Answers:

1. D
2. C
3. B
4. D
5. A
6. C
7. B
8. B
9. D
10. A
11. D
12. C
13. C
14. E 44π
15. B
16. A
17. B
18. D
19. C
20. D
21. B
22. E
23. B
24. D
25. C
26. B
27. D
28. A
29. D
30. A

Solutions:

1. First, find the lengths of AB, BC, and AC, and notate that AC is the side length opposite the angle ABC that you are seeking:

$$AB = \sqrt{(2-1)^2 + (3-2)^2 + (5-3)^2} = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{6}$$

<also named c in vernacular of naming side lengths of triangles>

$$BC = \sqrt{(5-2)^2 + (4-3)^2 + (3-5)^2} = \sqrt{3^2 + 1^2 + (-2)^2} = \sqrt{14}$$

<also named a in vernacular of naming side lengths of triangles>

$$AC = \sqrt{(5-1)^2 + (4-2)^2 + (3-3)^2} = \sqrt{4^2 + 2^2 + 0^2} = \sqrt{20}$$

<also named b in vernacular of naming side lengths of triangles>

Now, use the low of cosines to determine the measure of angle B:

$$b^2 = a^2 + c^2 - 2ac\cos B$$

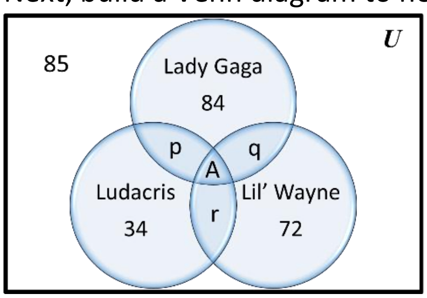
$$\cos B = \frac{b^2 - a^2 - c^2}{-2ac}$$

$$\cos B = \frac{\left(\sqrt{20}\right)^2 - \left(\sqrt{14}\right)^2 - \left(\sqrt{6}\right)^2}{-2\sqrt{14}\sqrt{6}} = \frac{20 - 14 - 6}{-2\sqrt{84}} = \frac{0}{-4\sqrt{21}} = 0$$

Cosine is only equal to 0 at 90° and 270°, and **90°** is the only allowable choice for a non-degenerate triangle.

2. Start off by determining your total amount of people. The easiest way is to start at $30^2 = 900$ and keep going: $31^2 = 961$, $32^2 = 1024$… too large, so the number of people is 961.

Next, build a Venn diagram to help with visualization:



We know that $p + q + r = 100$ and $34 + 84 + 72 + p + q + r + A = 964 - 85$. Seeing that there is $p + q + r$ in both equations, substitute $p + q + r = 100$ into the second equation:

$$34 + 84 + 72 + (100) + A = 961 - 85$$
$$290 + A = 876$$
$$A = \mathbf{586}$$

3. When multiplied out with *n* as the exponent, the units digit goes by the following pattern: *n*=0→1, *n*=1→7, *n*=2→9, *n*=3→3, *n*=4→1,… So a pattern is found to repeat in sets of four. 753 divided by 4 leaves a remainder of 1, so the last digit is **7**.

4. The area of an ellipse is $A = \pi ab$. Setting $\pi ab = \pi r^2$ yields $r = \sqrt{ab}$. So the diameter must be $\mathbf{2\sqrt{ab}}$.

5. Factoring leads to the following:

$$xy^4 + xy^2z^2 - 5y^4 - 5y^2z^2 + 3xy^2 + xz^2 - 15y^2 - 5z^2 + 2x - 10 = 0$$
$$xy^4 - 5y^4 + xy^2z^2 - 5y^2z^2 + 3xy^2 - 15y^2 + xz^2 - 5z^2 + 2x - 10 = 0$$
$$(y^4 + y^2z^2 + 3y^2 + z^2 + 2)(x - 5) = 0$$
$$(y^2 + 1)(y^2 + z^2 + 2)(x - 5) = 0$$

$y^2 = -1$ and $y^2 + z^2 = -2$ have no real solutions, there therefore the only solution left is $x = \mathbf{5}$.

