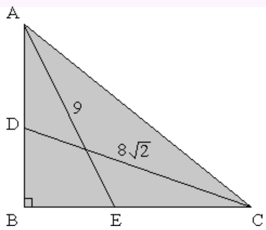
1. If the numbers 2^n and 5^n (where n is a positive integer) start with the same digit, what is this digit? The numbers are written in decimal notation, with no leading zeroes.

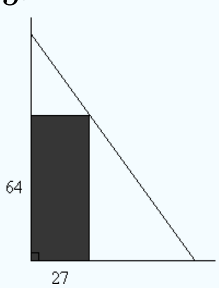
2. At a movie theater, the manager announces that a free ticket will be given to the first person in line whose birthday is the same as someone in line who has already bought a ticket. You have the option of getting in line at any time. Assuming that you don't know anyone else's birthday, and that birthdays are uniformly distributed throughout a 365 day year, what position in line gives you the best chance of being the first duplicate birthday?

3. Triangle ABC is right-angled at B. D is a point on AB such that $\angle BCD = \angle DCA$. E is a point on BC such that $\angle BAE = \angle EAC$. If $AE = 9$ inches and $CD = 8\sqrt{2}$ inches, find AC.



4. The terms of a sequence of positive integers satisfy $a_{n+3} = a_{n+2}(a_{n+1} + a_n)$, for $n = 1, 2, 3, \ldots$. If $a_6 = 8820$, what is a_7?

5.

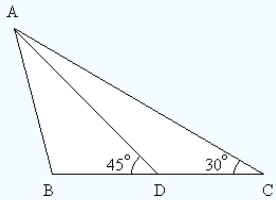


A ladder, leaning against a building, rests upon the ground and just touches a box, which is flush against the wall and the ground. The box has a height of 64 units and a width of 27 units.

Find the length of the ladder so that there is only one position in which it can touch the ground, the box, and the wall.

6. The sum of five real numbers is 7; the sum of their squares is 10. Find the minimum and maximum possible values of any one of the numbers, in standard decimal notation, with no leading zeroes.

7. In $\triangle ABC$, draw AD, where D is the midpoint of BC.



If $\angle ACB = 30°$ and $\angle ADB = 45°$, find $\angle ABC$.

8. The minute hand of a clock is twice as long as the hour hand. At what time, between 00:00 and when the hands are next aligned (just after 01:05), is the distance between the tips of the hands increasing at its greatest rate?

9. Find the area of the largest semicircle that can be inscribed in the unit square.

10. A confused bank teller transposed the dollars and cents when he cashed a check for Ms Smith, giving her dollars instead of cents and cents instead of dollars. After buying a newspaper for 50 cents, Ms Smith noticed that she had left exactly three times as much as the original check. What was the amount of the check? (Note: 1 dollar = 100 cents.)

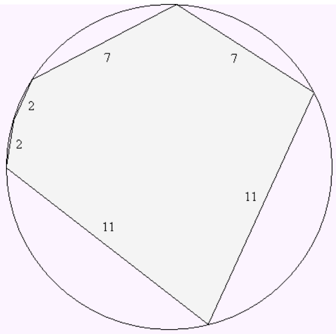
11. The sum of the reciprocals of two real numbers is -1, and the sum of their cubes is 4. What are the numbers?

12. Player A has one more coin than player B. Both players throw all of their coins simultaneously and observe the number that come up heads. Assuming all the coins are fair, what is the probability that A obtains more heads than B?

13. A rhombus, ABCD, has sides of length 10. A circle with center A passes through C (the opposite vertex.) Likewise, a circle with center B passes through D. If the two circles are tangent to each other, what is the area of the rhombus?

14. Find the smallest natural number greater than 1 billion (10^9) that has exactly 1000 positive divisors. (The term *divisor* includes 1 and the number itself. So, for example, 9 has three positive divisors.)

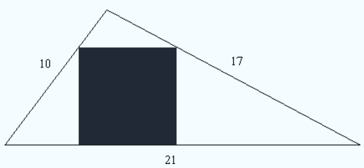
15. A hexagon with consecutive sides of lengths 2, 2, 7, 7, 11, and 11 is inscribed in a circle. Find the



radius of the circle.

16. A car travels downhill at 72 mph (miles per hour), on the level at 63 mph, and uphill at only 56 mph The car takes 4 hours to travel from town A to town B. The return trip takes 4 hours and 40 minutes. Find the distance between the two towns.

17. A triangle has sides 10, 17, and 21. A square is inscribed in the triangle. One side of the square lies on the longest side of the triangle. The other two vertices of the square touch the two shorter sides of the triangle. What is the length of the side of the square?



18. There are three Athletes (Alex, Brook and Chris) and their individual Coaches (Murphy, Newlyn and Oakley) standing on the shore. No Coach trusts their Athlete to be near any other Coach unless they are also with them. There is a boat that can hold a maximum of two persons. How can the six people get across the river?

