawaii Convention, reimbursed airfare and a $500 prize. They will be awarded a commemorative plaque during the Award Luau at the convention.

News from the President:

Greetings to all Mu Alpha Theta chapters. There are many exciting things happening for all members.

The Log 1 Contest was a wonderful competition this year with more students participating than last year. Students from 265 schools registered for the contest. The three rounds this year were all different. The first included a 15 question test for all students. The second round allowed students to choose one of three topic tests – Geometry, Equations & Inequalities, or Probability. The third and final competition was a ciphering event. Winners will be posted soon and awards will be sent out before the end of the year. Mu Alpha Theta will send other prizes chosen at random from participating schools that submitted scores. Our thanks to Tom Clymer of National Assessment and Testing for a job well done.

The Rocket City Math League continued for a second year. Divisions were divided into Pre- Algebra/Algebra I and Geometry/Algebra II. Awards for this contest will be posted and sent out before the end of the year, as well. The contest is run by student members of Grissom High School's Mu Alpha Theta aided by a grant from the National Office. 192 schools registered from 43 states, DC, Puerto Rico, Columbia in South America, Belgium, Germany, the Marshall Islands, Japan and Poland. More than 3600 students turned in scores for the first round alone.

National Convention in Hawaii is a convention students have been looking forward to for several years. The list of competitions and events looks outstanding. Information about the convention can be found online at www.mualphatheta.org under National Convention. Sponsors can download the Sponsor Information Packet there. Remember that the final registration date is May 15, 2005. Forms must be postmarked by that day and include payment. Sponsors must also registration information online. No registrations will be accepted after May 15.

National Convention Grants have been awarded to the following four schools:

Region I:	Ladue Horton Watkins High School, St. Louis, MO
Region II:	St. John High School, Plaquemine, LA
Region III:	Oxford High School, Oxford, AL
Region IV:	Granville High School, Granville, OH

These schools will receive two free registrations to the convention and up to $500 in reimbursed travel expenses.

International Science and Engineering Fair: Mu Alpha Theta will once again be giving a $1000 Award to an outstanding competitor in the area of mathematics at the final competition in Phoenix, AZ in May. Many students displaying in regional science fairs also received certificates from Mu Alpha Theta. Their names and the topics of their projects are posted on the Mu Alpha Theta website under Science Fair Winners. The Mu Alpha Theta Award is presented to the most challenging, thorough, and creative investigation of a problem involving modern mathematics. Over 650 winners in the United States and in many other countries are presented certificates each year.

As always, if you have any suggestions or ideas to share, please contact me, Susan Hiller – Mu Alpha Theta President, at susiehiller@bellsouth.net or Kay Weiss – Executive Director of Mu Alpha Theta at matheta@ou.edu. We are always looking for ways to enhance the membership of all students.

MU ALPHA THETA AT NCTM

NCTM held its Southern Regional Conference in New Orleans, Louisiana, November 4-6, 2004. The conference theme was "Math: Hot and Spicy." Visible at the conference were Mu Alpha Theta members from the Southern Region. These students served as conference hosts. Participants from Brother Martin High School, Cabrini High School, Benjamin Franklin High School, and Immaculotta High School were present and served as student hosts. Their responsibilities included: package handling and moving, assisting presiders at workshops, volunteering in NCTM booth, assisting at on-site registration, and serving as runners. The students were also able to attend some workshops and visit exhibits. They enjoyed the technology exhibits, some of the math games and materials, as well as the challenging textbooks.

Ms. Mattie White, Region II Governor, chaired the student host committee. According to Ms. White, the student hosts were recognized by other conference chairs and by conference participants, for the fine job they did at the conference. Sporting special t-shirts and student host badges, these students were easily recognized. Their support of the conference was very valuable. Congratulations and thanks to all participating Mu Alpha Theta Chapters.

2005 Andree Award Winner Announced

John Omundsen, a graduate of Fort Myers High School, is the recipient of this year's Andree Award. The Andree Award, named in honor of Mu Alpha Theta founders Richard and Josephine Andree, is given to a student interested in becoming a mathematics teacher. John is presently attending the University of Florida and is majoring in Mathematics and minoring in Secondary Education. In his essay, John spoke of his love of mathematics and how he hopes to help his future students see the wonder and beauty of the subject. He will accept his award at the Hawaii Convention.

National Office News from Kay:

Our newest version of the online order system now takes a purchase order number or a credit card. Login by pressing the blue button at www.mualphatheta.org on the left marked "Login to Order Certificates or Merchandise."You can View your Current Members, find another active chapter, change your password or email address, Purchase Certificates or Purchase Merchandise. Once purchased, go to Check Out to input billing information and just press the "Add Full Members" link to enter names by year of graduation. The one-time registration fee is still $5 per Full member.

