Evaluate:

\[ \cos 1° \cdot \cos 2° \cdot \cos 3° \cdot \ldots \cdot \cos 179° \]
#1 Alpha Ciphering  
MAØ National Convention 2015  

Solve for $x$:

$$|x - 2| - 5 \leq 2$$

Express your answer in interval notation.
Solve for the $2 \times 2$ matrix $M$:

$$M \begin{bmatrix} 5 & 11 \\ 3 & 3 \end{bmatrix} + 2M = \begin{bmatrix} 1 & 1 \\ 2 & 4 \end{bmatrix}$$
Two of the six $6^{th}$ roots of $-352 + 936i$ lie in the first quadrant on the complex plane. One of them is $3 + i$, the other can be expressed as $a + bi$. Find $a + b$. 
12 \cos 2x - 5 \sin 2x can be rewritten as 
\[ A \cos \left( Bx + \arcsin \left( \frac{c}{d} \right) \right) \] 
for integers \( A, B, C, D, \) 
where \( A, B > 0 \) and \( C, D \) relatively prime. Find the value of \( A + B + CD. \)
Find the area enclosed by the ellipse described by the polar graph

\[ r = \frac{6}{2 + \cos \theta} \]
Point $O$ is the center of a circle with radius 4. Point $A$ lies somewhere outside the circle, with $OA = 14$. Find the geometric mean of the lengths of all line segments connecting $A$ to a point on circle $O$. 
Find the remainder when \( 7^{2^3} \) is divided by 40.

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Let $A_n$ be the $n$th term in the Fibonacci-like sequence $1, 3, 4, 7, 11, 18,...$, where $A_1 = 1$, $A_2 = 3$, and $A_k = A_{k-1} + A_{k-2}$ for $k \geq 3$.

Let

\[ S_n = \sum_{k=1}^{n} A_k \]

Find

\[ S_{2015} - \sum_{k=1}^{2013} S_k \]
Find the product of the solutions of the equation:

$$4^{\ln x} - 6x \ln 2 + 8 = 0$$
A bag contains 5 coins. Four of the coins are fair, and the other is a coin with tails on both sides. Stephen draws a coin out of the bag at random, and flips it 3 times, with the coin coming up tails all 3 times. If the coin is flipped a fourth time, what is the probability it comes up tails on that fourth flip?
Let $a$, $b$, and $c$ be the three roots of $f(x) = x^3 - 7x + 5$. Find the value of

$$\frac{1}{a + 2} + \frac{1}{b + 2} + \frac{1}{c + 2}$$
Find the domain of the function, written in interval notation:

\[ f(x) = \sqrt{\ln(|x^2 - 1|)} \]