2009 Individual Theta Test Solutions

Solutions:

1. B

$$\left(\frac{1}{9}+\frac{1}{4}\right)^{-2} = \left(\frac{13}{36}\right)^{-2} = \left(\frac{36}{13}\right)^{2} = \frac{1296}{169}$$

2. A

	3	7	5	7	Remainder = 1
-2		-6	-2	-6	
	3	1	3	‖ 1	

3. A $64^x = ½ \rightarrow 2^{6x} = 2^{-1} \rightarrow 6x = -1 \rightarrow x = -1/6$

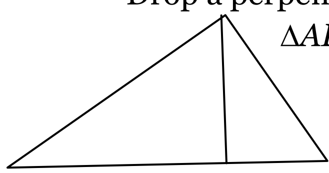
4. A $P = VF = -5$ units vertically and vertex = (-1, -4) so $y = -\frac{1}{20}(x+1)^2 - 4$

5. B 2009 ÷ 4 gives remainder of 1 so it is equal to $\boldsymbol{i}^1$ or $\boldsymbol{i}$

6. D $F(3) = 2(3)^2 = 18$ AND $G(18) = 18 + 5 = 23$

7. A $(1+\boldsymbol{i})^8 = [(1+\boldsymbol{i})^2]^4 = [2\boldsymbol{i}]^4 = 16$

8. C Drop a perpendicular from A to $\overline{BC}$ which forms two special right triangles.



ΔADC is 45-45-90 with $\angle C = 45°$ and AC = 12 so DC = AD = $6\sqrt{2}$

ΔBDA is a 30-60-90 with $\angle B = 30°$ and since

AD = $6\sqrt{2}$, BD = $6\sqrt{6}$.

Putting them together C gives $6\sqrt{2+6\sqrt{6}}$ for BC.

9. C To find the inverse, switch x and y and solve for y, so $x = \frac{2}{3}y - \frac{1}{2} \Rightarrow$

$$x + \frac{1}{2} = \frac{2}{3}y \rightarrow y = \frac{3}{2}x + \frac{3}{4}$$

10. B The new width is 1.6, or $\frac{8}{5}$, times the old width. to keep the area the same the new length must be multiplied by $\frac{5}{8}$. That is a reduction of $\frac{3}{8}$, or 37.5%.

11. D Isolate the radical, giving $\sqrt{x-3} = x - 3$. Square both sides: $x - 3 = x^2 - 6x + 9$. Solve this quadratic: $x^2 - 7x + 12 = 0$. Factors to $(x-3)(x-4) = 0$ Giving roots 3 and 4. Check the roots in the original equation: 0 – 3 = -3 and 1 – 4 = -3 are both true.

12. A Using heron's formula, $S = ½(5 + 10 + 13) = 14$

Area = $\sqrt{14(14-5)(14-10)(14-13)} = \sqrt{14 \cdot 9 \cdot 4 \cdot 1} = 6\sqrt{14}$

13. D — using sum AND product of roots, the quadratic whose roots are $1 + 2\boldsymbol{i}$ and $1 - 2\boldsymbol{i}$ Is $x^2 - 2x + 5$. the resulting polynomial is $(x - 2)(x^2 - 2x + 5) = 0$ and when multiplied out is $x^3 - 4x^2 + 9x - 10 = 0$.

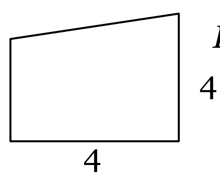
14. B — $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$ so $x^2 - xy + y^2 = 7$. $(x + y)^2 = x^2 + 2xy + y^2 = 16$ subtracting those two equations: $3xy = 9$ so $xy = 3$.

15. B — The area of the square is 10^2 or 100. the rhombus has an altitude of $5\sqrt{3}$ since the altitude makes a 30-60-90 triangle whose hypotenuse is 10. The rhombus area is $50\sqrt{3}$. rhombus : square = $50\sqrt{3} : 100 = \sqrt{3} : 2$

16. B — using the $d = \dfrac{3(5)+4(1)-4}{\pm\sqrt{9+16}}$; $d = 3$

17. D — Powers of 8 end in a pattern of 8, 4, 2, and 6. Powers of 3 end in a pattern of 3, 9, 7, and 1. Powers of 7 end in a pattern of 7, 9, 3, and 1. Since 2009 divided by 4 has a remainder of one, the units digits are 8, 3, and 7 which adds up to 18.

