Answers:

1. A
2. D
3. C
4. B
5. B
6. A
7. A
8. D
9. C
10. B
11. A
12. E
13. D
14. E
15. B
16. D
17. A
18. B
19. D
20. D
21. C
22. C
23. A
24. B
25. D
26. D
27. D
28. B
29. C
30. D

Solutions:

1. To convert the Cartesian point $\left(\frac{\pi}{\sqrt{3}}, \pi\right)$ to Polar coordinates, we must first find the radius by using the formula: $r^2 = x^2 + y^2$. $\boxed{\begin{array}{l} r^2 = \left(\frac{\pi}{\sqrt{3}}\right)^2 + (\pi)^2 = \frac{4\pi^2}{3} \\ r = \frac{2\pi}{\sqrt{3}} = \frac{2\pi\sqrt{3}}{3} \end{array}}$ Then to find the angle (argument) we can use the formula $\boxed{\begin{array}{l} \tan\theta = y/x \\ \tan^{-1}\left(y/x\right) = \theta \\ \tan^{-1}\left(\pi \Big/ \left(\frac{\pi}{\sqrt{3}}\right)\right) = \tan^{-1}\left(\sqrt{3}\right) = \pi/3 \end{array}}$ and the final answer is $\left(\frac{2\pi\sqrt{3}}{3}, \frac{\pi}{3}\right)$.
2. The angle between the two polar points is 90 degrees (a right angle), so to find the distance between the points we can simply use Pythagorean Theorem: $\sqrt{2^2 + 4^2} = 2\sqrt{5}$.
3. This question looks like question 2, but it made more complicated by the central angle between the two points. After graphing the coordinates, you can see that the central angle is 120 degrees. To find the distance between the polar coordinates we can utilize the Law of Cosines: $\boxed{\begin{array}{l} c^2 = a^2 + b^2 - 2ab\cos C \\ c^2 = 5^2 + 12^2 - 2(5)(12)\cos 120^\circ \\ c^2 = 5^2 + 12^2 - 2(5)(12)\left(\frac{-1}{2}\right) = 229 \end{array}}$. Distance= $\sqrt{229}$
4. The easiest way to approach this problem is my opinion is to convert the complex number to its trigonometric form. To do this, you graph the complex number on the Argand plane, find that its located in the fourth quadrant at an angle of -30 degrees. You can find the radius by using Pythagorean Theorem.
$\left(\sqrt{3} - i\right)^6 = \left[2cis\left(-30^\circ\right)\right]^6 = 64cis\left(-180^\circ\right) = (64, \pi)$
5. If the diameter is 10 units, then the radius is 5 units. With that information, plus the center being at (-5,5), I can write the equation of the circle in Cartesian coordinates: $(x+5)^2 + (y-5)^2 = 25$. By then using the equations to convert between polar and

rectangular, I can substitute to get: $\left(r\cos\theta+5\right)^2+\left(r\sin\theta-5\right)^2=25$. By then

simplifying: $\boxed{\begin{array}{l} r^2\cos^2\theta+10r\cos\theta+25+r^2\sin^2\theta-10r\sin\theta+25=25 \\ r^2\left(\cos^2\theta+\sin^2\theta\right)+10r\cos\theta-10r\sin\theta+25=0 \\ r^2+10r\cos\theta-10r\sin\theta+25=0 \end{array}}$

6. To find the directrix of a polar conic equation such as: $r=\dfrac{6}{2-2\cos\theta}$, we must first put the equation in the more familiar form by dividing by 2 on the numerator and denominator: $r=\dfrac{3}{1-\cos\theta}$. Now we can use the equation to find the eccentricity: $r=\dfrac{3}{1-\cos\theta}=\dfrac{ek}{1-e\cos\theta}$, which makes the eccentricity 1. A parabola, yayyy! Lucky for us, a parabola only has one directrix. Because that makes the k value equal to 3, and the sign in the equation is negative, and the trigonometric function is cosine, I can write the equation of the directrix as: $x=-3$.

