1. E – By the Sieve of Eratosthenes, we know that to find the lowest prime that divides a given number, it is sufficient to check all of the primes who square is less than the number. One can check that 13 and 17 do not divide 703. Additionally, $29^2 = 841 > 703$, so if 703 is not prime, then it is divisible by a number less than 29. Therefore, either 703 is divisible by a prime less than 29 that is not 13 or 17, in which case the answer is E, or 703 is prime, in which case the answer is again E.
2. C – Note that $(1+i)^2 = 1 + 2i + i^2 = 1 + 2i - 1 = 2i$, so $(1+i)^{12} = ((1+i)^2)^6 = (2i)^6 = 2^6 i^6 = -64$.
3. C – Since $A^{-1}A = I$, where I is the identity matrix, $A^{-1}AX = IX = X$. Therefore, multiplying both sides of the equation by A^{-1} gives us $X = A^{-1}B$. Since $A^{-1} = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$, this means that $X = A^{-1}B = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & -5 \\ -1 & 3 \end{bmatrix}$.
4. B – By De Moivre's Theorem, $z\left(\frac{5}{4}\right) = \left(\text{cis}\frac{3\pi}{5}\right)^{\frac{5}{4}} = \text{cis}\left(\frac{5}{4}\right)\left(\frac{3\pi}{5}\right) = \text{cis}\frac{3\pi}{4} = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$.
5. E – The answer is "go bucs". Since l is the 12$^{\text{th}}$ letter of the alphabet, $f(l) = 12$, so $f(l) - 5 = 7$. Since g is the 7$^{\text{th}}$ letter of the alphabet, we see that $f^{-1}(f(l) - 5) = f^{-1}(7) = g$. Similarly, we can compute the other values to get the result.
6. C – To find the greatest common divisor of 483 and 989, the quickest way is most likely to use the Euclidean algorithm. Since 989 = 2(483) + 23 and 483 = 21(23), the Euclidean algorithm tells us that 23 is the greatest common divisor of the two numbers.
7. A – Although one could solve this problem in numerous ways, one way to do so is to reduce the equations to reduced row echelon form. The matrix for the system of equations will look like $\left(\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 1 & 2 & -3 & 0 \\ -1 & -10 & 11 & -12 \end{array}\right)$. Subtracting the first row from the second row and adding it to the third row give us the matrix $\left(\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & 2 & -2 & -2 \\ 0 & -10 & 10 & -10 \end{array}\right)$. Multiplying the second row by $\frac{1}{2}$ and the third row by $\frac{1}{10}$ gives us $\left(\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & -1 & 1 & -1 \end{array}\right)$. Adding the second row to the third gives us $\left(\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & -2 \end{array}\right)$, which means that there is no solution since we have a zero row on the left equal to a non-zero entry.
8. B – Although there are many algorithms to solve this, there are also only 5 possible paths to F since one must follow the edges in the direction of the arrows. Therefore, it is easy to compute all 5 paths and see that the path from A to C to E to D to F, which has weight 3+5+1+2=11, has the smallest weight.
9. A – For Cramer's rule, one makes a matrix D whose entries are the coefficients of the x, y, and z terms. Additionally, to find x, one makes a matrix D_x whose entries are the same as D except that the column of the coefficients of the x terms are replaced by a column of the constants. Then x = (det D_x)/(det D). In other words, A is the correct choice here.
10. C – Since there is one change in signs of the coefficients, Descartes' rule of signs tells us that there is one positive real solution to the equation. When we plug in $-x$ for x into the equation, we get $0 = 10x^6 + 4x^4 + 13x^3 + 10x - 5$, so there is one change in the signs of the

coefficients. Again, by Descartes' rule of signs, this means that there is one negative real solution. Clearly, 0 is not a solution, so there are exactly two real solutions.