Associate members may be added at any time, without charge by pressing the "Add Associate Members" link on the left. An Associate member has completed at least two semester of math and is in their third semester. They have the GPA for membership and may compete in our national math competitions and attend the National Convention. These should not be members eligible for Full membership. When a member is eligible for Full membership, the $5 fee should be paid and you should move them up so they may list Mu Alpha Theta membership on their college resume.

The office is busiest with orders in October, March, April and May. Most orders go out the day the order arrives at the office but please give us some time to get it mailed back to you. Overnight orders can be processed with an additional charge, when possible.

Please check online for the latest merchandise prices and to download a form from Chapter Resources, Merchandise. We do not anticipate any price increases at this time but on July 1, 2005, all honor cords will cost $5 each. The 25 pack discount will no longer be available.

GOVERNING COUNCIL NEWS

The Governing Council of Mu Alpha Theta meets twice a year, during the summer convention and in February at the national office. During the February meeting the following issues were discussed:

- An effort is being made to encourage more two-year colleges to start chapters of Mu Alpha Theta.
- Four Convention Grants worth $1640 each were approved to be offered to chapters that had not attended a National Convention in at least three years.
- Dr. Irina Lyublinskaya, our NCTM representative, will be one of our judges at the International Science Fair competition in Phoenix, AZ in May and will present the $1000 Mu Alpha Theta Prize.
- The Log 1 Contest and the Rocket City Math League competitions seemed to be successful and should be continued next year.
- Mu Alpha Theta will continue to financially support the AMC and the Math Olympiad competitions.
- Up to $20,000 in grant money will be made available to student members wishing to do summer math research or enrichment.
- The 2006 Convention will be July 16 – 21. 2006 in Fort Collins, CO at Colorado State University.
- The winners of the Diane Rubin Award and the Andree Award were selected.
- The GC discussed ways to encourage more sponsors into becoming active in Mu Alpha Theta at the national level.
- Mu Alpha Theta and the Educational Foundation are in good financial shape. Continued membership growth combined with increased merchandise sales have resulted in a healthy financial prognosis.

Anyone having an issue to be brought up before the Governing Council should email or send it to the National Office before the August meeting. Recommendations and suggestions are always welcome and encouraged.

SPONSORS

TRI-TALLAHASSEE INVITATIONAL

In the first ever competition of its type, Lawton Chiles, Lincoln, and Rickards High Schools hosted the Tri-Tallahassee Invitational held on November 12 and 13, 2004 at Lawton Chiles High School. Five hundred fifty students from 19 schools and three states participated in the competition.

The feature part of the event was a ciphering round held on Friday night. Students competed in their own division, Algebra I, Geometry, Algebra II, Precalculus, and Calculus answering a series of 12 questions given a time limit of three minutes per question. Parents and teachers from the host schools served as proctors.

On Saturday, the more traditional competition of individual and bowl rounds occurred. In addition to the five division contests, a Statistics individual test was given. In order to make this competition more Nationals like, the sweepstakes scores included only the best of the Geometry and Algebra II division scores for each school. That score was added to the Precalculus and Calculus division scores to produce the overall sweepstakes score. Also, the Ciphering scores of the four team members were included with the team scores. No calculators were allowed except for on the Statistics test.

The top five in sweepstakes were in order, Vestavia Hills, Rickards, Gainesville Buchholz, Lawton Chiles, and Lincoln.

The top five schools for the Interschool Test were in order, Buchholz, Lawton Chiles, Tallahassee Maclay, Tallahassee Deerlake Middle School, and Charlotte (NC) Hopewell.

The division, Individual, and Ciphering winners were as follows: Calculus division – Rickards,
Individual – Bryan Starke (Lawton Chiles),
Ciphering – Ross Friedberg (Rickards)
Precalculus division – Vestavia Hills,
Individual – Zachariah Tyree (Rickards),
Ciphering – David Harris (Vestavia Hills)
Statistics Individual – Sunjae Kwon (Tallahassee Leon)
Algebra II division – Vestavia Hills,
Individual – Evan Zhao (Vestavia Hills),
Ciphering – Kevin Hu (Vestavia Hills)
Geometry division – Vestavia Hills,
Individual – Wei Yan (Vestavia Hills),
Ciphering – Pei-Ann Lin (Vestavia Hills)

We continue to work on our website at www.mualphatheta.org! See Chapter Resources for new features, and information is now available on our Educational Foundation. There are copies of old exams under National Convention, Past Tests. See information about our Online System and its features. We now offer an online Roster to find other Active Chapters in your city or state! Past copies of newsletters are also available. Our online system is secured to take a credit card for purchases of certificates and merchandise. If you need a password or chapter ID number to log in, email Kay Weiss at matheta@ou.edu.

