For each question, "E) NOTA" indicates that none of the above answers is correct.

1. Let $2.01\overline{6} = \frac{A}{B}$, with $A$ and $B$ relatively prime positive integers. Find $AB$.

A) $45150$ B) $181500$ C) $7260$ D) $465$ E) NOTA

2. Given the sequence $\frac{1}{6}, \frac{2}{7}, \frac{3}{8}, \cdots, \frac{n}{n+5}$ ..., find the product of the sequence's first $50$ terms.

A) $\frac{1}{3 \cdot 11 \cdot 13 \cdot 28 \cdot 53}$ B) $\frac{1}{9 \cdot 11 \cdot 13 \cdot 53 \cdot 56}$ C) $\frac{2}{9 \cdot 11 \cdot 13 \cdot 17 \cdot 53}$ D) $\frac{1}{11 \cdot 13 \cdot 17 \cdot 27 \cdot 53}$ E) NOTA

3. Evaluate:

$$\prod_{n=2}^{1023} \log_n(n+1)$$

A) $\log_2 10$ B) $\log_{1023} 10$ C) $\log_{1023} 1024$ D) $10$ E) NOTA

4. The interior angles of a convex hexagon are in an arithmetic progression. If the smallest angle is $105°$, find the cosine of the largest angle.

A) $\frac{\sqrt{2}}{2}$ B) $\frac{-\sqrt{3}}{2}$ C) $\frac{-1}{2}$ D) $\frac{-\sqrt{6}-\sqrt{2}}{4}$ E) NOTA

5. Let $t, u, v, w$, and $x$ be natural numbers in an arithmetic progression (in this order) with a common difference of 1. If $t + u + v + w + x = y^3$, and $u + v + w = z^2$ for some natural numbers $y$ and $z$, what is the least possible value for $v$?

A) $246$ B) $100$ C) $75$ D) $675$ E) NOTA

6. The infinite geometric series $1 + r + r^2 + r^3 + \cdots$ has the finite sum $S$. What is the sum of the following series in terms of $S$ and $r$?

$$1 + r^2 + r^4 + r^6 + \cdots$$

A) $\frac{S}{1-r}$ B) $\frac{S}{r}$ C) $Sr^2 + 1$ D) $\frac{S}{1+r}$ E) NOTA

7. The second term of a geometric sequence is 2 and the fourth term is 6. Which of the following is a possible first term in this sequence?

A) $\frac{-2\sqrt{3}}{3}$ B) $-\sqrt{3}$ C) $\frac{-\sqrt{3}}{3}$ D) 3 E) NOTA

8. Find the sum of an infinite geometric series with first term $\frac{17}{2}-\frac{3}{2}i$ and common ratio $\frac{1}{2}+\frac{1}{2}i$ where $i=\sqrt{-1}$.

A) $10+7i$ B) $20+14i$ C) $17-3i$ D) the sum diverges E) NOTA

9. Simplify and find the prime factorization of the following sum (the general term of the series is geometric):

$$\sum_{n=1}^{2016}\left(\frac{n}{7}+\frac{n}{14}+\frac{n}{28}+\frac{n}{56}+\cdots\right)$$

A) $2^4\cdot3^2\cdot2017$ B) $2^5\cdot3^2\cdot7\cdot2017$ C) $2^5\cdot3^2\cdot2017$ D) $2^5\cdot3\cdot7^2\cdot2017$ E) NOTA

10. The first term of an arithmetic sequence is $74$ and the last term is $2020$. If there are $319$ terms in the sequence, find the average value of these $319$ numbers.

A) $973$ B) $1047$ C) $937$ D) $1074$ E) NOTA

11. The expression $\log\left(\frac{1}{2}+1+\frac{3}{2}+2+\cdots+1008\right)$, where the series is arithmetic, is equivalent to which of the following?

A) $\log 504+\log 2017$ B) $\frac{1}{2}\log 2016+\frac{1}{2}\log 2017$
C) $(\log 2016)(\log 2017)$ D) $\log 2016+\log 2017-\log 2$ E) NOTA

12. In an arithmetic sequence, $a_1=-2016$ and $a_{2016}=2016$. Find the total number of positive and negative integral factors of $|\lfloor a_6\rfloor|$.

A) $8$ B) $4$ C) $16$ D) $12$ E) NOTA

13. Let $g_1,g_2,g_3,\cdots$ be an infinite geometric sequence of positive numbers. If this sequence has the property that $g_n=g_{n+1}+g_{n+2}$, which of the following could be the common ratio?

A) $\frac{-\sqrt{5}-1}{2}$ B) $\frac{\sqrt{5}}{2}$ C) $\frac{\sqrt{5}-1}{2}$ D) $\frac{2\sqrt{5}}{5}$ E) NOTA

14. Let $M =$ the number of consecutive zeroes at the end of $2016!$
Let $A =$ the highest integral number of 7's that evenly divides $2016!$
And let $\theta = [\sin(2016!\,°) - \cos(2016!\,°)]^{2016!}$
Find $M - A - \theta$.

A) $168$ B) $169$ C) $164$ D) $167$ E) NOTA

15. Find

$$-2i + \cfrac{1}{-2i + \cfrac{1}{-2i + \cfrac{1}{\cdots}}}$$

where $i = \sqrt{-1}$.

