1. Correct Answer: **C**, Solution: As Luke reaches the second wall, the walls are 15 (20*0.75) meters apart, but since both walls moved closer by 2.5 meters each, Luke actually traveled 17.5 meters before reaching it and turning around. By the time he reaches the first wall again, the walls are now 11.25 (15*0.75) meters apart, but since both walls moved closer together by 1.875 meters each, Luke actually traveled 13.125 meters. This continues ad infinitum, creating a geometric series, in which $a_1 = 17.5$ and $r = \frac{3}{4}$, thereby giving a total distance traveled of $\sum_{i=0}^{\infty} 17.5(\frac{3}{4})^i = \frac{17.5}{1-\frac{3}{4}} = 70$ meters. Since we are given Luke travels at a constant pace throughout of 5 meters per second, this means the total number of seconds elapsed is just $\frac{70\ meters}{5\ meters\ /\ second} = 14\ seconds$, or answer choice C.
2. Correct Answer: **E**, Solution: There are multiple ways to do this problem, but the most simple method is to realize that if the water fills half the volume in each orientation, the sum of the perpendicular heights must be equal to the total perpendicular height of the frustum, which was given to be 9. Since this is not any of the choices listed, the correct answer is E.
3. Correct Answer: **B**, Solution: Let A denote the event that you draw a red marble. Let B denote the event that you selected the first urn, and let B' denote the event that you selected the second urn. Then from Bayes' Theorem, we have:
$$P(B|A) = \frac{P(A|B)P(B)}{P(A|B)P(B)+P(A|B')P(B')}$$
$$= \frac{(\frac{3}{15})(\frac{1}{2})}{(\frac{3}{15})(\frac{1}{2})+(\frac{3}{10})(\frac{1}{2})} = \frac{2}{5}$$, which leads to answer choice B.
4. Correct Answer: **B**, Solution: For each suit, there are ten ways to create a straight flush. Since there are four suits, there are $10 * 4 = 40$ possible straight flushes in the entire deck, which leads to answer choice B.
5. Correct Answer: **D**, Solution: Upon multiplying the matrices, you get
$$\begin{bmatrix} -22 & 36 & -11 \\ -1 & 13 & 2 \\ 42 & -4 & 5 \end{bmatrix}$$
Since the sum of the eigenvalues is the same as the trace, this gives $-22 + 13 + 5 = -4$, from which you arrive at answer D.
6. Correct Answer: **A**, Solution: Since $x = \sqrt{-2i}$, x can either equal $(-1 + i)$ or $(1 - i)$ (which one you select does not matter because they both give the same final answer). Supposing you select $(1 - i)$, this gives $y = |x| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$. So $xy = \sqrt{2} - \sqrt{2}i$, and $|xy| = \sqrt{\sqrt{2}^2 + (-\sqrt{2})^2} = 2$, arriving at answer choice A.
7. Correct Answer: **B**, Solution: This problem is yet another variant of the dog tied to a rope problem. In this problem, the area Steve has to travel is dictated by the surface area of a sphere minus the area of a spherical triangle. Thus
$$total\ area = 4\pi r^2 - (A + B + C - \pi)r^2$$
$$= 4\pi(7)^2 - (2\pi - \pi)(7)^2$$

$= 147\pi$

By which you arrive at answer choice B.

8. Correct Answer: **C**, Solution: This problem can be solved by comparing the initial investment of $1000 with the payments of $50, adjusted to reflect the interest rate which got them to whatever time they occur at. This series looks like:

$$value = \frac{50}{1+i} + \frac{50}{(1+i)^2} + \ldots + \frac{50}{(1+i)^{24}}$$

$$= 50\left(\frac{1-\left(\frac{1}{1+\frac{1}{96}}\right)^{24}}{\frac{1}{96}}\right) = 1056$$

Since $1056 \geq 1000$,this is a worthwhile investment, so the correct answer is C.

9. Correct Answer: **E**, Solution: Since the 6’2” individual can throw it 518 meters, we have the equation:

$518 = 74(1 + tu)$

$7 = 1 + tu$

$tu = 6$

So, if the new force is $\frac{4}{3}t$ and the new wind factor is $\frac{3}{8}u$, we have

$distance = 66(1 + (\frac{4}{3}t)(\frac{3}{8}u))$

$= 66(1 + \frac{1}{2}tu)$

$= 66(4) = 264$

Since 264 is none of the choices, the correct answer is E, NOTA.