The Kalin Award

The Kalin Award was created by a grant from Dr. Robert Kalin of Florida State University, a former President of Mu Alpha Theta. Dr. Kalin is a nationally recognized mathematics educator and continues to be one of Mu Alpha Theta's strongest supporters.

The Kalin Award recognizes graduating seniors attending the National Mu Alpha Theta Convention who have excelled in mathematics and performed notable service to Mu Alpha Theta communities. The winner of the Award receives a $1000 prize and the finalists each receive $300. The Mu Alpha Theta Chapter of the winner receives $300.

Each Chapter is allowed to nominate one Mu Alpha Theta member for the Kalin Award per year. The Sponsor of the Chapter will determine how the Chapter's nominee is selected. The Application Form* consists of three parts. All three parts of the application must be postmarked by **June 15** and mailed to the National Office.

Our Mu Alpha Theta Chapter at Georgetown High School in Georgetown, TX raises about $2000 each year holding a Math Counts Tournament. Schools from all over the state of Texas come and participate in a Math Counts like Tournament. All tests are created by the Mu Alpha Theta students and the competition is run according to Math Counts regulations.

Sponsor: Michelle Brown

STUDENTS

Seeking Information about Careers in Mathematics?

One of Mu Alpha Theta's sponsoring organizations is SIAM, the Society for Industrial and Applied Mathematics. SIAM offers all sorts of information to students about the kinds of jobs at which mathematicians work. Free information can be found online at the link:
http://www.siam.org/students/career.htm .

SERVICE ACTIVITY

Many Mu Alpha Theta chapters are active in community service projects. One fun project is for Mu Alpha Theta members to become involved in the American Cancer Society Relay For Life. Relay For Life is a fun-filled overnight event designed to celebrate survivorship and raise money for research and programs of your American Cancer Society. There's plenty of music, food, and fun to go around. For more information call 1-800-ACS-2345 or visit their web site at www.cancer.org/relay

Maplesoft Contest at the Huntsville Convention

During the summer convention in Huntsville, AL last July, Maplesoft, our corporate partner, set up three computers in the lobby of the Hilton Hotel. Students from participating schools at the convention were able to try their luck taking an online math test powered by Maple TA. The computer algebra system, Maple 9.5,that runs in the background of Maple TA can determine if the student's answers are correct, even if the answer is correct but in a different form than the answer programmed into the computer. A different test was offered to each division: Theta, Alpha and Mu. A winner in each division was selected and won a free Student Version of Maple and a free registration to the 2005 Hawaii Convention. Mu Alpha Theta wants to thank Maplesoft for its continued support of our talented students. The winners of the contest were: Yao Yao Luo from Lincoln High School in Tallahassee, FL; Bernadette Durr from Vero Beach in Vero Beach, FL; and Yue Zhao from Hillsborough High School in Tampa, FL.

Honor your sponsor

Think about nominating your sponsor for a Mu Alpha Theta sponsor award. Download forms at http://www.mualphatheta.org under **Huneke or Sister Scolastica Awards**. Watch for the listed deadlines to ensure forms make it to the National Office on time!

You can also let your sponsor know how much you appreciate their time and efforts by making a donation to the Mu Alpha Theta Educational Foundation in honor of your sponsor. Donations support awards, grants, prizes and scholarships for Mu Alpha Theta students and help to defray the cost to participants at the yearly National Convention.

Students can also donate in honor of a graduating friend or a favorite math teacher. If Mu Alpha Theta has been a special part of your life, consider making a contribution to help other interested members participate in Mu Alpha Theta competitions or continue their education in Mathematics

Interested in Summer Math Enrichment?

Mu Alpha Theta is offering Summer Grants for up to $2000 per student to help you pay for a summer math enrichment program, math camp or to do math research with a professor or teacher. There is no deadline for applying. For details see

http://www.mualphatheta.org/Scholarships/MAO_Summer_Grants.htm

For information about summer programs:
See information about Summer Internships and Undergraduate Research opportunities at
http://www.phds.org/index.php?section=16

or summer math camp experiences at

http://www.ams.org/employment/mathcamps.html
Also check out your favorite college to see what summer programs might be offered for high school or undergraduates interested in math enrichment.

