"For all questions, answer choice "E. NOTA" means none of the above answers is correct."

1. What is the minimum number of roots a polynomial, f(x), must have if the graph of y=f(x) contains the following points: (-4, 7), (-3, 5), (-2, 6), (-1, -4), (0,-2), (1, -3), and (2, 1)?

A. 0 B. 2 C. 5 D. 6 E. NOTA

Answer: D The information indicates the graph has at least 5 turning points which means it would be at least a 6^{th} degree polynomial. Finite differences can only be calculated down to the 5^{th} degree (since only 7 points are given you would have 6 differences at the first degree, 5 at the second, and so on.)

2. The sum of the integers 1 through n can be modeled by a polynomial. What is the sum of the coefficients of that polynomial?

A. 0 B. ½ C. 1 D. 2 E. NOTA

Answer C. The formula is $\frac{n(n+1)}{2}$ which has two coefficients of ½.

3. How many times does the polar graph of $r(\theta) = \theta^2 - 7\theta + 1$ pass through the pole?

A. 0 B. 1 C. 2 D. 3 E. NOTA

Answer C. A polar graph passes through the pole when r=0 and this happens twice in the given equation.

4. What is the sum of the x-intercepts of $g(t) = e^{2t} + e^{t+ln5} - 14$

A. ln2 B. $\ln(\frac{2}{7})$ C. ln7 D. ln14 E. NOTA

Answer A. The substitution $x = e^t$ yields a factorable quadratic in x for which the roots are 2 and -7. Only one of these x values yields a real value of t (2) so the only x-intercept occurs at t = ln2.

5. Which of the following functions has y = x+2 as a slant asymptote?

A. $y = \frac{x^3+4x^2+6}{x^3+2x^2+3}$ B. $y = \frac{x^3}{x^2-2x}$ C. $y = \frac{4x^2+6}{2x^3+3}$ D. $All\ of\ the\ above$ E. NOTA

Answer B. Slant asymptotes occur when the degree of the numerator is one more than the degree of the denominator, ruling out all but B and E. One obtains the equation of the asymptote by polynomial division (ignoring the remainder term), thus showing B is the answer.

6. The function $f(x) = \frac{ax^2+x-7}{9x^2+bx+4}$ has a horizontal asymptote at y=c and exactly one vertical asymptote. If a is positive, what is $\frac{a}{bc}$?

A. $\frac{1}{16}$ B. $\frac{3}{4}$ C. $\frac{4}{3}$ D. $\frac{3}{2}$ E. NOTA

Answer B. The horizontal asymptote is given by the ratio of the leading coefficients, giving us the equation $\frac{a}{9} = c$. Therefore, $\frac{a}{c} = 9$. Then, since the function only has one vertical asymptote and the discriminant of the numerator is positive (1 +28a) we can conclude that there are no holes so the discriminant of the denominator must be zero, yielding 12 for the value of b. Therefore the fraction is $\frac{9}{12}$ which reduces to $\frac{3}{4}$.

7. What is the y-coordinate of the hole in the graph of $f(x) = \frac{x-5}{x^2-25}$?

A. 0 B. $\frac{1}{5}$ C. 1 D. 5 E. NOTA

Answer E. After simplification, removing the discontinuity, the expression is $\frac{1}{x+5}$ and direct substitution of x=5 (the x-coordinate of the discontinuity) yields the value $\frac{1}{10}$, which is not among the given choices.

8. If $f(x) = x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32$, what is $f^{-1}(-32)$?

A. -4 B. -2 C. 0 D. 2 E. NOTA

Answer A. $f(x)$ is the expansion of $(x+2)^5$. The inverse would be $\sqrt[5]{x} - 2$.

9. On what interval(s) is f(x) > 1, if $f(x) = \frac{x-7}{x^2+3}$?

A. $(7, \infty)$ B. $(-\infty, 7)$ C. $\left(\frac{1-\sqrt{17}}{2}, \frac{1+\sqrt{17}}{2}\right)$ D. $\left(-\infty, \frac{1-\sqrt{17}}{2}\right) \cup \left(\frac{1+\sqrt{17}}{2}, \infty\right)$ E. NOTA

Answer E. This function is not even positive until after x=7 (since the denominator is always positive) and by the point the denominator is far larger than the numerator and increasing at a faster rate.

