Alpha Apps Solutions

1. A

If we treat Vickie and Caroline as one person, there are (6-1)! = 120 ways that the group can be arranged. Since they can sit on either side of each other, there are a total of 240 arrangements.

2. C

$_{12}C_5$ = 782 gives the number of possible combinations of five points that can be chosen from the twelve (any five points will necessarily make a pentagon because they all lie on the same circle)

3. D

The reindeer can fly in an area equal to $\frac{7}{8}$ of a sphere. The radius of the sphere is 18 feet, which is 6 yards. Finding the volume: $\frac{7}{8} * \frac{4}{3} * \pi * 6^3 = 252\pi$.

4. A

We find the cross product of the two vectors; the area of the triangle will be half of the magnitude of the cross product. $\begin{vmatrix} i & j & k \\ -1 & 0 & 5 \\ 0-2 & 3 \end{vmatrix} = |-2k + 10i + 3j| = \sqrt{4 + 100 + 9} = \sqrt{113}.$ Thus, the area is $\frac{\sqrt{113}}{2}$.

5. C

The paths of the two objects make a triangle, with two adjacent sides being 4*3=12 and 7*3=21, with an angle of 30 degrees between them. We use the law of cosines to find the third side of the triangle. $c^2 = 12^2 + 21^2 - 2 * 12 * 21 * cos60 \rightarrow c = \sqrt{333}.$

6. B

The upstream part of the trip takes (4/5)*3 hours = 12/5 hours, and the downstream part of the trip takes 3-(12/5) hours = 3/5 hours. Thus, we have two equations and two unknowns, given by d=rt for both of the components of the trip:

$4 = (r - c) * \frac{12}{5}$ and $3 = (r + c) * \frac{3}{5}$. Solving this system for c gives c = 5/3 mph.

7. C

Between minute n and n+1, you will have 2 liters flow out of the tub, and, $6 * \frac{n+n+1}{2} = 3(2n+1)$liters flowing into the tub. Thus, after 1 minute, you have 3-2=1 liters; after 2 minutes, you have 1+(9-2)=8; after 3 minutes, you have 8+(15-2)=21 liters. So (to the nearest minute), it takes 3 minutes.

8. D

The number of ways to create the first team is given by 10C5; the other five people will make up the other team. $10C5 = \frac{10!}{5!5!} = 252$. However, making one team or its complements double counts the teams, so the answer should be half of this, or 126.

9. B

We are looking for the equation that does not have a period of frequency of 2π. The period of sin(at)+cos(bt) is given by the least common multiple of 2π/a and 2π/b. For choice B, the individual period are π and π/2, so the least common multiple is π. The other three have a period of 2π, which would cause resonance.

10. B

We have an infinite geometric series with first term $\frac{3}{64}$ and ratio $\frac{3}{4}$. The sum is thus $\frac{a}{1-r} = \frac{\frac{3}{64}}{1-\frac{3}{4}} = \frac{3}{64} * 4 = \frac{3}{16}$

11. E

The volume of the box has to be less than 64, because that is the volume that we would get if we had a 4x4x4 square. Since none of the answer choices are less than 64, the answer must be NOTA.

12. D

After n iterations, you have called 2+4+8+...+2^n people. Using the sum formula for geometric series gives

$$TOTAL = 2 * \frac{(1-2^n)}{1-2}$$
$$500 = 2 * \frac{(1-2^n)}{1-2}$$
$$500 = 2^{n+1} - 2$$
$$500 = 2^{n+1} - 2$$
$$502 = 2^{n+1}$$
$$251 = 2^n$$

249 is between 2^7 and 2^8, so you must have eight iterations, which will take 80 minutes.

13. D

If we split the membrane into four sections, we have $100 * x^4 = 62500$, so $x = 5$. This means that ¾ of the way up the membrane, the frequency is $100 * 5^3 = 12500$.

14. E

This curve is a circle with radius 1, centered at the point (3,4). When rotated about the origin, it creates a torus. To find the volume, unroll the torus into a cylinder: the radius of the base is 1, and the height is $2 * \pi * 5 = 10\pi$. Thus, the entire volume is $10\pi^2$.