7. The point is found in the second quadrant, and forms a triangle with equal length legs. This indicates a 45 degree reference angle. By using our 45-45-90 triangle ratios, I know the hypotenuse is $9\sqrt{2}$ and that all possible values would be: $\left(9\sqrt{2},{}^{3\pi}\!/_{4}+2k\pi\right)$.

8. $(-4,-4)$ appears to be a Cartesian coordinate pair, but it is Polar Coordinate pair, as stated in the problem. With a negative radius, that means we start on the "negative x" axis to graph, and then rotate in the negative angle direction (clockwise) four radians. (There is no degree symbol, so we must assume radians) 4 is between π (3.14) and $\dfrac{3\pi}{2}$ (4.71), so that means more than two quadrants around, but not as far as three. That lands us in Quadrant IV.

9. $(4,315^\circ)=\left(4,-1485^\circ\right)$

10. Rose – You can tell by the petals!

11. Because of the cosine in the equation, the symmetry will be about the horizontal axis.

12. The largest value that sine can ever be is 1. $r^2=4$, so the largest r can be is 2. The graph is a lemniscate, which can be determined by the equation.

13. The Spiral of Archimedes has an equation of $r = a\theta$.

14. For linear equations in polar coordinates, the equations can be of the form: $\theta = a$, and the slope of these lines can be found by taking the tangent of both sides, and converting to rectangular coordinates: $\tan\theta = \frac{y}{x}$. So to find which line has a slope of $\frac{-1}{\sqrt{2}}$, you would want to know what angle has a tangent of $\frac{-1}{\sqrt{2}}$. When you rationalize, you get: $\frac{-\sqrt{2}}{2}$. This is not a known value from the unit circle for the tangent of an angle.

15. From the given equation: $r = \frac{12}{4-3\cos\theta}$, the first step to classifying the conic is to get the equation in the form: $r = \frac{ek}{1-e\cos\theta}$. $r = \frac{12}{4-3\cos\theta} = \frac{3}{1-\frac{3}{4}\cos\theta}$. Once the equation is in this form, I can see that the eccentricity is ¾, which classifies it as an ellipse. Because the equation has a cosine, I know that the vertices will lie along the x-axis. To find the vertices, I will substitute the two unit circle values that are on the x-axis, 0 and π.

$$\boxed{\begin{array}{l} r = \dfrac{12}{4-3\cos(0)} = 12 \\ r = \dfrac{12}{4-3\cos(\pi)} = \dfrac{12}{7} \end{array}}$$

These coordinates are in polar, so we need to convert to rectangular: $\begin{array}{l}(12,0) \to (12,0) \\ \left(\frac{12}{7}, \pi\right) \to \left(-\frac{12}{7}, 0\right)\end{array}$.

16. To do this problem, I would convert all of the values to polar and then count how many are at the same location:

$\left(-2, 2\sqrt{3}\right) \to \left(4, 120^\circ\right)$

$(-4, 60^\circ) \to \left(4, 240^\circ\right)$

$\left[-2 + 2\sqrt{3}i\right] \to \left(4, 120^\circ\right)$

$4\left(\cos \frac{2\pi}{3} + i\sin \frac{2\pi}{3}\right) \to \left(4, 120^\circ\right)$

17. To find a vector from $(-1,4)$ to the point $(6,28)$, we must use: $\langle \text{terminal-initial} \rangle$.

$\langle 7,24 \rangle$ has a magnitude of $\sqrt{7^2+24^2}$ =25. To find the direction, we use $\begin{array}{c} \tan^{-1}\left({}^{y}\!/\!{}_{x} \right) = \theta \\ \theta = \tan^{-1}\left({}^{24}\!/\!{}_{7} \right) \end{array}$.

18. To convert $\left(-5, \dfrac{5\pi}{6}\right)$ to the trigonometric form of a complex number, we simply use the radius and angle given in the coordinate pair to write: $r(\cos\theta + i\sin\theta) = -5\left(\cos {}^{5\pi}\!/\!{}_{6}\right)$.

To raise this value to the fourth power:

$\left[-5\left(\cos {}^{5\pi}\!/\!{}_{6}\right)\right]^4 = 625cis\left(\dfrac{20\pi}{6}\right) = 625cis\left(\dfrac{10\pi}{3}\right) = 625cis\left(\dfrac{4\pi}{3}\right)$. That angle is in the third quadrant, so both x and y values must be negative.

