For all questions, NOTA means None Of These Answers.

(1) Which of the following vectors is **not** perpendicular to $3\hat{\imath} + 2\hat{\jmath} - \hat{k}$?

(a) $\hat{\imath} + \hat{\jmath} + 5\hat{k}$ (b) $-\hat{\imath} + \hat{\jmath} - \hat{k}$

(c) $\hat{\imath} - \hat{\jmath} - \hat{k}$ (d) $\hat{\imath} - \hat{\jmath} + \hat{k}$ (e) NOTA

(2) What is the center of the conic described by the equation $9x^2 + 4y^2 - 90x + 16y + 205 = 0$?

(a) $(5,-2)$ (b) $(5,2)$

(c) $(-5,-2)$ (d) $(-5,2)$ (e) NOTA

(3) What type of curve is described by the polar equation $r(\theta) = \frac{6}{3+2\cos(\theta)}$?

(a) Lemniscate (b) Hyperbola

(c) Parabola (d) Ellipse (e) NOTA

(4) What is the area enclosed by the quadrilateral with vertices at the Cartesian points $(1,0)$, $(2,2)$, $(0,3)$, and $(-1,-1)$?

(a) 6 (b) 2
(c) 4 (d) 12 (e) NOTA

(5) A plane going through the Cartesian points $(1,1,1)$, $(0,2,1)$, and $(1,0,2)$ also goes through the point $(3,1,Q)$. Find Q.

(a) 0 (b) -1

(c) 1 (d) 2 (e) NOTA

(6) What type of curve is defined by the polar equation $r(\theta) = 1 - 2\sin^2(3\theta)$?

(a) Limaçon with Inner Loop (b) Rose with Three Petals

(c) Lemniscate (d) Rose with Six Petals (e) NOTA

(7) A conic section is described by the equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$. This equation can be re-written using a matrix; the resulting equation is of the form $[x \quad y \quad 1]\boldsymbol{M}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = [0]$, where $\boldsymbol{M}$ is a 3×3 matrix. Which of the following matrices is $\boldsymbol{M}$?

(a) $\begin{bmatrix} A & \frac{D}{2} & \frac{E}{2} \\ \frac{D}{2} & B & \frac{F}{2} \\ \frac{E}{2} & \frac{F}{2} & C \end{bmatrix}$ (b) $\begin{bmatrix} A & \frac{B}{2} & \frac{C}{2} \\ \frac{B}{2} & D & \frac{E}{2} \\ \frac{C}{2} & \frac{E}{2} & F \end{bmatrix}$

(c) $\begin{bmatrix} A & \frac{E}{2} & \frac{D}{2} \\ \frac{E}{2} & C & \frac{B}{2} \\ \frac{D}{2} & \frac{B}{2} & F \end{bmatrix}$ (d) $\begin{bmatrix} A & \frac{B}{2} & \frac{D}{2} \\ \frac{B}{2} & C & \frac{E}{2} \\ \frac{D}{2} & \frac{E}{2} & F \end{bmatrix}$ (e) NOTA

(8) Vectors $\vec{v}$ and $\vec{w}$ are such that $\vec{v} \cdot \vec{w} = 5$ and $\|\vec{v} \times \vec{w}\| = 2$. If θ is the acute angle between $\vec{v}$ and $\vec{w}$, find $\sin(2\theta)$.

(a) $\frac{2}{5}$ (b) $\frac{2\sqrt{29}}{29}$

(c) $\frac{20}{29}$ (d) $\frac{5\sqrt{29}}{29}$ (e) NOTA

(9) A parabola has a latus rectum that corresponds to one of the latera recta of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$. What is the minimum possible distance between the directrix of the parabola and the center of the ellipse?

(a) $\frac{11}{5}$ (b) $\frac{18}{5}$

(c) $\frac{31}{10}$ (d) $\frac{9}{10}$ (e) NOTA

(10) Which of the following is the equation of a degenerate conic?

