Algebra 2 Hustle Solutions

1. $\sum_{k=1}^{20}(2k^2 - k)$ = $\frac{2(20)(21)(41)}{6} - \frac{20(21)}{2}$ = 20(7)(41) – 10(21) = 5740 – 210 = **5530**

2. $\left(\frac{1}{9}\right)^x = 27^{x-2}$ = 3^{-2x} = 3^{3x-6} → -2x = 3x – 6 → **6/5**

3. $\frac{1}{1+\frac{1}{1-\frac{1}{1+i}}} = \frac{1}{1+\frac{1}{\frac{i}{i+1}}} = \frac{1}{1+\frac{i+1}{i}} = \frac{1}{\frac{2i+1}{i}} = \frac{i}{2i+1} = \frac{-2-i}{-5} = \mathbf{\frac{2}{5} + \frac{i}{5}}$

4. $\log_{3\sqrt{3}} 729 = \log_{3^{\frac{3}{2}}} 3^6 = \mathbf{4}$

5. f(x) = $\frac{-20}{225}(x-15)(x+15)$ → f(6) = $\frac{-20}{225}(-9)(21)$ = **84/5**

6. A =13, C = $\sqrt{5}$, B = $\sqrt{A^2-B^2}$ = $\sqrt{164}$ = $2\sqrt{41}$. Therefore, the area is πab = $\mathbf{26\pi\sqrt{41}}$

7. (4 + i)(4 – i)(-6) = 17(-6) = **-102**

8. Simplify $\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca}$ into $\frac{c+a+b}{abc}$ and use Vieta's to obtain $\mathbf{\frac{7}{23}}$

9. $1 - \frac{\binom{45}{2}}{\binom{90}{2}} = 1 - \mathbf{\frac{22}{89} = \frac{67}{89}}$

10. f(x) = $6x^4 + 41x^3 + 88x^2 + 67x + 14$. f(-1) = 0 so -1 is a root. Use synthetic division to reduce fourth degree polynomial to $6x^3 + 35x^2 + 53x + 14$. This factors as (x + 2)(3x + 1)(2x + 7). Therefore, the four roots of this polynomial are **-1, -2,** $\mathbf{-\frac{1}{3}, and -\frac{7}{2}}$

11. 720 is the product of $2^4 \times 3^2 \times 5$. Therefore, we have 5(3)(2) = **30 factors**

12. 699_{15} is equal to 1494 in base 10. Converting this to base 8, the answer becomes $\mathbf{2726_8}$

13. $7^1 = 7$, $7^2 = 49$, $7^3 = 343$, $7^4 = 2401$. Tens digit is 0, 4, 4, 0, 0, 4, 4, 0, …. The tens digit of 7^{2018} is **4.**

14. .2 + .037 + .00037 + .0000037 + ….. = $.2 + \frac{\frac{37}{1000}}{\frac{99}{100}} = \mathbf{\frac{47}{198}}$

15. Using Reminder Theorem, the remainder will be 2(1) + 1 = **3**

16. $(x-1)^3 + x^3 + (x+1)^3 = (x+2)^3$ This rearranges to $0 = x^3 - 3x^2 - 3x - 4$,the only integer solution is x = 4, so the only set of consecutive integers is **{3, 4, 5, 6}**

17. S = $\frac{9(A_1+A_9)}{2} = \frac{9(A_5+A_5)}{2} = 9 * A_5$ = 9 *2 = **18**

18. $\sqrt{6+\sqrt{6+\sqrt{6+\ldots}}}$ → x = $\sqrt{6+x}$ → $x^2 = 6 + x$ → $x^2 - x - 6 = 0$, (x-3)(x+2) = 0, **x = 3**

19. $(2x + y - z)^{12}$ → $(2(1)+1-1)^{12} = 2^{12}$ = **4096**

20. The parabola has it's highest point at the vertex. The x-coordinate of the vertex is $\frac{-b}{2a}$ = **1**

21. $x^2 = 3x + 4 \rightarrow x^2 - 3x - 4 = 0$. Solution is **x = 4** since x = -1 is extraneous

22. Matrix C is the identity matrix which is the product of A and B. Therefore matrix B is just the inverse of matrix A. The determinant of A is 12.

$A^{-1} = \frac{1}{12}\begin{bmatrix} 4 & -1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{-1}{12} \\ \frac{1}{3} & \frac{1}{6} \end{bmatrix}$ **The sum of these entries is ¾**

23. Eccentricity of hyperbola is given by $\sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{4}{9}} = \sqrt{\frac{13}{9}} = \mathbf{\frac{\sqrt{13}}{3}}$

24. The sum of every four terms is 2 – 2i. Therefore, the entire sum is **8 – 8i**

25. The sum of the roots is -6 and the product of the roots is -45. The quadratic is $\mathbf{x^2 + 6x - 45}$**.**