19. Find all integer solutions of $y^2 = x^3 - 432$.

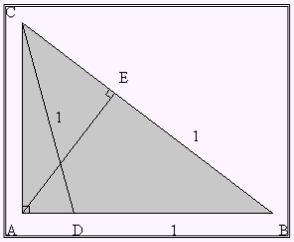
20. The sum of three numbers is 6, the sum of their squares is 8, and the sum of their cubes is 5. What is the sum of their fourth powers?

21. Find the smallest positive integer such that when its last digit is moved to the start of the number (example: 1234 becomes 4123) the resulting number is larger than and is an integral multiple of the original number. Numbers are written in standard decimal notation, with no leading zeroes.

22. Evaluate 2^{2004} (modulo 2004).

23. Solve the equation $\sqrt{4+\sqrt{4-\sqrt{4+\sqrt{4-x}}}} = x$

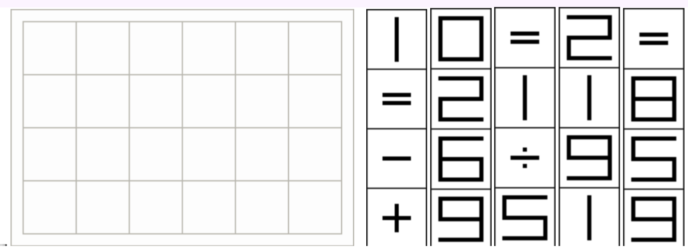
24. $\triangle ABC$ is right-angled at A. D is a point on AB such that $CD = 1$. AE is the altitude from A to BC. If $BD = BE = 1$, what is the length of AD?



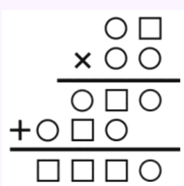
25. A sequence of positive real numbers is defined by

- $a_0 = 1$, $a_{n+2} = 2a_n - a_{n+1}$, for $n = 0, 1, 2, \ldots$. Find a_{2005}.

26. Arrange the 5 vertical strips within the rectangle above so that each row contains a true equation. Strips can be rotated 180°.



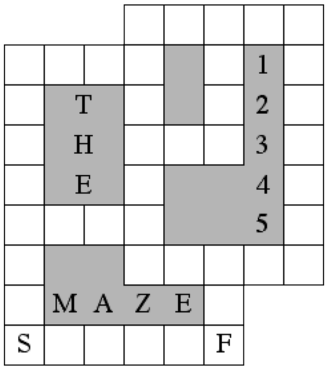
27. Place a digit in each circle and square so that the multiplications are correct. Each circle should contain an even digit, and each square should contain an odd digit. This puzzle has a unique solution.



28. Add some digits before or after the digits in the grid so that each row and column sums to 100.

3	9	8
3	9	6
7	2	1

29. This is the 12345 maze, in which you start on S. You can move 1 square in any direction, followed by 2 squares in any direction, then 3, then 4, then 5, and back to 1 again until you land on F.



30. Everyone knows that $2^4 = 4^2$. but hardly anyone knows $2^5 + 2^7 + 2^9 + 5^3 + 5^4 = 5^2 + 7^2 + 9^2 + 3^5 + 4^5$. Find two more equations like this: each side being the sum of single digits to a single digit power, with no terms repeated.

31. Using each of the following collections of 5 digits, along with the usual arithmetic signs: + − × / ^ ( ), make a total of 2008.
{1,3,8,8,8}, {1,4,4,4,8}, {2,3,8,8,8}, {2,5,9,9,9}, {2,7,8,8,8},
{2,8,8,8,9}, {3,4,8,8,8}, {4,4,4,8,9}, {4,5,8,8,8}, {5,6,6,6,8}.

32. What is the only positive integer n with the property that together $4n$ and $5n$ use each digit 1-9 exactly once?

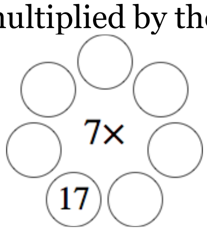
33. 722855133 has the peculiar property that all of its length 3 substrings are divisible by 19. Find the largest number so that all length 2 substrings are distinct and divisible by 19. Find the largest number so that all length 4 substrings are distinct and divisible by 19.

34. Find a 4×4 matrix whose entries are distinct positive integers less than 30 whose row sums are the same, and whose column products are the same.

35. A number that is the reverse of the sum of its proper substrings is 941, since 94 + 41 + 9 + 4 + 1 = 149. Find **other numbers** with this property.

36. Use the operations: + − × / ^ (and) to the digits 9,8 7 6 5 4 3 2,1 to make the result equal to 2009.

37. Put positive numbers in each small circle so that moving clockwise each number is either multiplied by the center number, or has one digit removed.