(a) $x^2 + xy + y^2 + x + y + 1 = 0$ (b) $xy - 1 = 0$

(c) $x^2 + xy + y^2 - 1 = 0$ (d) $x^2 + 3xy + 2y^2 + y - 1 = 0$

(e) NOTA

(11) Tyger accidently plots the polar point $\left(\pi, \frac{2\pi}{3}\right)$ as a Cartesian point. How far away is the point he plotted from the point he should have plotted?

(a) $\left(\frac{13}{6} - \frac{\sqrt{3}}{2}\right)\pi$ (b) $\frac{\pi}{3}\sqrt{17 - 6\sqrt{3}}$

(c) $\frac{\pi}{3}\sqrt{31 - 6\sqrt{3}}$ (d) $\frac{\pi}{3}\sqrt{13 - 6\sqrt{3}}$ (e) NOTA

Questions 12-14 relate to the non-degenerate conic defined by the equation

$$\boldsymbol{x^2 + xy + 2y^2 + 3x + 5y + 8 = 0.}$$

(12) What can be said about the eccentricity e of this conic?

(a) $e = 0$ (b) $0 < e < 1$

(c) $e = 1$ (d) $e > 1$ (e) NOTA

(13) A second $x'y'$-coordinate system is formed by rotating the xy-coordinate counterclockwise by an angle $\theta \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ such that in the new coordinate system, this conic has an equation of the form $A'x'^2 + C'y'^2 + D'x' + E'y' + F' = 0$. Find θ.

(a) $-\frac{\pi}{8}$ (b) $-\frac{\pi}{4}$

(c) $\frac{\pi}{8}$ (d) $\frac{\pi}{4}$ (e) NOTA

(14) A second $x'y'$-coordinate system is formed by rotating the xy-coordinate counterclockwise by an angle $\theta \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ such that in the new coordinate system, this conic has an equation of the form $A'x'^2 + C'y'^2 + D'x' + E'y' + F' = 0$. Find $\frac{A' + A'C' + C'}{F'}$.

(a) $\frac{5}{12}$ (b) $\frac{17}{56}$

(c) $\frac{5}{32}$ (d) $\frac{3}{8}$ (e) NOTA

(15) Find the area enclosed by a triangle with side lengths 15, 16, and 17.

(a) $26\sqrt{22}$ (b) $24\sqrt{14}$

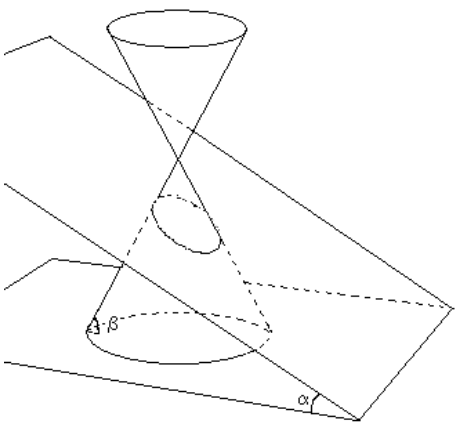
(c) $28\sqrt{13}$ (d) $24\sqrt{21}$ (e) NOTA

(16) The line going through the Cartesian points $(1,2,3)$ and $(-1,0,2)$ also goes through the point $(p,q,0)$. Find $p+q$.

(a) 9 (b) 10

(c) 11 (d) 12 (e) NOTA

(17)



A less popular definition of eccentricity relates directly to the geometric origin of the phrase "conic section." It turns out that if β is the angle of the slant generator of the double-napped cone with horizontal and α is the angle the plane forming the conic makes with horizontal, then the eccentricity $e = \frac{\sin(\alpha)}{\sin(\beta)}$. See picture, left. If the slant generator of a certain double-napped cone makes a $45°$ angle with the horizontal, what is the eccentricity of the conic formed when the plane $\sqrt{7}x + y + \sqrt{17}z + 7 = 0$ cuts this cone?

(a) $\frac{4}{5}$ (b) $\frac{\sqrt{17}}{5}$

(c) $\frac{2\sqrt{2}}{5}$ (d) $\frac{\sqrt{34}}{5}$ (e) NOTA

(18) Which of the following equations represents an asymptote of the hyperbola $\frac{x^2}{9} - \frac{(y-1)^2}{4} = 1$?