Chapter Highlight: Quantico Middle/High School, Quantico, VA

We attended our first Region IV competition in December in Salisbury, MD. It was a great experience for our new members. They are also very much involved in promoting math awareness in the school and community by providing math problems, riddles, jokes of the week to tutoring students and instructing students in the Middle school math Olympiad program competitions with practice sessions. Currently, they are running the million pennies project with funds raised going to charity.

The group helped with sponsoring a Geo/Techno Chinese Dragon to celebrate Chinese New Year and supported a math activity on sequences and patterns. In addition to having fun and making lots of noise around the school and community, the middle school students learned about patterns and sequences, moving from concrete objects to abstract, visualizing patterns in daily activities, and learning about the impact of Chaos theory on unstructured and structured patterns Sponsor: Rich Tom

Certificate of Appreciation

Mu Alpha Theta will now offer a Certificate of Appreciation that can be awarded to sponsors, speakers or anyone you wish to recognize for their Outstanding Contributions to Mathematics Education. These will be available for sale for $3 and have a space for the person's name, the date and the Chapter Sponsor's signature of the chapter presenting the certificate.

Convention Hosts and Test Coordinators Needed

Think about hosting a National Convention! Hosts are needed for 2008 and beyond. Mu Alpha Theta will help you organize, but we need sponsors to volunteer to host. At each convention, we also need someone to supervise test writing and proofing. For information about these important jobs, email Susan Hiller at Susie.hiller@bellsouth.net

2005 National High School Student $1000 Calculus Competition

Eligibility Requirements and Application Procedures

Any high school (or pre-high school) student in the United States is eligible to be nominated by a teacher.

The application deadline has been extended. Applications are now due on May 14, 2005.

Entry for National High School Calculus Competition:

Entries should be emailed to: calculusprize@yahoo.com

Use only standard text in your email. Supporting materials will be requested by the judges if needed - please do not mail or attach any such materials. There is no entry fee. (Employees and editors of calculus.org and close relatives may neither nominate candidates nor receive the award.) Go to www.mualphatheta.org for application process.

The Fibonacci Sequence and the Golden Ratio

(by Kay Weiss)

Whenever I start to teach a chapter on Sequences and Series, I send my math students to a wonderful webpage at http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fib.html to explore the Fibonacci Sequence and the Golden Ratio. I tell them to write me three paragraphs discussing how the series is generated, the meaning of the Golden Ratio and something interesting that they learned from this fascinating website. (I'm one of those writing across the curriculum people.) So, when I saw the following article in this month's College Mathematics Journal published by the MAA, one of our sponsoring organizations, I thought you would enjoy seeing it. With permission of the author, what follows are excerpts from the journal article. You can find the complete version scanned in the online edition of this Spring Newsletter at our homepage www.mualphatheta.org. **Teachers, please copy the article for your students or send them online to read it.**

For teachers who wish to explore further, some hands-on activities:

Have students find some snail shells or buy four or five nautilus shells from a seashell supplier and have your students get out their ruler, protractor and calculator. Have them measure or attempt to measure the shells to look at the ratios discussed in this article. Talk about measuring errors and accuracy. Can they verify the ratio found is the Golden Ratio? Have your students construct a regular pentagon with side 1 inch. Let them measure the diagonal and then use trigonometry to verify that the length of the diagonal in figure 10 is indeed the Golden Ratio. There are lots of ways to proceed with this lesson. Use your imagination or some hints about other regular polygons with an odd number of sides as suggested in the article.

With thanks to the Mathematical Association of America 2005 (All rights reserved) and to Clement Falbo.

The Golden Ratio—A Contrary Viewpoint

Clement Falbo

Clement Falbo (clemfalbo@yahoo.com; Sonoma State University, Rohnert Park, CA 94928) was born in San Antonio, Texas, in 1931. He served in the U. S. Navy from 1951 to 1954. He attended the University of Texas in Austin, where he earned his B.A., M.A., and Ph.D. in mathematics. He taught college mathematics for 35 years, mostly at Sonoma State. After his retirement, he and his wife Jean joined the U.S. Peace Corps and taught high school in Zimbabwe for two years. Since then, they have been traveling widely.

Introduction

Over the past five centuries, a great deal of nonsense has been written about the golden ratio, $\Phi = \frac{1+\sqrt{5}}{2}$, its geometry, and the Fibonacci sequence. Many authors make claims that these mathematical entities are ubiquitous in nature, art, architecture, and anatomy. Gardner [4] has shown that the admiration for this number seems to have been raised to cult status. Fortunately, however, there have been some recent papers, including Fischler [2] in 1981, Markowsky [7] in 1992, Steinbach [9] in 1997, and Fowler [3] in 1982, that are beginning to set the record straight. For example, Markowsky, in his brilliant paper "Misconceptions about the Golden Ratio," speaking about Φ, says:

> "Generally, its mathematical properties are correctly stated, but much of what is presented about it in art, architecture, literature and esthetics is false or seriously misleading. Unfortunately, these statements about the golden ratio have achieved the status of common knowledge and are widely repeated. Even current high school geometry textbooks ... make many incorrect statements about the golden ratio. It would take a large book to document all the misinformation about the golden ratio, much of which is simply repetition of the same errors by different authors."