38. Each empty white square in the grids below contains one of the numbers 1, 2, 3, . . . up to n, where n is the number of empty squares. Each of the horizontal and vertical equations should be true. Each number will be used exactly once.

	+		−		=	
+	■	÷	■	+	■	■
	+		−		=	
−	■	÷	■	+	■	■
	×		×		=	
=	■	=	■	=	■	■
	■		■		■	■

39. 

How much does the baby weigh if the mother weighs 100 pounds more than the combined weight of the baby and the dog, and the dog weighs 60 percent less than the baby? (the scale reads 170).

40. A balloon propelled by some mechanical device travels five miles in ten minutes with the wind, but requires one hour to go back again to the starting point against the wind, how long would it have taken to go the whole ten miles in a calm, without any wind?

41. In how many ways can 10 different marbles be placed in 3 different urns if each urn must contain at least one marble?

42. 1. 26 L of the A	15. 3 W on a T	29. 64 S on a C B
2. 7 D of the W	16. 100 C in a D	30. 9 P in S A
3. 7 W of the W	17. 11 P in a F (S) T	31. 6 B to an O in C
4. 12 S of the Z	18. 12 M in a Y	32. 1000 Y in a M
5. 66 B of the B	19. 13 is U F S	33. 15 M on a D M C
6. 52 C in a D (W J)	20. 8 T on an O	34. 10 N on a T
7. 13 S in the U S F	21. 29 D in F in a L Y	35. 50 S on the A F
8. 18 H on a G C	22. 27 B in the N T	36. 60 D in a S
9. 39 B of the O T	23. 365 D in a Y	37. 16 C in a G
10. 5 D on a F	24. 13 L in a B D	38. 20 Y in a S
11. 90 D in a R A	25. 52 W in a Y	39. 4 P in a B
12. 3 B M (S H T R)	26. 9 L of a C	40. 6 C in a S
13. 32 is the T in D F at which W F	27. 60 M in an H	41. 7 S in the B D
14. 15 P in a R T	28. 23 P of C in the H B	42. 8 L and 8 E has a S

43. If r is the remainder when each of the numbers 1059, 1417, 2312 is divided by d, where d is an integer greater than 1, find the value of $d + r$.

45. What positive number(s) x satisfy the equation $(log_a x)(log_x b) = log_a b$?

46. In an acute triangle with sides of lengths a, b, and c, $(a + b + c)(a + b - c) = 3ab$ Find the measure of the angle opposite the side of length c.

47. Find the sum of the 17^{th} powers of the 17^{th} roots of the equation $x^{17} - 3x + 1 = 0$.

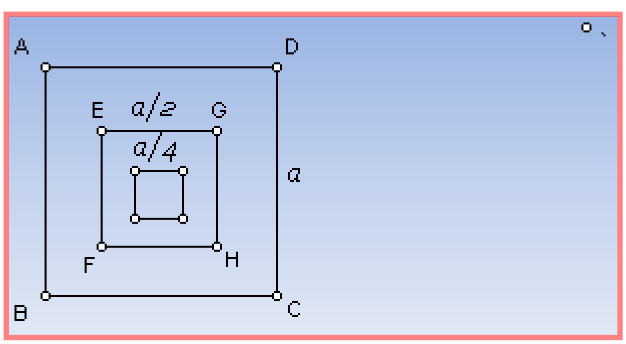
48. Find all ordered pairs of positive integers (x, y) that satisfy: $x^2 + x + 29 = y^2$.

49. Ardith, Crystal, Francine, Gilbert, and Herbert, whose last names are Evermore, Jinson, Miller, Namursky, and Tomlin, have different house numbers (1020, 1030, 1045, 1055, 1070) and live on streets whose first word is "Park" (Boulevard, Drive, Lane, Road, Street). The universities they attended(one for each person) are California Institute of Technology(CIT), Michigan State University (MSU), Texas Christian University (TCU), University of Utah (UU) and Yale (Y).

Read the clues and match everything

1. Crystal and Jinson have never been farther west than Oklahoma. 2. Herbert has never been further south than Kansas 3. The person who lives on Park Drive, who is not Gilbert, has a higher house number than Namursky and a lower house number than the person who attended UU. 4. Herbert whose house number is 15 higher Than Namursky's does not live on Park Drive, neither does Jinson	5. Miller has a lower house number than Gilbert but a higher house number than Tomlin, who did not attend MSU. 6. Neither Francine nor the person who attended MSU lives on Park Street, and the person who attended UU doesn't either. 7. The house number on Park Road is higher than that on Park Lane but lower than the house number of the person that attended TCU.

50.



A square ABCD, of side a, is cut out of paper. Another square EFGH is cut out of paper with side length $a/2$ and placed on the square ABCD. A third square is cut out of paper with side length $1/2$ that of square EFGH. <u>Assume that this pattern can be repeated indefinitely</u>. Find:

1. the total perimeter of all the squares

2. the total area of all the squares.