11. D – By the Fundamental Theorem of Algebra, every polynomial has the same number of complex solutions as its degree. In other words, since this is a sixth degree polynomial, it has 6 complex solutions.
12. A – Since $\lambda\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}-\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}=\begin{bmatrix}\lambda-\cos\theta & \sin\theta\\ -\sin\theta & \lambda-\cos\theta\end{bmatrix}$, its determinant is $(\lambda-\cos\theta)^2+\sin^2\theta=\lambda^2-2\lambda\cos\theta+\cos^2\theta+\sin^2\theta=\lambda^2-2\lambda\cos\theta+1$. We know that a quadratic has two unique solutions if and only if its discriminant is non-zero. The discriminant of this polynomial is $(2\cos\theta)^2-4=4\cos^2\theta-4$, which is zero if and only if $\cos\theta=\pm1$. In the interval $[0,2\pi)$, this means that $\theta=0,\pi$.
13. B – To find a_2, we use the recursive formula to see that $a_2=\frac{2a_1}{1}=2$. Similarly, $a_3=\frac{3a_2}{2}=3$. Therefore, it is easy to see that in general $a_n=n$.
14. D – Since there are 15 numbers, 11 is the middle number. This is greater than 7, so we now choose the first half of the list to try to find 7. In other words, we now restrict our search to the list -10, -8, -7, -2, 0, 4, 7. Since there are 7 numbers in this list, -2 is the middle number. Since it is less than 7, we now look at the list to the right of -2. In other words, the list 0, 4, 7. 4 is the middle number now, which is less than 7, so we look at the list to the right of 4, which is the singleton 7. 7 is the middle number now, so we have found it on the fourth iteration.
15. C – Since Zach chose three of the numbers and there are 7 numbers total, the probability that his number was one of the three is simply $\frac{3}{7}$.
16. D – It is easy to check that any number can be found within three iterations of the binary search algorithm. For example, if $x=25$, then we would first choose the middle of the list 20. Since 20 < 25, we would look at the right list, which has middle number 31. Since 31 > 20, we would then look at the left list, which now has the singleton 25.
17. B – One can simply compute up to the 11th value using the recursive definition of the Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89
18. C – $\begin{vmatrix}3 & -4 & 2\\ 0 & -1 & -2\\ 1 & -1 & 3\end{vmatrix}=3\begin{vmatrix}-1 & -2\\ -1 & 3\end{vmatrix}-(-4)\begin{vmatrix}0 & -2\\ 1 & 3\end{vmatrix}+2\begin{vmatrix}0 & -1\\ 1 & -1\end{vmatrix}=3(-5)+4(2)+2(1)=-5$
19. B – One could convert from base 2 to base 10 and then base 16. However, it is also known that for converting from base 2 to base 16, one can just convert every group of 4 digits of the base 2 number into its respective base 16 number. In other words, since the base 2 number 1001 is just 9 in base 16 and the base 2 number 0100 is just 4 in base 16, 10010100 in base 2 is the same as 94 in base 16.
20. E – Letting $\frac{-1}{x^2(x-1)}=\frac{A}{x^2}+\frac{B}{x}+\frac{C}{x-1}$, we see that this is equivalent to $-1=A(x-1)+Bx(x-1)+Cx^2$. By combining like terms in this equation, we see that $-1=(B+C)x^2+(A-B)x-A$. Therefore, setting the constant terms equal, we get that $A=1$. By setting the x terms equal, we see that $A-B=0\to B=A=1$. Setting the x^2 terms equal, we see that $B+C=0\to C=-B=-1$. Therefore, $\frac{-1}{x^2(x-1)}=\frac{1}{x^2}+\frac{1}{x}-\frac{1}{x-1}$.
21. A – Since p is true, $\neg p$ is false. Since q is false, this means that $\neg p \vee q$ is false. Therefore, without even evaluating the second expression, we can determine that $((\neg p\ \vee q)\ \wedge\ (p\ \vee$

$\neg q))$ must be false. Additionally, since this entire expression is false, we can determine that $((\neg p \vee q) \wedge (p \vee \neg q)) \Rightarrow q$ must be trivially true.

22. A – By the definition of the binary operation, $\pi * e = \pi + e^{i\pi} = \pi - 1$.
23. A – By the chart, we see that $g(1) = 0$. Again, by the chart, we see that $g(0) = -1$. Since f is an odd function and $f(1) = 4$, $f(-1) = -4$. Since g is even and $g(4) = -5$, $g(-4) = -5$. Therefore, again using the fact that f is odd and that $f(5) = 12$, $f(-5) = -12$.
24. E – First, note that $s(65536) = 6 + 5 + 5 + 3 + 6 = 25$. Therefore, $s^2(65536) = s(s(65536)) = s(25) = 2 + 5 = 7$. Additionally, $s(7) = 7$, so applying s any additional number of times will just return 7.
25. D – $|3 - 4i| = \sqrt{3^2 + (-4)^2} = \sqrt{25} = 5$.
26. B – Two vectors are perpendicular if and only if their dot product is 0. The dot product of the two vectors is $x + 6 - 2 = x + 4$, which is 0 exactly when $x = -4$.
27. B – One may use a greedy algorithm to determine the answer. In other words, we can determine the answer by just taking the largest number of possible coins for the largest valued coin still remaining at each step. For example, we may take up to three quarters and still have no more than $0.87. Additionally, our next highest valued coin is the dime, which we can take one more to get to the value of $.85. We cannot take any nickels since that would bring us to 90 cents. Finally, we may obtain two more pennies to get to $0.87. For this problem, the greedy algorithm produces the best possible result. In other words, the three quarters, the one dime, and the two pennies are the fewest possible number of coins that we can use to get to $.87.
28. D – Using the recursive definition, we see that the third point that Julia goes to will have a first coordinate of -2+1 = -1. Then the next point will have a first coordinate of -1-2 = -3. The next one will have a third coordinate of -3-1=-4, and so on. It is clear at this point that all the first coordinates from this point on will be negative since the last two first coordinates were negative. Therefore, the first coordinate can never be positive, so Julia can never reach the origin.
29. D – Since the first coordinates of Evan's first two points are simply 0, the first coordinate of the third point to which he will travel is 0+0=0. Additionally, since the second coordinate of his first point is -1 and the second coordinate of his second point is 1, the second coordinate of his next point is $-\sin\frac{\pi(1-(-1))}{4} = -\sin\frac{\pi}{2} = -1$. Therefore, his third point is (0,-1). Similarly, for his fourth point, we see that its first coordinate is 0 since the first coordinate of his second and third points was also 0. Additionally, since the second coordinate of his second point is -1 and the second coordinate of his third point is -1, the second coordinate of his fourth point is $-\sin\frac{\pi(-1+1)}{4} = 0$. Therefore, his fourth point is (0,0). Evan's points degenerate after that in that they contain radicals that end up being a part of the coefficient of pi in the y-coordinate formulation.
30. A – Since the first coordinates of Will's first two points are 0, the first coordinate will always remain 0 by the recursive definition. Additionally, for any number θ, $|e^{i\theta}| = |\text{cis}\,\theta| = \sqrt{\cos^2\theta + \sin^2\theta} = 1$, so independent of the y_1 and y_2, the recursive formula gives us that the second coordinate of the remaining points will be 1. Therefore, the recursive formula tells Will o go to (0,1) for the remaining points. Therefore, including the first two points that Will visits, this shows that Will only have visited the points (0,-1) and (0,1) for the entire time.