It is remarkable that prior to Fischler's and Markowsky's papers, there seemed to have been no set standards for obtaining measurements of artwork. Often, a proponent of the golden ratio will choose to frame some part of a work of art in an arbitrary way to create the appearance that the artist made use of an approximation of Φ. Markowsky shows an example in which Bergamini [1] arbitrarily circumscribes a golden rectangle about the figure of St. Jerome in a painting by Leonardo Da Vinci, cutting off the poor fellow's arm in order to make the picture fit.

It is frequently asserted that the golden ratio occurs in nature as the shape of spirals in sea shells. We can easily test this claim by first providing a protocol for measuring the spirals. One requirement should be to allow for some error in the measurements. Markowsky [7] suggests an error bar of $\pm 2\%$, which seems to be quite adequate. Measuring under this protocol, we find that spirals in sea shells do not generally fit the shape of the golden ratio. This is true despite the numerous articles on the Internet and elsewhere, in which pictures apparently have been stretched to fit the Φ ratio—"stretching the truth"—so to speak.

The golden ratio is associated with the Fibonacci sequence in a very simple way. The sequence is an example of a quadratic recursive equation sometimes used to describe various scientific and natural phenomena such as age-structured population growth. In order to define the general quadratic recursive formula, let x_0, x_1, p, and q be fixed positive numbers, and for any integer $n \geq 2$, define x_n as

$$x_n = px_{n-1} + qx_{n-2}. \tag{1}$$

Murthy [8] provides a number of theorems for this general recursive equation. It is clear that many of the features that are proclaimed to be unique to the Fibonacci sequence are, indeed, common to all second-order recursive equations. For example,

$$\lim_{n\to\infty} \frac{x_{n+1}}{x_n} = r, \tag{2}$$

where r is the positive root of the quadratic equation

$$x^2 - px - q = 0, \tag{3}$$

obtained by assuming that $x_n = x^n$ is a solution to (1). One of our objectives in this paper is to show that if $q = 1$ and r is the limit in (2), then the pair (r, p) has all of the geometric and algebraic properties that are often ascribed as being unique to the pair $(\Phi, 1)$. For example, we have $r - p = 1/r$ corresponding to the property $\Phi - 1 = 1/\Phi$.

For equation (1) to be useful in describing aged-structured population growth in plants and animals, the coefficients p and q must be determined by some niche or fecundity properties of the organism being studied. In other applications, such as phyllotaxy, we may use a second-order recursive equation such as (1) to predict and explain the evolution of leaf placement on a stem in terms of maximizing the gathering of sunlight. However, we should not expect the complexities of natural systems to yield to the easy-to-compute Fibonacci sequence, and there seems to be no unbiased evidence favoring the Fibonacci sequences over all other possible sequences. If one expends great effort in looking only for this special sequence, then it may be perceived, whether or not it is there. This is succinctly illustrated in terms of statistical analysis by Fischler [2], who shows that careless computations and misused formulas produce the golden number when it isn't there.

In a popular new book, Livio [6] draws upon the information developed in Markowsky and others to discuss the protocol violations that are the source of claims that Φ occurs in classical architecture, such as the Parthenon and the Egyptian pyramids. Livio presents a well-written explanation of misleading claims concerning these classics, as well as various paintings and other art work. Livio calls the advocates for these claims the "golden numberists." It seems, however, that he believes that certain constructions (such as Kepler's triangle) or arithmetic equations (such as a continued fraction representation of Φ) are significant and unique enough for him to subtitle his book "The Story of Phi, the World's Most Astonishing Number."

Origins of the golden ratio

The golden ratio is the solution to a problem given by Euclid (c. 300 BC) in his *Elements*, Book VI, Proposition 30:

To cut a given finite line in extreme and mean ratio.

