The Sudoku Epidemic

By Robin Wilson

The sudoku craze has become an epidemic. Originating in the US and Japan, and hardly known a year ago, it has since swept the world, infecting much of Europe, Australia and Asia before returning to the US. So what is sudoku, where did it come from, and how does one solve it?

What is sudoku?

Sudoku (or su doku) is a number puzzle, usually consisting of a 9 x 9 grid divided in nine 3 x 3 boxes, into which numbers already appear in a few cells. Here's an example (Puzzle 1):

9					7		1	
	7	4			2		6	5
	1		8	9		4		
2				8		5	9	
1			2		3			6
	4	5		1				2
		7		4	1		2	
3	2		9			8	5	
	6		5					7

Puzzle 1

The object of the puzzle is to complete all the remaining cells with the numbers from 1 to 9, so that:

- *each row contains all the numbers from 1 to 9*
- *each column contains all the numbers from 1 to 9*
- *each 3 x 3 box contains all the numbers from 1 to 9*

The completed pattern is thus a 9 x 9 Latin square with an extra condition on the 3 x 3 boxes.

A well-constructed sudoku puzzle has only one solution which one can find by proceeding in a logical step-by-step manner. The solution to Puzzle 1 is given on page 26, but you may wish to try it for yourself first.

Because of the simplicity of its rules, sudoku has caught the imagination of young and old alike. Its popularity arises partly from the fact that the puzzles can vary in difficulty from the easy to the extremely hard, depending on how many numbers are given and where they're placed, so solvers can choose a puzzle to match their level of ability.

Why 'sudoku'?

Despite its name, sudoku isn't of Japanese origin. Claims have been made that the puzzle can be traced back to Euler in the 18th century, but these claims are incorrect – Euler certainly worked on Latin squares, but never considered the extra condition on the 3 x 3 boxes. In fact, the first known sudoku puzzle appeared in a New York puzzle magazine in 1979, under the name of *Number place*.

The puzzles didn't reach Japan until the mid-1980s. The puzzle company Nikoli saw an American number place problem and introduced it to the readers of their puzzle magazine in 1984 under the name ***Su**nji wa **doku**shin ni kagiru*, shortened to *sudoku* – meaning 'single number' because just one number appears in each cell. They soon became highly popular, and currently some 15 Japanese sudoku magazines and 30 books are published monthly, catering to half a million readers.

In 1997, Wayne Gould, a retired High Court Judge living in Hong Kong, discovered sudoku in a Tokyo puzzle book. Fascinated, he spent the next few years designing computer programs to produce sudoku puzzles of varying difficulty which he then provided for newspapers around the world.

In 2004, Gould sent one to *The Times* in London, who published their first puzzle in November – his puzzles still appear, in four levels of difficulty (easy, mild, difficult, and fiendish). Other newspapers followed suit, and by Easter 2005 a huge cottage industry had developed, with most British newspapers including daily sudoku puzzles and rivalling each other to offer prizes for solutions to puzzles of ever-increasing complexity and originality. These now include 16 x 16 sudoku grids, grids of five linked 9 x 9 sudoku puzzles (none completable on its own), and *The Times* 'killer puzzles' in which the numbers in certain subsets of cells have specified sums. There is even a three-dimensional sudoku puzzle in the form of a 9 x 9 x 9 cube.

In May 2005 the sudoku craze suddenly spread around the world. Within a month sudoku puzzles had appeared in newspapers in Australia, Canada, Israel, Eastern Europe, and India, and they have recently returned to haunt the United States, now appearing in many dozens of American newspapers.

Further historical notes can be found in [**5**].

Solving a sudoku puzzle

Suppose that you are given the sudoku puzzle on the right to complete (Puzzle 2).

We'll call the rows *A*, *B*, . . . , *I*, the columns *a*, *b*, . . .,*i*, and the 3 x 3 boxes ❶,❷, . . . ,❾, proceeding horizontally from top-left (box ❶) to bottom-right (box ❾). We denote cells by their row and column letters – for example, *Cf* is the cell in row *C* and column *f* (containing 9).

There are several techniques that you can use to solve sudoku puzzles, such as: *scanning rows and columns*, *filling in the gaps*, and *filling in single cells*; further details can be found in [**3**]:

Scanning rows and columns

Look first at columns *d*, *e*, *f* of Puzzle 2. In these columns, each number from 1 to 9 must appear three times in total, once in each column and once in each 3 x 3 box; for example, the number 3 appears in cells *Bf* (box ❷ and column *f*), *Ee* (box ❺ and column *e*), and *Gd* (box ❽ and column *d*). It follows that:

- the third appearance of 9 must be in cell *Id*
- the third appearance of 5 must be in cell *Ad*

In the same way, we notice that:

- in rows *G*, *H*, *I*, 5 must be in cell *Ii* (since 5 already appears in column *h*)
- in columns *g*, *h*, *i*, 6 must be in cell *Hi* (since we have just put 5 in cell *Ii*)
- in box ❺ , 6 cannot be in rows *D* or *E*, so must be in row *F*, in cell *Fd*; similarly, 7 must be in cell *Ff*

More subtle is the following:

