For all questions, answer choice "E. NOTA" means none of the above answers is correct.

1. What is the measure of angle ABC, to the nearest degree, for the points A(1, 2, 3), B(2, 3, 5), and C(5, 4, 3)?

   A. 30° B. 45° C. 60° D. 90° E. NOTA

2. A survey showed the following results:
   84 people like the music of Lady Gaga only
   34 people like the music of Ludacris only
   72 people like the music of Lil' Wayne only
   A total of 100 people like only two of the three artists.
   85 disliked all three.
   The number of people surveyed is the largest perfect square less than 1000. How many people like the music of all three artists?

   A. 876 B. 686 C. 586 D. 296 E. NOTA

3. If 2137^{753} is multiplied out, what would the units' digit be?

   A. 1 B. 7 C. 9 D. 3 E. NOTA

4. What is the diameter of a circle which has the same enclosed area as $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$?

   A. ab B. $(ab)^2$ C. $\sqrt{ab}$ D. $2\sqrt{ab}$ E. NOTA

5. If x, y, and z all represent real numbers, solve for x in the following equation:

$$xy^4 + xy^2z^2 - 5y^4 - 5y^2z^2 + 3xy^2 + xz^2 - 15y^2 - 5z^2 + 2x - 10 = 0$$

   A. 5 B. 2 C. 1 D. cannot be determined E. NOTA

6. What is the domain of the following function?

$$f(x) = \frac{x+6}{x-7}$$

   A. $\{x\epsilon\mathcal{R}|x > -6 \; or \; x < -6\}$ B. $\{x\epsilon\mathcal{R}|x > -7 \; or \; x < -7\}$
   C. $\{x\epsilon\mathcal{R}|x > 7 \; or \; x < 7\}$ D. $\{x\epsilon\mathcal{R}|x > 6 \; or \; x < 6\}$ E. NOTA

7. Find the sine of the acute angle between the vectors $\boldsymbol{i} + 2\boldsymbol{j} - 5\boldsymbol{k}$ and $3\boldsymbol{i} - 4\boldsymbol{j} - 5\boldsymbol{k}$.

A. $\frac{2\sqrt{165}}{15}$ B. $\frac{\sqrt{165}}{15}$ C. $\frac{2\sqrt{15}}{15}$ D. $2\sqrt{15}$ E. NOTA

8. Evaluate to following expression:

$$\lim_{x\to+\infty} \frac{2x - \frac{1}{3x}}{5x - \frac{1}{6x}}$$

A. 0 B. $\frac{2}{5}$ C. 1 D. ∞ E. NOTA

9. In a nine-element data set of integers from 0 to 100 inclusive, the mode is 14, the median is 38, and the range is 77. What is the largest possible value of the mean?

A. $\frac{499}{9}$ B. 54 C. 55 D. $\frac{497}{9}$ E. NOTA

10. What is the sum of the last two digits of $\sum_{n=1}^{2016} n^2$?

A. 15 B. 10 C. 13 D. 6 E. NOTA

11. Evaluate the expression: $\begin{bmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} + \begin{bmatrix} -6 & -2 & 7 \\ 3 & 1 & 1 \\ -9 & 4 & 5 \end{bmatrix}$

A. $\begin{bmatrix} -96 & -82 & 77 \\ 63 & 51 & 41 \\ -39 & 24 & 15 \end{bmatrix}$ B. $\begin{bmatrix} 15 & 10 & 0 \\ 3 & 4 & 3 \\ 12 & -2 & -4 \end{bmatrix}$

C. $\begin{bmatrix} -93 & 18 & 106 \\ -57 & 9 & 67 \\ -21 & 0 & 28 \end{bmatrix}$ D. $\begin{bmatrix} 3 & 6 & 14 \\ 9 & 6 & 5 \\ -6 & 6 & 6 \end{bmatrix}$ E. NOTA

12. Given that $a = \log 2$ and $b = \log 3$, expand $\log x$, where x is the number of distinct five -letter permutations of the letters in the term WASHU.

A. $a + 2b + 1$ B. $2a + 2b + 1$
C. $2a + b + 1$ D. $a + b$ E. NOTA

13. How many intersection points over the interval $[-2\pi, 2\pi]$ do the graphs of $y = \cos(2x)$ and $y = \frac{1}{2}$ have?

A. 2 B. 4 C. 8 D. 16 E. NOTA

14. Find the area of the region defined by the parametric equations $x = 11\cos(\theta) - 8$ and $y = 4\sin(\theta) + 7$.

