All uppercase letter variables are positive integers unless otherwise stated. All fractions containing uppercase letter variables are in lowest terms. NOTA means "None of the Above."

1) Mettaton is sliding along the x-axis with velocity $3t^2 - 4$. At time $t = 1$ seconds, Mettaton is at the point $(-1,0)$. Where is Mettaton at time $t = 4$?

(A) $(30,0)$ (B) $(34,0)$ (C) $(44,0)$ (D) $(50,0)$ (E) NOTA

2) The volume of a cube is increasing at a rate of 6 units cubed per second. If the longest diagonal of the cube is increasing at a rate of $A \cdot 3^{-B/C}$ units per second when the volume of the cube is 81 units cubed, find $A + B + C$.

(A) 9 (B) 15 (C) 21 (D) 24 (E) NOTA

3) Find the instantaneous rate of change of the value of $\int_x^{x^2} (t^2 - 1)\, dt$ at $x = 2$.

(A) $\frac{50}{3}$ (B) 36 (C) 42 (D) 54 (E) NOTA

4) Consider the function $f(x) = x^3 - 6x^2 + 11x + 9$. Let $f^{\theta}(x)$ be the graph obtained by rotating $f(x)$ θ degrees clockwise about the function's inflection point. Find the maximum value of $\alpha \leq 90$ such that for all $\theta \in [0, \alpha)$, $f^{\theta}(x)$ is a function.

(A) 30 (B) 45 (C) 60 (D) 90 (E) NOTA

5) A right circular cone has height 6 and radius 8. If the greatest volume of an inverted square pyramid that can be inscribed in the cone is equal to $2^A/3^B$, find $A + B$.

(A) 11 (B) 12 (C) 13 (D) 14 (E) NOTA

6) In the dark, distant future of Mu Alpha Theta, tests will be 100 questions long and each question will have 20 answer choices, with only one choice being correct. Radleigh decides to guess on every question because the reduced time limit of 10 minutes just isn't enough time for him. The probability that Radleigh gets every question wrong is closest to which of the following?

(A) e^{-2} (B) e^{-5} (C) e^{-10} (D) e^{-100} (E) NOTA

7) A 140-degree piece of metal is placed in a 20-degree room. After 10 minutes, it has temperature 80 degrees. During which of the following time intervals will the metal reach 25 degrees, given that Newton's Law of Cooling applies?

(A) 10-20 minutes (B) 20-30 minutes (C) 30-40 minutes (D) 40-50 minutes (E) NOTA

8) The longest horizontal ladder that can pass through (make the right turn) the perpendicular junction of two identical long hallways with width 8 is equal to $\sqrt{A}$. Find the sum of the digits of A.

(A) 7 (B) 8 (C) 14 (D) 19 (E) NOTA

9) How many values of k in the range $[0,2018\pi]$ satisfy the equation $\int_0^k \sin x \; dx = \frac{1}{2}$?

(A) 2017
(B) 2018
(C) 4035
(D) 4036
(E) NOTA

10) What is the x-intercept of the tangent line to $y = 6x^3 - 3x^2 + 4x - 12$ at $x = 2$?

(A) 0
(B) $\frac{1}{2}$
(C) 1
(D) $\frac{3}{2}$
(E) NOTA

11) Which of the following statements are true?

I: If $f(x) > 1$ for all $x \in \mathbb{R}$, then $\lim_{x \to 0} f(x) > 1$.

II: If $f'(k) = 0$, x is not constant, and $f(x)$ is not constant, then $f(x)$ has a relative extremum or inflection point at $x = k$.

III: $x = 0$ is not a function.

(A) III ONLY
(B) I and II ONLY
(C) I and III ONLY
(D) II and III ONLY
(E) NOTA

12) Bradley's Bagels sells bagels for \$1 each. Market analysis shows that for every 5-cent decrease in the price of his bagels, Bradley will sell 100 more bagels. If Bradley can sell 1600 bagels for a dollar each, what should Bradley change the price per bagel to in order to earn the most money?