10. If the roots of $3x^4 - 8x^3 + 9x - 17$ are a, b, c, and d, find $a + b + c + d$.

A. -3 B. $-\frac{8}{3}$ C. $\frac{8}{3}$ D. 8 E. NOTA

Answer C. After factoring, the polynomial will be $3(x-a)(x-b)(x-c)(x-d)$. The x^3 term of the expansion will be $3(-a-b-c-d)x^3$. Factoring out a -3 gives you the desired expression, so we divide the coefficient -8 by -3.

11. $f(x) = x^7 - 4x^6 - 6x^5 + 7x^4 + x^3 - x - 5$ and a is the maximum number of positive zeroes guaranteed by Descartes' Rule of Signs and b is the maximum number of negative zeroes guaranteed by the rule, what is a-b?

A. -3 B. -1 C. 1 D. 3 E. NOTA

Answer B. The number of sign changes in the coefficients of f(x) give a and the number of sign changes in f(-x) give b. These are 3 and 4 respectively, hence B.

12. If $f(x) = (-3x+2)(x-5)^2(x-11)$ on what interval(s) is f(x) strictly positive?

A. $\left(-\infty, \frac{2}{3}\right) U(11, \infty)$ B. $(\frac{2}{3}, 5)$ C. $(5, 11)$ D. $(\frac{2}{3}, 11)$ E. NOTA

Answer E. The function has end behavior tending toward negative infinity as x tends toward both negative and positive infinity and has zeroes at 2/3, 5, and 11. Since the zero at 5 is a double zero the function doesn't change sign there as at the other zeroes, but since it still is equal to zero at 5 the correct answer would be $\left(\frac{2}{3}, 5\right) U(5, 11)$.

13. What is the remainder when $x^4 + 3x^3 - 4x^2 + 5x - 6$ is divided by $x - 1$?

A. -17 B. -1 C. 1 D. 6 E. NOTA

Answer B. The answer can easily be found by plugging in 1 for x.

14. The function $f(x) = \frac{ax^2+4x+c}{3x^2+5x-7}$ has a horizontal asymptote of $y = 3$ and a zero at $x = 1$. Find ac.

A. -117 B. -21 C. -5 D. -4 E. NOTA

Answer A. The asymptote requires that $a = 9$ and, given that, the zero requires $c = -13$

15. For what value(s) of θ does the polar function $r(\theta) = \theta^3 - \pi\theta^2 - \frac{\pi^2}{4}\theta + \frac{\pi^3}{4}$ pass through the pole?

A. $-\pi$ B. $-\frac{\pi}{2}$ & $\frac{\pi}{2}$ C. π D. $-\frac{\pi}{2}, \frac{\pi}{2},$ & π E. NOTA

Answer D. Polar functions cross the pole when $r = 0$. Simple factoring by grouping reveals the answer is D.

16. If $\cos^2(\theta) + \frac{64}{65}\cos(\theta) + \frac{3}{13} = 0$, what is the sum of all possible values of $\csc(\theta)$ if $0 \leq \theta \leq \pi$?

A. -7 B. -2 C. 2 D. 7 E. NOTA

Answer E. Solving the quadratic for $\cos(\theta)$ reveals the two possible values for it are $-\frac{3}{5}$ and $-\frac{5}{13}$. Given the restrictions on θ and using Pythagorean triples, we can get that the possible values for $\sin(\theta)$ are $\frac{4}{5}$ and $\frac{12}{13}$ respectively. After reciprocating for $\csc(\theta)$ and adding we get 7/3.

17. How many asymptotes (of any kind) are on the graph of $(x) = \frac{x-2}{x^3-7x}$?

A. 0 B. 1 C. 2 D. 3 E. NOTA

Answer E. There are no slant asymptotes, 3 vertical (at the zeroes of the denominator), and one horizontal (y=0).