6. The denominator for this function cannot equal zero, or the function would be undefined at that point. Thus setting $x - 7 = 0$, we find that $x = \mathbf{7}$ would be not included in the set.

7. $\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|cos\theta \rightarrow (1)(3) + (2)(-4) + (-5)(-5) = 20 =$ $\left[\sqrt{(1)^2 + (2)^2 + (5)^2}\right]\left[\sqrt{(3)^2 + (-4)^2 + (-5)^2}\right][cos\theta] \rightarrow cos\theta = \frac{2\sqrt{15}}{15} \therefore sin\theta = \boldsymbol{\frac{\sqrt{165}}{15}}$

8. As $x \to \infty$, both $\frac{1}{3x}$ and $\frac{1}{6x}$ approach 0. Thus what remains is $\lim_{x\to+\infty} \frac{2x}{5x} = \boldsymbol{\frac{2}{5}}$.

9. Since the mode is 14, the range is 77, and the median is 38, the following has to be the set of the nine numbers:

$$\{14,14,36,37,38,88,89,90,91\}$$

Adding up all of the numbers equals 497, so the mean is $\boldsymbol{\frac{497}{9}}$.

10. $\sum_{n=1}^{2016} n^2 = \frac{(2016)(2017)(4033)}{6}$. The last two digits of the sum are only dependent on the last two digits of the numbers of the product. So, $\frac{16*17*33}{6} = \frac{8976}{6} = 1496$. So $9 + 6 =$ $\mathbf{15}$.

11. Add the elements of each matrix: $\begin{bmatrix} 9+-6 & 8+-2 & 7+7 \\ 6+3 & 5+1 & 4+1 \\ 3+-9 & 2+4 & 1+5 \end{bmatrix} = \begin{bmatrix} \mathbf{3} & \mathbf{6} & \mathbf{14} \\ \mathbf{9} & \mathbf{6} & \mathbf{5} \\ \mathbf{-6} & \mathbf{6} & \mathbf{6} \end{bmatrix}$

12. Permutation calculation: $5! = 120$, so $\log x = 2\log 2 + \log 3 + \log 10 = \boldsymbol{2a + b + 1}$

13. The period of $y = \cos(2x)$ is π, the absolute value portion flips everything below the x-axis to above it, and $y = \frac{1}{2}$ strikes the curve **8** times, as noted in the figure below:

14. Solve each parametric equation for the trigonometric function:

$$\cos(\theta) = \frac{x+8}{11}$$
$$\sin(\theta) = \frac{y-7}{4}$$

Square both sides of both equations, and add them together:

$$\cos^2(\theta) + \sin^2(\theta) = 1$$
$$\left(\frac{x+8}{11}\right)^2 + \left(\frac{y-7}{4}\right)^2 = 1$$

$$\frac{(x+8)^2}{121}+\frac{(y-7)^2}{16}=1$$

This is an ellipse is area $A=\pi ab=\pi(11)(4)=\mathbf{44\pi}$

15. Use the quadratic formula to find the roots of the equation:

$$x=\frac{2\sin(\theta)\pm\sqrt{4\sin^2(\theta)-\sin^2(2\theta)}}{2}$$

$$x=\sin(\theta)\pm\frac{\sqrt{4\sin^2(\theta)-\sin^2(2\theta)}}{2}$$

Then use trigonometric identities and simplify:

$$x=\sin(\theta)\pm\frac{\sqrt{4\sin^2(\theta)-4\sin^2(\theta)\cos^2(\theta)}}{2}$$

$$x=\sin(\theta)\pm\sqrt{\sin^2(\theta)-\sin^2(\theta)\cos^2(\theta)}$$

$$x=\sin(\theta)\pm\sqrt{\sin^2(\theta)\left[1-\cos^2(\theta)\right]}$$

$$x=\sin(\theta)\pm\sqrt{\sin^4(\theta)}$$

$$x=\sin(\theta)\pm\sin^2(\theta)$$

The maximum root occurs over the interval when $\sin(\theta)$ is maximized. So, using this interval, the maximum value would occur at $\frac{\pi}{2}$. Thus, the maximum possible root is $\sin\left(\frac{\pi}{2}\right)+\sin^2\left(\frac{\pi}{2}\right)=1+1^2=\mathbf{2}$.