18. B — $V = \frac{1}{3}\pi h(r^2 + rR + R^2)$ where r and r are the radii of the bases of the frustum.



$R = 1$ and $R = 4$ and $H = 4$; $V = \frac{1}{3}\pi \cdot 4(1 + 4 + 16) = 28\pi$

19. A — The line through (1, 1) and (5, 4), the corners of the trapezoid that is revolved, is $y + 1 = \frac{3}{4}(x - 1)$. The vertex of the cone will be the point where this line intersects the x-axis. That point is (-1/3, 0).

20. D — $$\frac{\frac{x^2-9}{6x-12}}{\frac{x+3}{3x^2+3x-18}} = \frac{(x-3)(x+3)}{6(x-2)} \cdot \frac{3(x+3)(x-2)}{x+3} = \frac{(x-3)(x+3)}{2}$$

21. C — $16(x^2 - 6x + 9) + 9(y^2 + 4y + 4) = -36 + 144 + 36 = 144 \rightarrow \frac{(x-3)^2}{9} + \frac{(y+2)^2}{16} = 1$ so $a = 4$ and $b = 3$. The area of an ellipse is πab, or 12π.

22. C — The sequence pattern is $1^{-6}, 2^{-5}, 3^{-4}, 4^{-3}, 5^{-2}, 6^{-1}, 7^0$ so 3^{-4} is the missing term.

23. D — Two conditions can make the expression equal 1: the base is 1 or the exponent is 0. $x^2 + 6x + 6 = 1$ when x = -1 or -5 and $x^2 - 7x + 6 = 0$ when $x = 6$ or 1 The sum of the solutions is (-1) + (-5) + 6 + 1 = 1.

24. A If $3 - 4\boldsymbol{i}$ is a root, then $3 + 4\boldsymbol{i}$ must be a root and a quadratic factor is $x^2 - 6x + 25$. including the 3rd root, r, we get $(x - r)(x^2 - 6x + 25) = x^3 + ax + b$. Since the quadratic term is missing, $-6x^2 - rx^2 = 0$ and $r = -6$.
$y = (x + 6)(x^2 - 6x + 25) = x^3 - 11x + 150$ gives $a = -11$ and $b = 150$
so $\frac{-11 \cdot 150}{25} = -66$.

25. B $S = \frac{10}{1 - 0.5} = \frac{10}{0.5} = 20$

26. D $|A| = 0 + 60 - 4 - 0 + 24 + 10 = 90$

27. C Using exponent rules: $9^{2.3-(-1.8)+7.6} = 9^{11.7}$ $(9^{11.7})^{\frac{1}{5}} = 9^{2.34}$

28. B Let $x = \sqrt{12 + \sqrt{12 + \sqrt{12 + \cdots}}}$.

Squaring both sides gives $x^2 = 12 + \sqrt{12 + \sqrt{12 + \sqrt{12 + \cdots}}}$ or $x^2 = 12 + x$
solving $x^2 - x - 12 = 0$ gives the roots $x = 4$ and $x = -3$.

29. C $\log_3(x + 2) + \log_3(x - 4) = 3 \quad \Rightarrow \quad \log_3(x^2 - 2x - 8) = 3$
so, $x^2 - 2x - 8 = 3^3 = 27 \rightarrow x^2 - 2x - 35 = 0 \rightarrow x = 7$ or $x = -5$
since we cannot take the log of a negative number, only 7 will check.

30. B $a_{13} = 97$ means $a_1 + 12d = 97$
$S_{25} = \frac{25}{2}[a_1 + a_{25}] = \frac{25}{2}[2a_1 + 24d] = \frac{25}{2}[2(a_1 + 12d)] = \frac{25}{2}[2 \cdot 97] = 2425$

TIEBREAKERS

1. $y = \sqrt[3]{x+6} + 7$ Switching x and y $\rightarrow$ $x + 6 = (y - 7)^3 \rightarrow y - 7 = \sqrt[3]{x+6} \rightarrow$
$y = \sqrt[3]{x+6} + 7$

2. Symmetry about the origin or "if $f(a) = b$, then $f(-a) = -b$."

3. $4\sqrt{6}$ difference of focal radii $= 2a = 2(2\sqrt{6}) = 4\sqrt{6}$