(A) \$0.80
(B) \$0.85
(C) \$0.90
(D) \$0.95
(E) NOTA

13) Eridan is painting his room with purple and pink paint. Purple paint costs $\$(27x + 4)$ total for x gallons, and pink paint costs $\$(2x^2 - x + 1)$ total for x gallons. How many gallons of purple paint should Eridan buy to minimize cost if he needs 20 gallons of paint to paint his room?

(A) 10
(B) 11
(C) 12
(D) 13
(E) NOTA

14) If $I = \frac{\pi}{M} - \sqrt{N}$, find $M + N$.

$$I = \int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} \sqrt{\frac{1-x}{1+x}} \arcsin x \; dx$$

(A) 6
(B) 8
(C) 9
(D) 12
(E) NOTA

15) The area inside the inner loop of the limaçon $r = 2 + 4\cos\theta$ is equal to $A\pi - B\sqrt{C}$ where C is squarefree. Find the value of $A + B + C$.

(A) 8
(B) 10
(C) 11
(D) 13
(E) NOTA

16) In an alternate universe, the Sphinx doesn't ask Oedipus a question about man and instead poses the following riddle to Jackson: "*There once was a man from Nantucket / Who lived in a cylindrical bucket. / With a radius of 4, / And height 8 and no more, / If water is entering the bucket at a rate directly proportional to the amount of water already in the bucket and is flowing into the bucket at a rate of 16 cubic units per second when it overflows, how many seconds does it take to fill the bucket if the bucket initially has* 8π *units cubed of water in it… ucket?*" The Sphinx coughs and apologizes for her wordy limerick, but demands the answer anyways. What correct answer does Jackson need to give to not be eaten by the Sphinx?

(A) $32\pi \ln 2$
(B) 32π
(C) $64\pi \ln 2$
(D) 64π
(E) NOTA

17) Samir drops a ball from the top of a 45-meter tall building. Given that the acceleration due to gravity is 10 meters per second squared and air resistance is ignored, how long in seconds does it take for the ball to hit the ground?

(A) 2
(B) 2.5
(C) 3
(D) 3.5
(E) NOTA

18) Sameer throws a ball straight down from the same building 1 second after Samir does. Given that the two balls hit the ground at the same time, with what initial velocity does Sameer throw the ball with, in meters per second?

(A) $\frac{15}{2}$
(B) $\frac{25}{2}$
(C) 15
(D) 25
(E) NOTA

19) Find R.

$$R = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{\ln(i+n) - \ln n}{i+n}$$

(A) $\frac{\ln^2 2}{2}$
(B) $\frac{\ln 2}{2}$
(C) $\ln^2 2$
(D) $\ln 2$
(E) NOTA

20) Find S.

$$S = \sum_{n=1}^{\infty} \int_0^{\frac{1}{e^n}} \left((\log_x e)^2 - \log_x e\right) dx$$

(A) $1 - \ln(e-1)$
(B) $\ln(e-1)$
(C) 1
(D) $\ln(e+1)$
(E) NOTA

21) Paloma the parrot is flying along the curve $r = 2e^{\theta}$ starting at $\theta = 0$. How far does Paloma travel in her first full rotation around the polar plane?

(A) $\sqrt{2}(e^{\pi} - 1)$
(B) $2\sqrt{2}(e^{\pi} - 1)$
(C) $\sqrt{2}(e^{2\pi} - 1)$
(D) $2\sqrt{2}(e^{2\pi} - 1)$
(E) NOTA

22) On his daily route, a postman delivers packages to people with houses on the graph of $f(x) = x^4 - 2x^3 - 3x^2 + 3x + 7$. Today, he notices that two of his deliveries go to the residents of two special houses. These houses are the intersections of $f(x)$ and the unique line that is tangent to $f(x)$ at two distinct points. What is the sum of the ordinates of these two points?