18. How many zeroes does the function $f(x) = \cos\left(\frac{1}{x}\right)$ have on the open interval $(0,1)$?

A. 0 B. 1 C. 2 D. infinitely many E. NOTA

Answer D. As x approaches zero the argument to the cosine function approaches infinity, generating an infinite number of zeroes for cosine along the way.

19. What is the largest x value for which $f(x) = \cos\left(\frac{1}{x}\right)$ has a zero?

A. $\frac{1}{\pi}$ B. $\frac{2}{\pi}$ C. $\frac{5}{2\pi}$ D. $\frac{\pi}{2}$ E. NOTA

Answer B. As x gets larger the argument of the cosine function approaches zero. This means that the x value which generates the smallest argument of the cosine function which still results in a zero is what we are looking for. That argument is $\frac{\pi}{2}$, therefore the x value needs to be $\frac{2}{\pi}$.

20. What happens to the distance between consecutive zeroes of $f(x) = \sin(x^2)$ as x increases over the interval $(-100, -50)$?

A. it always decreases B. it remains constant C. it always increases
D. it decreases then increases E. NOTA

Answer C. Since the function is even, the phenomenon may be more easily considered on the interval (50, 100). As x increases on this interval, the argument for sine is increasing at a polynomial rate. When it increases at a linear rate the distance remains constant, so here the distances will decrease since we are cycling through the sine values at a faster rate. Then we apply this logic to the mirror image on the given interval, only we are travelling the opposite direction, so they are always increasing.

21. What is the sum of the coefficients of the expansion of $(x-5)^9$?

A. -5^9 B. -2^9 C. 0 D. 2^9 E. NOTA

Answer E. Expanding the polynomial will yield an expression of the form $x^9+bx^8+cx^7+\cdots+hx-5^9$. Substituting 1 in for x yields the sum of the coefficients, so one may do the same for the expression in the problem, which is equivalent, yielding -4^9.

22. $f(x)$ and $g(x)$ are both polynomial functions such that $f\big(g(x)\big)=\ g\big(f(x)\big)=\ x$. If a is the degree of $f(x)$ and b is the degree $g(x)$ what is $a+b$?

A. 0 B. 1 C. 2 D. 3 E. NOTA

Answer C. In order for f and g to be inverses and both be polynomials, they must be linear.

23. $f(x)$ and $g(x)$ are both polynomial functions of degree greater than 2 such that the degree of g is one more than the degree of f. If the leading coefficient of f is positive and odd and the leading coefficient of g is negative and odd, what is the end behavior of $g\big(f(-x)\big)$?

A. As $x\to\ \infty, y\to\ \infty$ and as $x\to\ -\infty, y\to\ \infty$
B. As $x\to\ \infty, y\to\ -\infty$ and as $x\to\ -\infty, y\to\ -\infty$
C. As $x\to\ \infty, y\to\ \infty$ and as $x\to\ -\infty, y\to\ -\infty$
D. Not enough information
E. NOTA

Answer B. Because the degrees of f and g are only one apart $g\big(f(-x)\big)$ is going to have an even degree, giving us end behavior in the same direction. Since exponents are performed before multiplication, the leading coefficient of the composite function will still be negative, since any leading coefficient of the inner function would either be positive already (if the degree of f was even) or would be raised to an even power after being placed in g (if the degree of f was odd) and then subsequently multiplied by the negative leading coefficient of g.

24. $f(x)$ and $g(x)$ are both polynomial functions of degree greater than 2 such that they both have a leading coefficient of 1 and the degree of f is n and the degree of g is $2n$. The function $h(x) = \frac{g(x)}{f(f(x))}$ approaches an asymptote as $x \to \infty$. What is this asymptote and from which side does it approach?

A. y = 0 from the bottom B. y = 0 from the top
C. y = 1 from the bottom D. y = 1 from the top E. NOTA

Answer B. The denominator is going to be of degree n^2 which is greater than 2n for all n > 2, therefore the asymptote is y = 0. Since both f and g have positive end behavior in that direction, the rational function h is going to be positive for sufficiently large values of x, therefore it is approaching from the top.