- in box ❾, 8 is in column *g*; in box ❻, 8 cannot be in column *g* or row *E*, so must be in column *i*, in cell *Di* or *Fi*; thus, in box ❸, 8 must be in column *h*, in cell *Ah*

Filling in the gaps

When rows, columns and boxes are almost complete, we can sometimes fill in the missing entries; for example:

- in column *d*, only 1 is now missing – so put 1 in cell *Dd*
- in box ❺, only 4 is now missing – so put 4 in cell *Df*
- in column *f*, 1 and 2 are missing, but we cannot yet decide where to put them

Filling in single cells

	a	b	c	d	e	f	g	h	i
A			4				6		
B		2		4		3		1	
C	1			8		9			4
D		7			5			6	
E				2	3	8			
F		1			9			5	
G	5			3		6			7
H		9		7		5		2	
I			7				8		

Puzzle 2

Look at cell *Bi*. It cannot contain 1, 2, 3 or 4 (already in row *B*), 5, 6 or 7 (in column *i*) or 8 (in box ❸), so must contain 9; we can then put 9 in cell *Aa*.

Similarly, cell *Ab* cannot contain 4, 5, 6 or 8 (already in row *A*), or 2, 7, 1, 9 (in column *b*), so must contain 3.
Note also that in column *b*, the only place that we can put 8 is in cell *Gb*; we can then put 8 in cell *He*.

Using small numbers

We could continue like this, but another approach is to write in each square the possible numbers that can be inserted. For our puzzle the current position is:

	a	b	c	d	e	f	g	h	i
A	9	3	4	5	1 2 7	1 2	6	8	2
B	6 7 8	2	5 6 8	4	6 7	3	5 7	1	9
C	1	5 6	5 6	8	2 6 7	9	2357	3 7	4
D	2 3 8	7	2389	1	5	4	2 3 9	6	2 3 8
E	4 6	4 5 6	5 6 9	2	3	8	1479	4 7 9	1
F	2348	1	2 3 8	6	9	7	2 3 4	5	2 3 8
G	5	8	1 2	3	1 2 4	6	1 4 9	4 9	7
H	3 4	9	1 3	7	8	5	1 3 4	2	6
I	2346	4 6	7	9	1 2 4	1 2	8	3 4 9	5

Puzzle 2 (continued)

Note that:

- in cells *Ai* and *Ei*, only one number can be inserted, thereby enabling us to fill in further squares
- in box ❶, the cells *Cb* and *Cc* must contain 5 and 6 (in some order), so cell *Bc* must contain 8 and cell *Ba* must then contain 7

Continuing in this way, you should be able to complete the puzzle – the solution is on page 26.

Some mathematics

A number of interesting mathematical problems arise in connection with sudoku. The number of possible 9 x 9 Latin squares is

5,524,751,496,156,892,842,531,225,600;

how many of these are valid sudoku grids? In June 2005, this number was found by Felgenhauer and Jarvis [**1**] to be

6,670,903,752,021,072,936,960.

Later, the number of 'essentially different' grids (once relabelling of the nine symbols and obvious row and column permutations have been eliminated) was found by Russell and Jarvis [**2**] to be 5,472,730,538.

It is also natural to ask for the maximum and minimum number of filled-in cells for a sudoku puzzle to have a unique solution. The maximum number is 78 – if from the solution of Puzzle 2 the 1s and 9s in positions *Aa*, *Af*, *Ca* and *Cf* are erased, leaving 77 filled-in cells, the puzzle cannot be uniquely completed. The minimum number seems to be 17. There are several known examples with 17 filled-in cells which can be completed uniquely, but no-one has found one with just 16 filled-in cells, or proved that none can exist. Further information can be found in [**5**].

Summing up

It only remains to wish you all the best in your sudoku explorations. Solving these puzzles can be very frustrating at times – but it can also be enormous fun! We conclude with a puzzle (Puzzle 3) containing nine letters – if you complete it correctly, a hidden mathematical word will appear in one of the rows or columns; the solution is given on page 26. Further word puzzles can be found in [**4**].

				H	T		G	
T								
		R	M			H		
H			L		I	O		
G								A
		L	T		R			M
		O			H	I		
								L
	M		O	R				

Puzzle 3

Robin Wilson is a Professor of Pure Mathematics at the Open University, Gresham Professor of Geometry in London (the oldest mathematical chair in England, dating from 1597) and a Fellow of Keble College, Oxford University. He has written and edited about thirty books on subjects ranging from graph theory and the history of mathematics to music, philately and sudoku. He is internationally known for his bright clothes and his awful jokes.

References

1. B. Felgenhauer and F. Jarvis, Enumerating possible Sudoku grids, http://www.shef.ac.uk~pm1afj/sudoku/sudoku.pdf.
2. E. Russell and F. Jarvis, There are 5472730538 essentially different Sudoku grids …, http://www.shef.ac.uk/~pm1afj/sudoku/sudgroup.html.
3. R. Wilson, *How to Solve sudoku: A step-by-step guide*, Infinite ideas, 2005.
4. R. Wilson, *Hidden Words sudoku*, Infinite ideas, 2005.
5. A useful website that gives information about all aspects of sudoku and is updated regularly is http://en.wikipedia.org/wiki/Sudoku.