(A) 1
(B) 2
(C) 5
(D) 6
(E) NOTA

23) Find the maximum volume of the solid formed by rotating the region bounded by the ellipse $25x^2 + 9y^2 - 150x + 72y + 144 = 0$ about a line that passes through the point $(-5,2)$.

(A) $150\pi^2$
(B) $180\pi^2$
(C) $240\pi^2$
(D) $300\pi^2$
(E) NOTA

24) Find L.

$$L = \lim_{x\to 0} \frac{\sqrt{1+\sin^2(x^2)} - \cos^3(x^2)}{x^3 \tan x}$$

Hint: l'Hospital's Rule is not required for this problem.

(A) 1
(B) 2
(C) 3
(D) DNE
(E) NOTA

25) Given the following equation, find $f(4)$.

$$\int_0^{x^2} f(t)\,dt = x^2 \sin \pi x$$

(A) $\frac{\pi}{4}$
(B) $\frac{\pi}{2}$
(C) π
(D) 2π
(E) NOTA

26) Aaron is standing some distance away from a vertical screen that is 24 feet tall and 14 feet off the ground. If Aaron's eyes are 6 feet above the ground, how many feet away should Aaron stand from the screen to maximize his viewing angle (the angle between the ray extending from Aaron's eyes to the top of the screen and the ray extending from Aaron's eyes to the bottom of the screen)? You may assume that the ground is flat and that Aaron is looking directly at the screen.

(A) 8
(B) $8\sqrt{2}$
(C) 16
(D) $16\sqrt{2}$
(E) NOTA

27) Saaketh and Sameera play a game where they each independently choose a random real number (s_1 and s_2 respectively) in the range $(0,1)$. Saaketh wins if the value of ${}^{s_1}/_{s_2}$ rounded to the nearest integer is odd. What is the probability that Saaketh wins?

(A) $\frac{\ln 2}{2}$
(B) $\frac{1}{2}$
(C) $\frac{\pi-1}{4}$
(D) $\frac{\pi-2}{2}$
(E) NOTA

28) Daniel, Max, and Vlad are standing on the points $(-2,0)$, $(0,\sqrt{2})$, and $(2,0)$ respectively. Each of them simultaneously begins running towards Cassie at the point $(0,-2)$ at a rate of $\sqrt{2}$ units per second. At the time 1 second after they begin running, how fast is the area of the triangular region formed by Daniel, Max, and Vlad decreasing per second?

(A) $\sqrt{2}$
(B) 2
(C) $1+\sqrt{2}$
(D) $2\sqrt{2}$
(E) NOTA

29) Trevor and Sanjoy are having an integration competition. Going into the final question, the two are tied in points. The last question reads as follows:

$$\text{If } \int_1^{\infty}\left(\arcsin\left(\frac{1}{x}\right)-\frac{1}{x}\right)\,dx = A+\ln B-\frac{\pi}{C}\text{, find } A+B+C.$$

Sanjoy says that the integral is impossible because splitting the terms and subtracting one integral from the other results in an $\infty-\infty$ indeterminate form. Trevor disagrees, saying that like in some limits, the "indeterminate form" has a true value. Who is correct, and if there is a true value, what is $A+B+C$?

(A) Trevor is correct, and $A+B+C=4$.
(B) Trevor is correct, and $A+B+C=5$.
(C) Trevor is correct, and $A+B+C=7$.
(D) Sanjoy is correct, and the integral does not exist.
(E) NOTA

30) An infinite number of mathematicians walk into a bar. The first one orders 1 orange juice. The second one orders $\frac{1}{2}$ of an orange juice. The third one orders $\frac{1}{4}$ of an orange juice. The fourth one orders $\frac{1}{8}$ of an orange juice, and so on. After all of the mathematicians have been served their orange juices, how many total orange juices will the bartender have poured?

(A) 1
(B) 2
(C) 4
(D) ∞
(E) NOTA