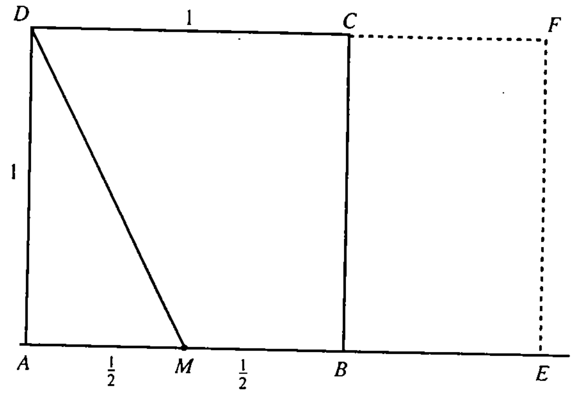

Figure 1. The mean proportion $AE : AB :: AB : BE$

That is, given a segment $\overline{AE}$, find the point B for which $AE/AB = AB/BE$. Euclid had already solved this in a previous theorem (Book II, Proposition 11). We show his construction in Figure 1 in order to generalize it later.

Start with the unit square $ABCD$ and let M be the midpoint of $\overline{AB}$. Construct the line segment $\overline{MD}$. Draw a circle with center at M and radius $\overline{MD}$ so that it cuts $\overrightarrow{AB}$ at the point E. So, $MD = ME$. In modern notation, we have the lengths,

$$ME = \frac{\sqrt{5}}{2}, \quad MB = \frac{1}{2}, \quad \text{and} \quad BE = \frac{\sqrt{5}-1}{2}.$$

Now, since $AE = AB + BE$, or $AE = 1 + BE$, we can write: $AE = (1 + \sqrt{5})/2$. Thus, we can easily show that $AE/AB = AB/BE$.

The length AE, $(1 + \sqrt{5})/2$, is denoted by Φ, and is called the *golden ratio*, or the *divine proportion*. In the above figure, the rectangle $AEFD$ is called the *golden rectangle*. The history of the golden ratio pre-dates Euclid. As early as 540 BC, the Pythagoreans had studied it in their work with the pentagon. We discuss some of the ratios that appear in the pentagon and all other odd polygons later.

The golden ratio Φ can be used to construct a beautiful logarithmic spiral, shown in Figure 2. This graph can be obtained by fitting the polar equation $\rho = be^{c\theta}$ to selected points on the golden rectangle. We can, however, also plot a logarithmic spiral inscribed in a rectangle with a ratio other than Φ. Figure 3 shows a spiral with a 1.33 to 1 ratio.

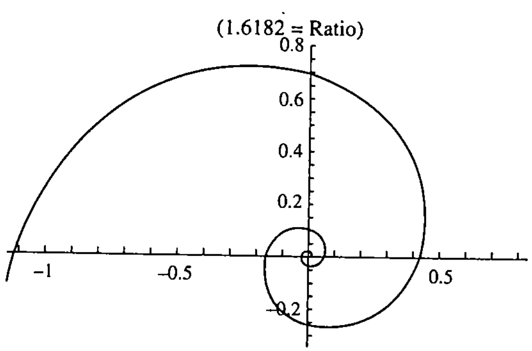

Figure 2. A spiral with the golden ratio

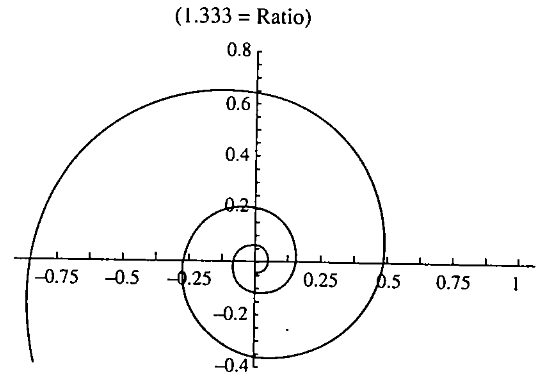

Figure 3. A spiral with a 1.33 to 1 ratio

Spirals in sea shells

Basically, the two types of sea shells that many of us are familiar with are the *cephalopod* (head-foot) and the *gastropod* (stomach-foot), or with apologies to my biologist friends, the octopus and the snail. The nautilus, a cephalopod, is an octopus in a shell that consists of a series of chambers, each sealed from the previous one. This animal lives in the latest chamber with its eight tentacles sticking out. In Figure 4 is a photo that I took of a longitudinal section of a nautilus.