19. From the graph: the directrix is at $y=-2$, the vertex is at $(0,-1)$, and because it is a parabola, the eccentricity is known to be 1. The equation of the directrix tells me the k value is 2, and that the sign of the equation should be negative, and use a sine function.

$$r = \frac{(2)(1)}{1-(1)\sin\theta} = \frac{2}{1-\sin\theta}$$

20. The equation that Everett is walking on is a circle, so the portion of the circle that he travels will just be a fraction of the circumference. From ${}^{\pi}\!/\!{}_{6}$ to ${}^{2\pi}\!/\!{}_{3}$ is ${}^{\pi}\!/\!{}_{2}$ or ½ of a circle (since the circle is traced twice on the interval from 0 to 2π. From the equation, I know the diameter is 12 units, so the circumference would be 12π. ½ of 12π is 6π.

21. $\left(-5, {}^{\pi}\!/\!{}_{3}\right)$ is located in the third quadrant, at a reference angle of 60 degrees. The only other value with those characteristics is $\left(5, {}^{-2\pi}\!/\!{}_{3}\right)$.

22. The parametric equations $\begin{array}{l} x = 4\cos\theta \\ y = 12\sin\theta \end{array}$ represent an ellipse. Here is how to convert the equations to rectangular and find the enclosed area:

$$\boxed{\begin{array}{l} x = 4\cos\theta; \cos\theta = \dfrac{x}{4} \\ y = 12\sin\theta; \sin\theta = \dfrac{y}{12} \\ \cos^2\theta + \sin^2\theta = 1 \\ \left(\dfrac{x}{4}\right)^2 + \left(\dfrac{y}{12}\right)^2 = 1 \\ A = ab\pi = (4)(12)\pi = 48\pi \end{array}}$$.

23. $\dfrac{\left(4cis\,\pi/3\right)^{-3}\left(5cis\,\pi/6\right)^{4}}{cis\pi}=\dfrac{\left(5cis\,\pi/6\right)^{4}}{\left(4cis\,\pi/3\right)^{3}cis\pi}=\dfrac{625cis\dfrac{4\pi}{6}}{64cis\pi\left(cis\pi\right)}$, the modulus is just the length of the radius, which can be found by using the coefficients and not needing to simplify the angles. $625/64$

24. To find the distance between vertices of $r=\dfrac{16}{2-8\sin\theta}$, we can start by finding the coordinates of the vertices of the conic. Since the equation has a sine function, we know the vertices lie along the y axis. By substituting $\pi/2$ and $3\pi/2$ into the equation, I find the polar coordinates of the vertices to be: $\left(-8/3,\pi/2\right)$ and $\left(8/5,3\pi/2\right)$. Converting those to rectangular coordinates, I get: $\left(0,-8/3\right)$ and $\left(0,-8/5\right)$. The distance between those points is: $\dfrac{8}{3}-\dfrac{8}{5}=\dfrac{40}{15}-\dfrac{24}{15}=\dfrac{16}{15}$.

25. The length of the loop can be found by subtracting the a and b in the equation. D has the largest difference (2) between the values, and thus has the largest length of the loop.

26. A) $\tan\theta=\tan 7\pi/4=\Delta y/\Delta x=-1$

B) $\tan\theta=\tan -3\pi/4=\Delta y/\Delta x=1$

C) $\boxed{\begin{array}{l} r=\dfrac{\pi}{4}\csc\theta \\ r\sin\theta=\dfrac{\pi}{4} \\ y=\dfrac{\pi}{4};m=0 \end{array}}$

D) $\tan\theta=\tan -2\pi/3=\Delta y/\Delta x=\sqrt{3}$

27. The rose $r=6\cos(3\theta)$ has 3 petals starting at 0 degrees, and would be evenly space at: 0, 120, and 240. The lemniscate $r^2=36\sin(2\theta)$ has propellers at 45 degrees and 225 degrees, so those two graphs never intersect.

28. B

29. C – Conchoids of Nicomedes

30. D