15. A

$$\begin{bmatrix}2 & 0\\0 & 3\end{bmatrix}^5 = \begin{bmatrix}2^5 & 0\\0 & 3^5\end{bmatrix} = \begin{bmatrix}32 & 0\\0 & 243\end{bmatrix}$$

16. C

The largest sphere will have radius 4. Its volume is thus $\frac{4}{3}\pi * 3^3 = 36\pi$.

17. A

Medication A can never have an amount that is greater than 3 times the amount of medication B for the two hours, and there should be 30 mg of Medication B in the body at the start (since B is decreasing more quickly than A, the ratio of B to A will be decreasing). We set A*(2)^(-t) < 3*30*(2)^(-t/.75). We plug in t=3, because the ratio will be decreasing and needs to remain above 1:3 until at least t=23 Solving gives A < 45.

18. B

The decagon has (10)(10-3)/2 = 35 diagonals. The ones that go through the center of the circle will be the ones with vertices that are opposite. There are five pairs of opposite vertices, so there will be 30 remaining diagonals that do not go through the center of the circle.

19. B

There are 2520 total permutations = 7!/2!. 6!/2! = 360 of these begin with A, so the 360th one will be the last one beginning with A, which is ARPLECC.

20. A

Probability that she will get it through on the first shot = 1/6*1/5=1/30

Probability that she will miss the first shot but get it through on the second shot = 29/30*1/30=29/900

These are the only (mutually exclusive) options, so the probability is 1/30+29/900=59/900.

21. D

We first find the number of paths from (0,0) to (3,5), and then the number of paths from (3,5) to (7,10). For the first step, we must have three "right" steps and five "up" steps. The number of different orderings of these steps is given by 8!/3!5! = 56. The second stage is given by (4+5)!/4!5! = 126. Multiplying the two together gives 7056.

22. E

We set a = 1/x and b = 1/y. Solving the system a+2b=5 and 3a+b=-2 using elimination gives b = 17/5. This means that y = 5/17.

23. A

$e^{i\theta} = cis\theta$, so $e^{\frac{3i\pi}{2}} = cis\left(\frac{3\pi}{2}\right) = cos\frac{3\pi}{2} + isin\frac{3\pi}{2} = -1$. The magnitude is thus 1.

24. B

This forms a circle with radius two; the area is thus 4π.

25. A

This is an infinite geometric series whose common ratio has absolute value less than 1. Therefore, the sum is $\frac{a}{1-r} = \frac{1}{1+\frac{i}{2}} = \frac{2}{2+i}$. Simplifying by multiplying by $\frac{2-i}{2-i}$ gives $\frac{4-2i}{5}$

26. C

The point 5-12i will be at the coordinates (5,-12). By the Pythagorean Theorem, this is 13 away from the origin.

27. D

The total impedance is equal to the impedance of the resistor plus the impedance of the inductor $I = 1 + i(2)\left(\frac{\sqrt{3}}{2}\right) = 1 + i\sqrt{3}$. (The frequency of the voltage source is $\frac{4\pi}{2\pi} = 2$). Thus, our variation on Ohm's law gives the total complex current as $\frac{10}{1+i\sqrt{3}}$.

28. D

Multiplying by the conjugate gives $\frac{10}{1+i\sqrt{3}} * \frac{1-i\sqrt{3}}{1-i\sqrt{3}} = \frac{10-10i\sqrt{3}}{4} = \frac{5}{2} - \frac{5}{2}i\sqrt{3} = 5\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) = 5e^{\frac{5i\pi}{3}}$ by Euler's Theorem.

29. B

$I_T(t) = I_C * e^{i\omega t} = 5e^{\frac{5i\pi}{3}} * e^{2it} = 5e^{i\left(\frac{5\pi}{3}+2t\right)}$, so $\varphi = \frac{5\pi}{3}$

30. B

The answer from above can be written in as $I_T(t) = 5cis(\frac{5\pi}{3} + 2t)$. The real part of this equation will be given by taking only the cosine component of the cis: $I(t) = 5cos(\frac{5i\pi}{3} + 2t)$