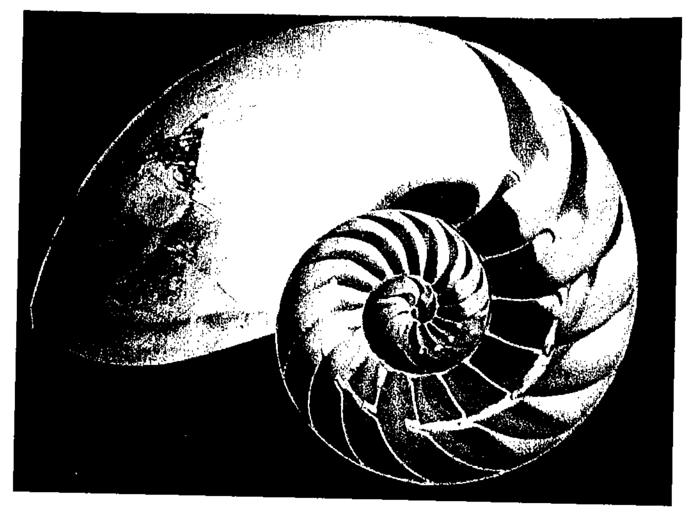

Figure 4. A digital photograph of a longitudinal section of a nautilus

The nautilus is definitely not in the shape of the golden ratio. Anyone with access to such a shell can see immediately that the ratio is somewhere around 4 to 3. In 1999, I measured shells of *Nautilus pompilius*, the chambered nautilus, in the collection at the California Academy of Sciences in San Francisco. The measurements were taken to the nearest millimeter, which gives them error bars of ± 1 mm. The ratios ranged from 1.24 to 1.43, and the average was 1.33, not Φ (which is approximately 1.618). Using Markowsky's $\pm 2\%$ allowance for Φ to be as small as 1.59, we see that 1.33 is quite far from this expanded value of Φ. It seems highly unlikely that there exists any nautilus shell that is within 2% of the golden ratio, and even if one were to be found, I think it would be rare rather than typical.

A ratio of different color, $\sqrt{2}$

A spiral of a different ratio is in the shape of the "silver ratio" $\sqrt{2}$ to 1. (Some authors call $1 + \sqrt{2}$ the silver ratio.) A spiral based on $\sqrt{2}$ is considerably closer to the shape of the nautilus than Φ is (see Figure 5).

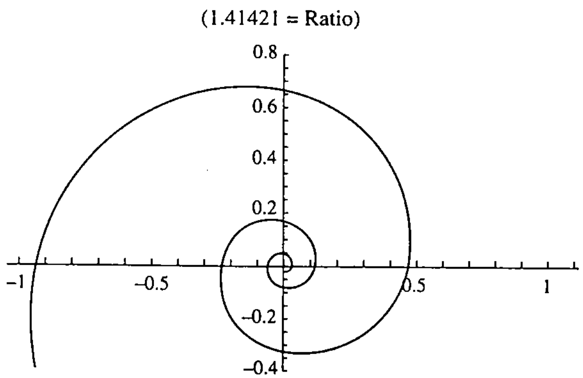

Figure 5. A spiral with the ratio $\sqrt{2}$ to 1

It is well known that the golden rectangle can be subdivided into subrectangles (each similar to the original) forming a kind of "spiral" of smaller and smaller similar rectangles. This interesting property happens to be shared by every rectangle (except the square). For example, if you take a $\sqrt{2}$-by-1 rectangle and cut it in two (in the natural way), you get two rectangles with sides in the same ratio. Continuing this, we get similar rectangles as shown in Figure 6.

(This rectangle was actually used in a field study in biology. In an article in *Science*, Harte, Kinzig, and Green [5] used Figure 6 when they wanted to determine a relation between species distribution and geographical area. They were interested in studying regions of varying sizes, but they wanted to maintain similarity in the shape of the region so as not to introduce a bias against rare species. One of their protocols was to start with a rectangle of known dimensions and then to generate a sequence of smaller similar subrectangles in which to collect samples. They chose the rectangle with the ratio of $\sqrt{2}$ to 1.)

Pentagons, heptagons, nonagons, ...

Regular polygons with an odd number of sides have some interesting properties related to the ratios we have been discussing. Steinbach makes the case for other polygons in his paper; he opens with this:

> "One of the best-kept secrets in plane geometry is the family of ratios of diagonals to sides in the regular polygons. So much attention has been given to one member of this family, the golden ratio Φ in the pentagonal case, that the others live in undeserved obscurity. But the wealth of material that pours from the pentagon—proportional sections, recursive sequences, and quasiperiodic systems—can be matched wonder-for-wonder by any other polygon .... "

It is well known that the golden ratio occurs in the regular pentagon with unit sides. In Figure 10, the length $AC = 2\cos(\pi/5)$, which is Φ.