A. 15π B. 30π C. 45π D. 137π E. NOTA

15. Determine the maximum possible real root for the following equation over the interval $\frac{\pi}{6} \leq \theta \leq \frac{\pi}{2}$:

$$0 = x^2 - 2\sin(\theta)\, x + \frac{\sin^2(2\theta)}{4}$$

A. 0 B. 2 C. $\frac{3+2\sqrt{3}}{4}$ D. $\frac{2+2\sqrt{3}}{3}$ E. NOTA

16. Solve for x: $\frac{\log(0.0625)}{\log(0.\overline{1})} = \frac{\log(4^x)}{\log(9)}$

A. 2 B. -2 C. 1 D. -1 E. NOTA

17. What is the sum of the elements in the thirteenth row of Pascal's Triangle? The first row consists of a single 1.

A. 2048 B. 4096 C. 1024 D. 8192 E. NOTA

18. Express the base-5 number $(6102)_5$ in base-8.

A. 1141_8 B. 1114_8 C. 1414_8 D. 1411_8 E. NOTA

19. What is the maximum number of points of intersection for a set of five distinct circles?

A. 10 B. 15 C. 20 D. 25 E. NOTA

20. What is the largest eigenvalue of $\begin{bmatrix} 10 & -8 \\ 6 & -4 \end{bmatrix}$?

A. 1 B. 2 C. 3 D. 4 E. NOTA

21. The equation $V = 2\pi^2 Rr^2$ yields the volume of a torus (commonly referred to as a "donut"), a circle that has been rotated 360° about the z-axis, where R is the distance from the z-axis to the center of the torus (also known as the "central circle"), and r is the radius of the torus's cross-section, the rotated circle from which the torus is derived. A "donut-hole" is a sphere with a diameter equal to the diameter of the hole in the torus that it was removed from. If a particular torus has $R = 3.6$" and $r = 1.8$", how many "donut-holes," to the nearest whole number, must Paul eat to consume a volume of dough equivalent or greater than one "donut"?

A. 8 B. 9 C. 10 D. 11 E. NOTA

22. Mr. Hallman drops a bouncy ball from a balcony of height 6 feet. Every time the ball bounces, it bounces to two-thirds of its previous height. How far does the ball travel before it comes to rest?

A. 6 B. 12 C. 24 D. 36 E. NOTA

23. What is the positive x-intercept of $y = -2 + \ln(x^2 - \frac{1}{9})$?

A. $\pm\left(e + \frac{1}{3}\right)$ B. $\sqrt{e^2 + \frac{1}{9}}$ C. $e + \frac{1}{3}$ D. $\sqrt{e + \frac{1}{3}}$ E. NOTA

24. At Washington University in St. Louis, Michael's drive from his apartment to school has 10 stoplights, each with a 30% chance of delaying him 3 minutes. The same drive takes him 13 minutes with no stoplights slowing him. Every day, he leaves for school 15 minutes before his 8 A.M. class at WashU. What is Michael's expected amount of (average) time between when he arrives at school and when his first class starts?

A. 4 minutes B. 3 minutes C. 2 minutes D. he is late by 7 minutes E. NOTA

25. If $0.223\overline{81}$ … is expressed as a reduced fraction of positive integers in the form of $\frac{p}{q}$, then what is $q - p$?

A. -6731 B. 6731 C. 4269 D. -4269 E. NOTA

26. What are the coordinates of the point on the curve $(x - a)^2 + (y - b)^2 = r^2$ with the minimum x-value?

A. $(x - r, y)$ B. $(a - r, b)$ C. $(r - a, b)$ D. $(r - x, y)$ E. NOTA

27. At what value(s) of x do the graphs $2x^2 + y^2 = 4$ and $y = 5x^2$ intersect?

A. $\frac{-1+\sqrt{101}}{25}$ B. $\frac{1}{5}\sqrt{-1+\sqrt{404}}$
C. $\pm\frac{1}{5}\sqrt{\frac{-1+\sqrt{404}}{25}}$ D. $\pm\frac{1}{5}\sqrt{-1+\sqrt{101}}$ E. NOTA

28. A Swiss Bank ATM pin code consists of five digits, each an integer from 0 to 9, inclusive. In order to be more secure, all five digits must be distinct, but the pin code may not consist of strictly increasing or strictly decreasing numbers when reading the pin code from left to right (so, for example, neither 24689 nor 98642 is an acceptable pin code, but 26489 is acceptable, as is 98462). How many such pin codes are possible?

A. 29,736 B. 118 C. 14,868 D. 252 E. NOTA

29. Given the trigonometric form of a complex number to be $-4cis\left(-\frac{5\pi}{4}\right)$, find its rectangular form.

A. $-2\sqrt{2} - 2\sqrt{2}i$ B. $-2\sqrt{2} + 2\sqrt{2}i$
C. $2\sqrt{2} + 2\sqrt{2}i$ D. $2\sqrt{2} - 2\sqrt{2}i$ E. NOTA

30. Find the volume, in cubic feet, of the largest regular tetrahedron that can be inscribed in a cube with edge length 2 feet.

A. $\frac{8}{3}$ B. $\frac{16}{3}$ C. $\frac{4}{3}$ D. $\frac{2\sqrt{2}}{3}$ E. NOTA