10. Correct Answer: **C**, Solution: Because interest is compounded twice yearly at a rate of 5%, for a given year the interest rate is equivalent to 1.05^2. So, if we write an equation of value at the end of ten years we have:

$$Value_{10} = 10 + 10(1.05)^2 + 10(1.05)^4 + \ldots + 10(1.05)^{18}$$
$$= 10(1 + 1.05^2 + (1.05^2)^2 + \ldots + (1.05^2)^9)$$

which is a geometric series with $r = 1.05^2$, whose sum can be given by the expression $Value_{10} = 10(\frac{1-1.05^{20}}{1-1.05^2})$, thereby yielding answer choice C.

11. Correct Answer: **C**, Solution: This is the equation for the spiral of Archimedes, thus yielding answer choice C.

12. Correct Answer: **A**, Solution: First, you need to recognize that 613 is prime. There are many ways to do this, the most straightforward being attempting to divide 613 by all primes less than or equal to the square root of 613. From there, the the question is an application of Wilson’s Theorem:

(for prime n), $(n-1)! \equiv -1(mod\ n)$

$(612)! \equiv -1(mod\ 613)$

$(612)! \equiv 612(mod\ 613)$

So the remainder is 612, or answer choice A.

13. Correct Answer: **D**, Solution: This is a question of combinations and permutations. The total number of recipes he can create is given by

$({}_7C_3)({}_5C_2)({}_6P_4)({}_2C_1) = 35 * 10 * 360 * 2 = 252000.$

However, $\frac{1}{4}$of these recipes, or 63000 are lethal, leaving 189000 possible nonlethal recipes, leading to answer choice D.

14. Correct Answer: **B**, Solution: You should immediately see that because the probabilities of each of the four possible movements sum to 1, there are no alternative movements which can be made. Each time Cyrus moves, his expected location on the x-axis is changed by $(\frac{1}{4})(1)+(\frac{3}{8})(-1)=-\frac{1}{8}$, and his expected location on the y-axis is changed by $(\frac{1}{3})(1)+(\frac{1}{24})(-1)=\frac{7}{24}$. Since his decisions do not depend on his location, six such movements will just multiply the expected single turn locations by six: $6(-\frac{1}{8},\frac{7}{24})=(-\frac{3}{4},\frac{7}{4})$, thus the correct answer is B.

15. Correct Answer: **C**, Solution: $cot(2\theta)=\frac{A-C}{B}=\frac{3}{4}$
$\Rightarrow cos(2\theta)=\frac{3}{5}$
$\Rightarrow cos(\theta)=\sqrt{\frac{1+\frac{3}{5}}{2}}=\frac{2\sqrt{5}}{5}, sin(\theta)=\sqrt{\frac{1-\frac{3}{5}}{2}}=\frac{\sqrt{5}}{5}$
$\Rightarrow x=\left(\frac{2\sqrt{5}}{5}x'-\frac{\sqrt{5}}{5}y'\right), y=\left(\frac{\sqrt{5}}{5}x'+\frac{2\sqrt{5}}{5}y'\right)$
Plugging in these values to the original equation yields:
$\frac{x\prime^2}{120}+\frac{y\prime^2}{3}=1$, which is an ellipse, or answer choice C.

16. Correct Answer: **A**, Solution: This problem immediately appears to be solvable by the Chinese Remainder Theorem, and this is indeed one way to solve it. However, inspection of the available answers leads one immediately to 80, or answer choice A.

17. Correct Answer: **B**, Solution: Since the domain of $3f(x)=[-1,8]$, the domain of $f(x)=[-1,8]$, and thus the domain of $f(4x-3)=[-1,8]$, and so finally the domain of $2f(4x-3)=[-1,8]$. $-1\le 4x-3\le 8\Rightarrow \frac{1}{2}\le x\le \frac{11}{4}$, which leads you to answer choice B.

18. Correct Answer: **C**, Solution: By partial fraction decomposition, the expression is equal to $\frac{1}{x+3}+\frac{2}{x+5}+\frac{1}{x-2}$, thus $A=C=1, B=2, j=3, k=5, l=-2$, and $ABC-jkl=32$, or answer choice C.

19. Correct Answer: **A**, Solution: $xy=\frac{(6)(5)(4)(3)(2)(1)}{(3)(2)(1)(2)(1)}*\frac{(9)(8)(7)(6)(5)(4)(3)(2)(1)}{(6)(5)(4)(3)(2)(1)}$. The numerator of the expression for x cancels with the denominator for the expression for y, so the answer is a nine letter word in which one letter is repeated twice and one letter is repeated three times. This is answer choice A.