A) $\pm i$ B) $0$ C) $\pm\sqrt{2} + i$ D) $\pm\frac{\sqrt{3}}{2} - \frac{1}{2}i$ E) NOTA

16. What is the units digit of $5 + 5^2 + 5^3 + \cdots + 5^{2016}$?

A) $0$ B) $1$ C) $2$ D) $5$ E) NOTA

17. Phil the penguin is waddling across the Cartesian plane. He starts his journey at the origin and moves $625$ units to the right to the point $(625,0)$. Being tired, he then turns $90°$ to his right and walks only one-fifth as far to the point $(625, -125)$. If he continues to turn $90°$ to his right and walk one-fifth as far as he did on his previous leg of the trip indefinitely, how far from his starting point will he end up?

A) $\infty$ B) $\frac{625\sqrt{26}}{26}$ C) $\frac{15625\sqrt{26}}{26}$ D) $\frac{3125\sqrt{26}}{26}$ E) NOTA

18. What is the smallest positive integer $z$ such that the sum of the arithmetic series $z + 2z + 3z + \cdots + 99z + 100z$ is a perfect square?

A) $202$ B) $5050$ C) $100$ D) $101$ E) NOTA

19. An infinite geometric series has first term $17$ and common ratio $4x - 8$. Find the range of values of $x$ for which the sum converges to a finite value.

A) $\left[\frac{7}{4}, \frac{9}{4}\right]$ B) $\left(\frac{7}{4}, \frac{9}{4}\right]$ C) $\left(\frac{7}{4}, \frac{9}{4}\right)$ D) $\mathbb{R}$ E) NOTA

20. Evaluate:

$$\prod_{n=0}^{100}(1+i)^n$$

A) $-2^{2525}i$ B) $2^{5050}$ C) $-2^{2525}$ D) $2^{2525}i$ E) NOTA

21. What is the sum of all the numbers on the interior of Pascal's triangle in the first 2016 rows? (Note: The interior of Pascal's triangle includes all numbers $\neq 1$).

A) $2^{2015}-4031$ B) $2^{2015}-4033$ C) $2^{2016}-2015$ D) $2^{2016}-4032$ E) NOTA

22. Simplify: $\frac{6}{3}+\frac{6}{8}+\frac{6}{15}+\frac{6}{24}+\cdots+\frac{6}{n(n+2)}+\cdots$

A) $4$ B) $5$ C) $\frac{9}{2}$ D) $\frac{14}{3}$ E) NOTA

23. Alex, the fisherman, always catches 3 more fish each day than the previous day. If he caught 47 fish on June 10th, how many fish did he catch in total during the month of June?

A) $1905$ B) $2015$ C) $1860$ D) $1815$ E) NOTA

24. Let $A=$ the 10th term of the arithmetic sequence: $15, 22, 29, \cdots$
Let $B=$ the common ratio of the geometric sequence: $\frac{2}{9}, \frac{-4}{81}, \frac{8}{729}, \cdots$
Let $C=$ the sum of the squares of the least ten natural numbers
Let $D=$ the first term of an arithmetic sequence with 12th term 76 and common difference 7.
Find

$$\begin{vmatrix} C-A & B \\ \frac{81}{2} & D \end{vmatrix}$$

A) $-316$ B) $316$ C) $-2447$ D) $-298$ E) NOTA

25. The roots of the function $y=\frac{7}{5}x^5-\frac{217}{40}x^4+\frac{217}{32}x^3-\frac{217}{64}x^2+mx-\frac{7}{160}$ are in a geometric progression with $R_{n+1}=R_n\cdot r$ where $R_1, R_2, \cdots, R_5$ are the roots and $r$ is the common ratio of the geometric progression. If $R_1=\frac{1}{8}$ and $R_1<R_2<\cdots<R_5$, find $6R_1+7R_2+7R_3+7R_4+7R_5$ .

A) $\frac{217}{8}$ B) $27$ C) $\frac{757}{20}$ D) $32$ E) NOTA

26. Amy and Bo are playing a dice game with the following rules: If you roll a $6$, you win on that turn. If you roll a $1$, you lose on that turn. If you roll a 2,3,4, or 5, the other player then has to take a roll. (Note: there is only one winner and one loser at the end of the game). If Amy goes first, what is the probability she wins?

A) $\frac{2}{3}$ B) $\frac{3}{10}$ C) $\frac{3}{4}$ D) $\frac{1}{2}$ E) NOTA

27. Let the sequence $x_{-101}, x_{-100}, x_{-99}, \cdots, x_{100}, x_{101}$ represent an arbitrary arrangement of the numbers $-101, -100, -99, \cdots, 100, 101$. Then the product:

$$(x_{-101} - (-101))(x_{-100} - (-100)) \cdots (x_{99} - 99)(x_{100} - 100)(x_{101} - 101)$$

is always which of the following?

A) even B) odd
C) $0$ D) indeterminate (not enough information) E) NOTA

28. The sum of $18$ consecutive positive integers is a perfect square. What is the smallest value this sum could be?

A) $169$ B) $225$ C) $289$ D) $324$ E) NOTA

29. Let $D_n$ denote the smallest number greater than 1 which when divided by any of the least $n$ distinct positive prime integers always leaves a remainder of 1. Find the value of:

$$\frac{D_5 - D_4}{D_2 D_1}$$

A) $80$ B) $\frac{3465}{16}$ C) $100$ D) $\frac{60}{7}$ E) NOTA

30. Let $ABCD$ be a convex quadrilateral with the Fibonacci property: $m\angle A = m\angle B$, $m\angle A + m\angle B = m\angle C$, and $m\angle B + m\angle C = m\angle D$. Let $\frac{x}{y} = m\angle D$ expressed as an irreducible fraction of positive integers. Find $y - x$.

A) $353$ B) $673$ C) $763$ D) $1073$ E) NOTA