25. Let s be the sum of the terms in the 28th row of Pascal's triangle. Find $\sqrt[3]{s}$

A. 512 B. $512\sqrt[3]{2}$ C. 1024 D. $1024\sqrt[3]{4}$ E. NOTA

Answer A. The rows of Pascal's triangle are related to the coefficients of binomial expansions. Following a reasoning similar to that in # 21 above, one can deduce that the sums of these rows are increasing powers of 2, starting with 2^0 in the first row. S is therefore $2^{27}$and the cuberoot of s is 2^9, which is 512.

26. Which of the following represents the function $f(x) = x^2$ with holes at 2 and -2?

A. $g(x) = \frac{x^4-4x^2}{x^2}$ B. $g(x) = \frac{x^2(x-2)^2}{(x-2)^2}$ C. $g(x) = \frac{x^2}{x^2(x-2)^2}$
D. $g(x) = \frac{x^2-4}{x^4-4x^2}$ E. NOTA

Answer E. Answer D comes closest since it is the only one for which the denominator and numerator of the rational function are zero at those two x values, but when simplified it yields $\frac{1}{x^2}$ rather than x^2.

27.Find $\sum_{n=0}^{\infty} \frac{n^2-3n+3}{2^n}$

A. 1 B. $\frac{3}{2}$ C. 3 D. 6 E. NOTA

Answer D. Writing out a few terms and taking advantage of the polynomial numerator to employ finite differences is perhaps the easiest solution method. Let us call the sum x, the first few terms of which are $3 + \frac{1}{2} + \frac{1}{4} + \frac{3}{8} + \frac{7}{16} + \cdots$ If we divide by 2 on both sides we get $\frac{1}{2}x = \frac{3}{2} + \frac{1}{4} + \frac{1}{8} + \frac{3}{16} + \cdots$ Subtracting these two (in order to employ finite differences to eventually yield a geometric series) we get $\frac{1}{2}x = 3 - \frac{2}{2} - \frac{0}{4} + \frac{2}{8} + \frac{4}{16}$ (I have purposely left these unsimplified to more clearly show that we now have a linear pattern on the numerator rather than quadratic). Repeating this process once more we get $\frac{1}{4}x = 3 - \frac{5}{2} + \frac{2}{4} + \frac{2}{8} + \frac{2}{16} + \cdots$ Aside from the first two terms, the rest of the numerators are 2, which means you have a geometric series (you can test later terms if you wish) the sum of which is 1. This simplifies the equation to $\frac{1}{4}x = 4 - \frac{5}{2}$ from which point the solution should be obvious.

28. The function $g(x) = \frac{1}{f(x)}$ has three vertical asymptotes at x = a, b, and c. $f(x)$ is a cubic polynomial with leading coefficient 1. What is the y-coordinate of the y-intercept of $g(x)$?

A. abc B. $\frac{1}{abc}$ C. $a + b + c$ D. $\frac{1}{a+b+c}$ E. NOTA

Answer E. This means a, b , and c are the zeroes of f(x), which means the constant term of f(x) is –abc. Because of this the answer is the negative of answer B.

29. How many *possible* rational zeroes are there for $f(x) = 4x^7 + 2x^6 - 9x^4 + x^3 - 72$

A. 7 B. 18 C. 36 D. 72 E. NOTA

Answer C. The Rational Zeroes Theorem states that all possible rational zeroes for a polynomial function must be of the form $\frac{p}{q}$ where p is a factor of the constant term and q is a factor of the leading coefficient. 72 has 1, 2, 3, 4, 6, 8, and their "partners" (9 is "partner" to 8 to make 72, etc.) giving us a 12 positive factors (24 factors total). Only 1, 3, and 9 generate different possible zeroes when divided by 2, which adds 6 possible zeroes to our count (including negatives). The same occurs when dividing by 4 (because the possible zeroes counted when dividing by two take care of any other factor producing new possible zeroes). This gives us a total of 36.

30. Find the product of all solutions for x in the following equation: $\frac{x^2 + 3x+2}{x^2 + 2x+1} = \frac{x+2}{x^2+4x+3}$

A. -4 B. -2 C. 2 D. 4 E. NOTA

Answer B. After factoring and simplifying one will find $x = -2$ and, depending on the method, $x = -1$. The latter solution, however, is extraneous.