20. Correct Answer: **B**, Solution: Write the equation as $(sin(x)-1)(sec(x)+1)=0$. When $sin(x)=1$, $x=\frac{\pi}{2}$, and the original equation is undefined. If $sec(x)=-1$, then $x=\pi$, which is then the only solution, leading you to answer choice B.

21. Correct Answer: **C**, Solution: you will recognize this is one definition of how a cardioid is generated, causing you to arrive at answer choice C.

22. Correct Answer: **D**, Solution: $\left(\frac{x+y}{x-y}\right)^2=\frac{x^2+2xy+y^2}{x^2-2xy+y^2}=\frac{10xy}{6xy}=\frac{5}{3}$, so $\frac{x+y}{x-y}=\sqrt{\frac{5}{3}}=\frac{\sqrt{15}}{3}$, or answer choice D.

23. Correct Answer: **B**, Solution: First, recognize $\sqrt{(1-x^2)}=(1-x^2)^{\frac{1}{2}}$. By the binomial theorem, then,

$(1-x^2)^{\frac{1}{2}} = 1 - \frac{x^2}{2} - \frac{x^4}{8} - \frac{x^6}{16} - \ldots$, so the correct answer is B.

24. Correct Answer: **C**, Solution: This sum can be rewritten as $\frac{1}{2}+\frac{2}{4}+\frac{3}{8}+\frac{4}{16}+\frac{5}{32}+\frac{6}{64}+\ldots$, where the numerators are an increasing arithmetic sequence, and the denominators form an increasing geometric sequence. If we set
$S=\frac{1}{2}+\frac{2}{4}+\frac{3}{8}+\frac{4}{16}+\frac{5}{32}+\frac{6}{64}+\ldots$, then
$\frac{1}{2}S=\frac{1}{4}+\frac{2}{8}+\frac{3}{16}+\frac{4}{32}+\frac{5}{64}+\ldots$, and subtracting, we have
$\frac{1}{2}S=\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\frac{1}{64}+\ldots$, which is an infinite geometric series with $a_1=\frac{1}{2}$and $r=\frac{1}{2}$. Thus $\frac{1}{2}S=1$, and $S=2$, yielding answer choice C.

25. Correct Answer: **D**, Solution: First, rewrite the equation as $2(e^{3-x})^2-3(e^{3-x})-20=0$. Making the substitution $y=e^{3-x}$, this equation becomes the quadratic $2y^2-3y-20=0$. This factors as $(y-4)(2y+5)=0$. Since y cannot be negative (this has no real solutions), we have
$e^{3-x}=4$
$\Rightarrow 3-x=ln(4)$
$\Rightarrow x=3-ln(4)$, or answer choice D.

26. Correct Answer: **B**, Solution: Completing the square yields that all are degenerate with the exception of answer choice B, which is a hyperbola.

27. Correct Answer: **A**, Solution: This answer is given by ${}_7C_4$ / ${}_{15}C_4$ = $\frac{1}{39}$, or answer choice A.

28. Correct Answer: **E**, Solution: Since the first digit in the number will never be four, there are four possible digits which can occupy this place (6,7,8,9). For the remaining three places in our four digit number, one of them needs to be a four, and the other two need to not be a four. Thus there are $1*9*9$ possible combinations of the last three digits, so there are $81*4=324$ total numbers which meet the criteria. Since this is none of the options given, the correct answer is E.

29. Correct Answer: **B**, Solution: This is the graph of a circle with diameter 1. However, since θgoes from 0 to 2π, the circle's diameter is traced twice by the path of the curve. Since the circumference of the circle is 2π, the total curve length is 4π, or answer choice B.

30. Correct Answer: **A**, Solution: By the shoelace formula, the area of a convex polygon with five sides, given its five coordinates (this equation also works for other convex polygons), if we let the coordinates be denoted $(x_1,y_1),(x_2,y_2)$, and so forth, is given by:

$$Area = \left|\frac{(x_1y_2-y_1x_2)+(x_2y_3-y_2x_3)+(x_3y_4-y_3x_4)+(x_4y_5-y_4x_5)+(x_5y_1-y_5x_1)}{2}\right|$$

$$= \left|\frac{(-10)+(-13)+(-7)+(-7)+(-13)}{2}\right|$$

$= 25$, which leads to answer choice A.