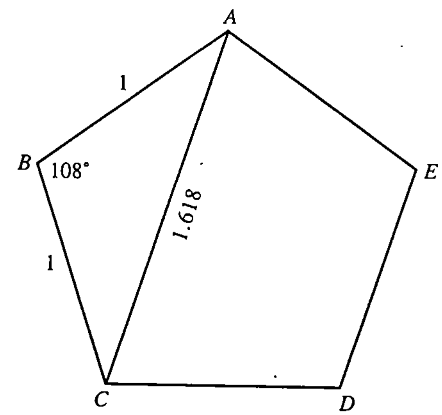

Figure 10. The pentagon and the golden ratio, $AC - AB = 1/AC$

Much of the publicity given to Φ has come from the special place that the pentagon had among the Pythagoreans. In this case, the equation $AC - AB = 1/AC$ says that $\Phi - 1 = 1/\Phi$. But, if we look at other regular odd polygons, we see that some pair of diagonals also satisfy the same relationship. Indeed, it is an interesting exercise to prove the following result.

Let $\mathcal{P}$ be any regular odd polygon with n sides, $n \geq 5$. If r is the length of a longest diagonal of $\mathcal{P}$ and p is the length of a second longest diagonal, then $r - p = 1/r$.

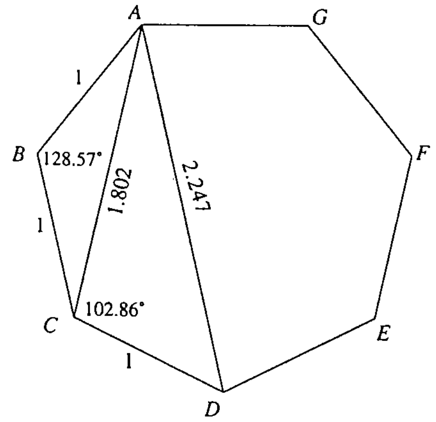

Figure 11. The heptagon and two related ratios. If $AD = r$, then $AC = r - 1/r$

Example. Behold the heptagon! See Figure 11. The diagonals $\overline{AD}$ and $\overline{AC}$ have the relationship described:

$$AD = 4\cos^2(\pi/7) - 1 \approx 2.247 \quad \text{and} \quad AC = 2\cos(\pi/7) \approx 1.802$$

That is, $AD - AC = 1/AD$.

Algebraic properties of $x_n = px_{n-1} + x_{n-2}$

We want to consider a modest generalization of the Fibonacci sequence, equation (4) below. The paper by Murthy presents substantially more general recursive equations that include (4).

Again, let $r > 1$ and $p = r - \frac{1}{r}$. Let $x_0 = 1$, $x_1 = 1$, and for $n \geq 2$, let

$$x_n = px_{n-1} + x_{n-2}. \tag{4}$$

Then the general term of the sequence is

$$x_n = c_1 r^n + c_2\left(-\frac{1}{r}\right)^n, \tag{5}$$

where

$$r = \frac{p + \sqrt{p^2+4}}{2}, \qquad \frac{-1}{r} = \frac{p - \sqrt{p^2+4}}{2},$$

and

$$c_1 = \frac{2 - p + \sqrt{p^2+4}}{2\sqrt{p^2+4}}, \qquad c_2 = \frac{p - 2 + \sqrt{p^2+4}}{2\sqrt{p^2+4}}.$$

Because of the way that p is defined, r and $\frac{-1}{r}$ are the roots of the quadratic equation

$$x^2 - px - 1 = 0. \tag{6}$$

A few terms of this sequence, obtainable from either equation (4) or equation (5), are

$$x_2 = p + 1,$$
$$x_3 = p^2 + p + 1,$$
$$x_4 = p^3 + p^2 + 2p + 1.$$

If $p \neq 1$, this is not Fibonacci. We now point out several properties that any such solution r of (6) would have and that the quadratic equation is sufficient to imbue r with these properties.

Conclusions

We have shown that in certain instances, the claim that the golden ratio has a special place among numbers as a valid description of nature is unsupported. Furthermore, it has been well refuted that this ratio is somehow exceptionally pleasing and that it occurs frequently in art and architecture. For example, from taking measurements, we find that there is no basis for saying that Φ is the ratio that naturally occurs in sea shells. In particular, there is no basis for the assertion that it occurs in the nautilus. I disagree with Livio's implication that Φ is "The World's Most Astonishing Number." The interesting properties of Φ as a positive root to the quadratic equation (6) (with $p = 1$) are matched by the positive root to (6) for any positive number, $p \neq 1$. The wonderful geometric properties of Φ as an extreme mean are matched by the other means discussed here. Also, finding Φ, as $2\cos(\pi/5)$, in the pentagon is exciting, but no more so than finding $4\cos^2(\pi/7) - 1$ in the heptagon. Perhaps, Φ deserves to be included in a list along with e, π, and other numbers because it can be used to simplify certain formulas. In that sense, it is interesting, perhaps even very interesting, but not entirely astonishing.

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