16. Given the equation…

$$\frac{\log(0.0625)}{\log(0.\bar{1})}=\frac{\log(4^x)}{\log(9)}$$

…rewrite the equation to where you take the logarithm of the same number:

$$\frac{\log\left(\frac{1}{16}\right)}{\log\left(\frac{1}{9}\right)}=\frac{\log(4^x)}{\log(3^2)}$$

$$\frac{\log\left(\frac{1}{4}\right)^2}{\log\left(\frac{1}{3}\right)^2}=\frac{\log(4^x)}{\log(3^2)}$$

$$\frac{\log(4)^{-2}}{\log(3)^{-2}}=\frac{\log(4^x)}{\log(3^2)}$$

$$\frac{-2\log(4)}{-2\log(3)}=\frac{x\log(4)}{2\log(3)}$$

$$x=\mathbf{2}$$

17.

			1				sum=1	2^0	first row
		1		1			sum=2	2^1	second row
	1		2		1		sum=4	2^2	third row
1		3		3		1	sum=8	2^3	fourth row
							…	…	…
								2^{12}	thirteenth row

Therefore, $2^{12}=(2^4)(2^4)(2^4)=(16)(16)(16)=(256)(16)=$**4096**

18. Convert 2016_5 to base-10: $2(5^0)+0(5^1)+1(5^2)+6(5^3)=777$. Then, using the stair-step division method, read the remainders downward, as shown:

0 R1
$8\overline{)1}$ R4
$8\overline{)12}$ R1
$8\overline{)97}$ R1
$8\overline{)777}$

19. Each circle can intersect with each other circle a maximum of 2 times. There are $\binom{5}{2}$ ways (equal to 10) to choose a pair of circles, and each pair can intersect twice, so $(2)(10)=\mathbf{20}$ possible intersections.

20. To find eigenvalues, follow the format of the determinant $|\boldsymbol{A}-\lambda\boldsymbol{I}|=0$:

$$\begin{vmatrix}10-\lambda & -8\\ 6 & -4-\lambda\end{vmatrix}=0$$
$$(10-\lambda)(-4-\lambda)-(-8)(6)=0$$
$$(-40-6\lambda+\lambda^2)-(-48)=0$$
$$\lambda^2-6\lambda+8=0$$
$$(\lambda-4)(\lambda-2)=0$$
$$\lambda=4,2$$

The greater of these values is **4**.

21. The radius of a donut-hole is R-r, and its volume is $\frac{4}{3}\pi(R-r)^3$. The general equation for the donut to donut-hole ratio is $(2\pi^2Rr^2):\frac{4}{3}\pi(R-r)^3$. For this particular donut (with $R=3.6$" and $r=0.9$". thus $4r=R$ and $R-r=r$), Paul must eat 3π donut-holes. Round to **9**.

22. The ball will travel 6 feet plus twice (because it goes up and down) the sum of the geometric series $4+\frac{4}{3}+\frac{4}{9}+\cdots$. The sum of this series is $\frac{4}{\left(1-\frac{2}{3}\right)}=12$. So, $6+2(12)=\mathbf{30}$.

23. To find intercepts, set the equation equal to zero and solve for x:

$$0 = -2 + \ln\left(x^2 - \frac{1}{9}\right)$$
$$2 = \ln\left(x^2 - \frac{1}{9}\right)$$
$$e^2 = x^2 - \frac{1}{9}$$
$$e^2 + \frac{1}{9} = x^2$$
$$\pm\sqrt{e^2 + \frac{1}{9}} = x$$

Since we are looking for the positive intercept, look at the positive value: $\boldsymbol{\sqrt{e^2 + \frac{1}{9}}}$.