(a) $r = \frac{1}{2\cos(\theta) - 3\sin(\theta)}$ (b) $r = \frac{3}{2\cos(\theta) + 3\sin(\theta)}$

(c) $r = \frac{1}{2\cos(\theta) + 3\sin(\theta)}$ (d) $r = \frac{3}{2\cos(\theta) - 3\sin(\theta)}$ (e) NOTA

(19) An ellipse with eccentricity $e = \frac{2}{3}$ has a focus at the origin and a corresponding directrix (the one closer to that focus) with the equation $y = -2x + 3$. Which of the following could be the vertex of the ellipse lying between this focus and directrix?

(a) $\left(\frac{4}{5}, \frac{2}{5}\right)$ (b) $\left(\frac{2}{5}, \frac{4}{5}\right)$

(c) $\left(\frac{6}{25}, \frac{3}{25}\right)$ (d) $\left(\frac{12}{25}, \frac{6}{25}\right)$ (e) NOTA

(20) Find the volume of the parallelepiped with edges corresponding to the vectors $\langle 1,1,2\rangle$, $\langle 0,2,1\rangle$, and $\langle 3,1,0\rangle$.

(a) 2 (b) 11

(c) 10 (d) 5 (e) NOTA

(21) Find the area bounded by the conic represented by the parametric equations

$x(t) = 5\sin(2t) - 2$ and $y(t) = 7\cos(2t) + 3$.

(a) ∞ (b) 6π

(c) 70π (d) 35π (e) NOTA

(22) Find the equation of the slant asymptote of $y = \frac{3x^2+5x-2}{x+1}$.

(a) $y = 5x - 2$ (b) $y = 3x + 2$

(c) $y = 5x - 4$ (d) $y = 3x + 5$ (e) NOTA

(23) What is the period of the graph of $y = \sin(5x) + \sin(3x)$?

(a) 2π (b) $\frac{2\pi}{3}$

(c) $\frac{\pi}{2}$ (d) $\frac{2\pi}{5}$ (e) NOTA

(24) What is the amplitude of the graph of $y = \sqrt{3}\sin(3x) + \sqrt{13}\cos(3x)$?

(a) $\sqrt{3}$ (b) 4

(c) $\sqrt{13}$ (d) $\sqrt{10}$ (e) NOTA

(25) Find the distance from the point $(3,-1,2)$ to the plane $4x - y + 3z + 2 = 0$.

(a) $\frac{21\sqrt{26}}{26}$ (b) 21

(c) $\sqrt{26}$ (d) $\frac{26\sqrt{21}}{21}$ (e) NOTA

(26) How many vertical and horizontal asymptotes does the graph of the function $f(x) = \frac{e^x+e^{-x}}{e^x-e^{-x}}$ have?

(a) 1 (b) 2

(c) 3 (d) 4 (e) NOTA

(27) Find the area inside the curve defined by the polar equation $r = 2\cos(\theta)$.

(a) 4π (b) π

(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$ (e) NOTA

(28) A triangle in the Cartesian plane has vertices $(0,0)$, $(0,k)$, and $(h,0)$, where $h,k > 0$. What is the area of the largest rectangle that can be inscribed in this triangle?

(a) $\frac{hk}{8}$ (b) $\frac{hk}{2}$

(c) $\frac{hk}{3}$ (d) $\frac{hk}{4}$ (e) NOTA

(29) I have a curve that is the locus all points that are exactly $\frac{1}{2}$ as far away from the point $(0,2)$ as they are from the x-axis. This coincides with the locus of points for which the sum of the distances to $(0,2)$ and a point Q is constant. Find Q.

(a) $(0,4)$ (b) $\left(0,\frac{8}{3}\right)$

(c) $\left(0,\frac{10}{3}\right)$ (d) $\left(0,\frac{4}{3}\right)$ (e) NOTA

(30) The 2015 complex roots of the equation $x^{2015} - 4 = 0$ are plotted in the complex plane. Which of the following is closest to the area enclosed by the convex polygon that has these points as vertices?

(a) 0 (b) e

(c) π (d) 2π (e) NOTA