24. The expected wait time is calculated by the following:
(10 lights)*(-3 min) * (.3 probability) = -9 minutes.
Subtracting the original 13 min. and adding 15 minutes
gives a **7** minute late expected "wait" time.

25. Take then decimal and convert it to fractional form by performing the following:

$$\frac{p}{q} = 0.223\overline{81}$$
$$(1000)\frac{p}{q} = (0.223\overline{81})(1000)$$
$$(1000)\frac{p}{q} = 223.\overline{81} = 223 + \frac{81}{99} = 223 + \frac{9}{11}$$
$$(11)(1000)\frac{p}{q} = \left(223 + \frac{9}{11}\right)(11)$$
$$(11000)\frac{p}{q} = 2462$$
$$\frac{p}{q} = \frac{2462}{11000} = \frac{1231}{5500}$$

So $q - p = 5500 - 1231 = \mathbf{4269}$.

26. The circle has a radius of r units and is centered at the point (a, b). Hence, the minimum x-value is r units below the center of the circle. This point is therefore located at $(\boldsymbol{a - r}, \boldsymbol{b})$.

27. Substitute $y = 5x^2$ into the equation for the ellipse.

$$2x^2 + (5x^2)^2 = 4$$

This forms a quadratic (almost) that can be solved.

$$2x^2 + (5x^2)^2 = 4$$
$$2x^2 + (25x^4) - 4 = 0$$

Rewrite this equation into a form that is quadratic using $h = x^2$, taking into account that only the positive y-value is considered here since the parabola opens up above the x-axis:

$$25h^2 + 2h - 4 = 0$$
$$h = \frac{-2 + \sqrt{(2)^2 - (4)(25)(-4)}}{2(25)} = \frac{-2 + \sqrt{404}}{50} = \frac{-2 + 2\sqrt{101}}{50}$$
$$= 2\left(\frac{-1 + \sqrt{101}}{50}\right) = \frac{-1 + \sqrt{101}}{25}$$

Now, since h is solved for, and $h = x^2$, take the square root of h to find your answers for x:

$$h = \frac{-1 + \sqrt{101}}{25}$$
$$h = x^2$$
$$x = \pm\sqrt{h} = \pm\sqrt{\frac{-1 + \sqrt{101}}{25}} = \pm\frac{\mathbf{1}}{\mathbf{5}}\sqrt{\mathbf{-1 + \sqrt{101}}}$$

28. First, find the number of ways to select five distinct digits:

$$\binom{10}{5} = 252$$

Then, to have five digits, you have $5! = 120$ ways to order them.
However, two orderings for each set of five distinct numbers must be thrown out since, in one way, they are strictly increasing and, in another way, they are strictly decreasing. Therefore, the total number of possible pin codes that satisfy all conditions is found by the following:

$$252(120 - 2) = 252(118) = \mathbf{29,736}$$

29. The cis function has a definition of $cis(x) = \cos(x) + i\sin(x)$. So, substitute and simplify:

$$-4cis\left(-\frac{5\pi}{4}\right) = -4\left[\cos\left(-\frac{5\pi}{4}\right) + i\sin\left(-\frac{5\pi}{4}\right)\right] = -4\left[\left(\frac{-1}{\sqrt{2}}\right) + i\left(\frac{1}{\sqrt{2}}\right)\right]$$
$$= -4\left[\left(\frac{-\sqrt{2}}{2}\right) + i\left(\frac{\sqrt{2}}{2}\right)\right] = \mathbf{2\sqrt{2} - 2\sqrt{2}}\boldsymbol{i}$$

30. The tetrahedron, having four vertices, is placed where each vertex at each corner of the cube, where no two vertices of the tetrahedron share an edge. The six edges of the tetrahedron are all face diagonals of the cube. There, each edge of the tetrahedron has a length of $2\sqrt{2}$. Therefore, the volume of the tetrahedron is calculated by the following:

$$\frac{\left(2\sqrt{2}\right)^3}{6\sqrt{2}} = \frac{\left(2\sqrt{2}\right)^3\sqrt{2}}{12} = \frac{32}{12} = \frac{\mathbf{8}}{\mathbf{3}}$$