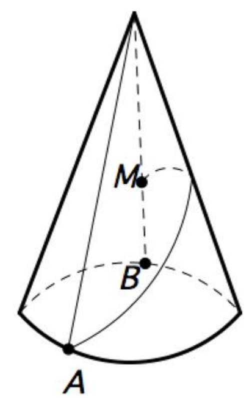
1. Suppose that $x^4+2x^3-23x^2+px+q=0$ has two sets of double roots. Find the value of $\frac{p+q}{p}$, when written in lowest terms.

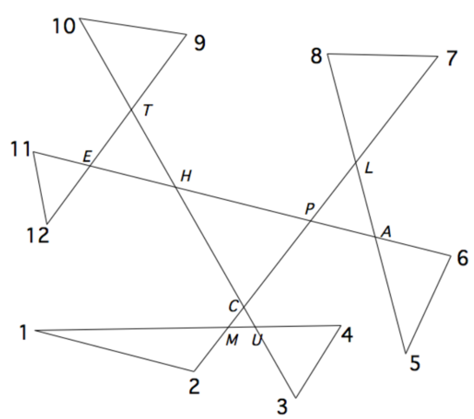
A. -5 B. -7 C. 7 D. 5 E. NOTA

2. A bug is at point A on a right circular cone. If the bug crawls to the point M on the surface of the cone by the shortest route, how far does it crawl? The lengths of the line segments in the diagram are $OA=OB=15$ and $OM=8$, and the diameter of the base of the cone is 15. (O is the vertex of the cone, and B is diametrically opposite A on the circular base)



A. $9\sqrt{2}$
B. 13
C. 8π
D. $9\sqrt{3}$
E. NOTA

3. Find the sum of the angles marked 1 through 12, all of which are less than 180 degrees. Answers are in degrees.



A. 720 B. 1080 C. 1440 D. 600 E. NOTA

4. Two substances, Eddybrite and Charlieshine, each contain two ingredients, flurrium and zachium. One pound of Eddybrite contains 3 oz of flurrium and 4 oz of zachium. One pound of Charlieshine contains 2 oz of flurrium and 6 oz of zachium. A manufacturer will combine quantities of Charlieshine and Eddybrite to obtain a mixture that contains at least 9 oz of flurrium and 20 oz of zachium. The cost of Eddybrite is \$3 per pound and the cost of Charlieshine is \$4 per pound. If the minimum cost A comes from using B oz of Eddybrite and C oz of Charlieshine, find the value of $4(A+B+C)$.

A. 76 B. 72 C. 86 D. 80 E. NOTA

5. The roots of $3375x^3-525x^2-70x+8=0$ are in geometric progression. Let the roots be, in order, c, e, and f, with $c>f$. If an infinite geometric series is formed with first term f and common ratio c, what is the sum of this infinite geometric series?

A. $\frac{2}{27}$ B. $-\frac{4}{75}$ C. $-\frac{2}{25}$ D. $\frac{1}{9}$ E. NOTA

6. Circle M contains two points, J and L, that are endpoints of a diameter of the circle. Two points A and K are chosen at random from the same semicircular arc created using J and L as endpoints of the diameter. What is the probability that $m\angle AMK$ is an acute angle?

A. $\frac{1}{2}$ B. $\frac{2}{3}$ C. $\frac{5}{6}$ D. $\frac{3}{4}$ E. NOTA

7. In how many ways can five men and three women sit at a circular table so that no two women sit side by side?

A. 240 B. 720 C. 1200 D. 1440 E. NOTA

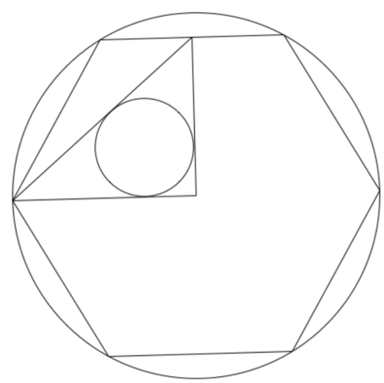
8. Find the number of distinct permutations of the letters in STLOUISRAMS.

A. $\frac{12!}{3!3!2!}$ B. 665,280 C. $\frac{11!}{2!2!}$ D. 19,958,400 E. NOTA

9. It takes Paula 12 days to build a house. If Jacob works with Paula, it takes 8 days to finish the house. If Paul and Jacob work together, it takes 16 days to finish. If Paul works by himself and completes one-sixth of the house, then Jacob works by himself and completes one-third more of the house, and then Paula finishes the house alone, how much total time, in days, was spent on the house? Assume all people work cooperatively and independently.

A. 16 B. 22 C. 28 D. 36 E. NOTA

10. In the diagram, a regular hexagon is inscribed in a circle. A smaller circle is inscribed in the right triangle which has one vertex at the center of the outer circle and one vertex on a vertex of the hexagon, as in the diagram. A point is randomly selected inside the larger circle. Find the probability that the point also lies inside the smaller circle.



A. $\frac{2+\sqrt{3}+\sqrt{7}}{4}$

B. $\frac{777-168\sqrt{21}}{400}$

C. $\frac{7-\sqrt{21}+2\sqrt{3}-2\sqrt{7}}{4}$

D. $\frac{7-\sqrt{21}+2\sqrt{3}-2\sqrt{7}}{8}$

E. NOTA

11. The interior angles of a hexagon are in the ratio 3:5:6:7:7:8 and the exterior angles of a pentagon are in the ratio 1:2:3:4:5. Find the positive difference between the measure of the largest interior angle of the pentagon and the measure of the smallest exterior angle of the hexagon. All answers are in degrees.

A. 4 B. 40 C. 136 D. 156 E. NOTA

12. The price of a used car is displayed (in dollars only) on four cards on the windshield. If the card with the thousands digit blows off, the apparent price of the car would drop to one forty-ninth of the intended value. What number is on the card that blows off?

A. 3 B. 5 C. 7 D. 9 E. NOTA

13. A colony of the bacteria *S. giffordides* is growing in a petri dish. The colony's area A (in cm^2) can be modeled by $A(t)=\dfrac{50}{1+30\left(10^{-2t}\right)}$, where t is measured in days. To the nearest hour, after how many hours does the area's growth rate begin to decrease?

A. 12 B. 18 C. 21 D. 25 E. NOTA

14. The graph of $\left||x|-3\right|+\left||y|+4\right|\le 8$ is a polygon of n sides with enclosed area A units². Find the value of $n+A$.

A. 62 B. 66 C. 68 D. 70 E. NOTA

15. The largest cross-section of an ice cream cone is an isosceles triangle and sitting at the top of the cone is a semiellipse (the cross section of the ice cream). The top of the cross-section of the cone is the minor axis of the semiellipse, and the ice cream is as tall as it is wide. If the area of the cross-section of the ice cream equals the area of the cross-section of the cone, and if the vertex angle of the cone is θ, what is the value of $\tan\left(\frac{1}{2}\theta\right)$?

A. $\dfrac{1}{2\pi}$ B. $\dfrac{\pi}{4}$ C. $\dfrac{1}{\pi}$ D. $\dfrac{2}{\pi}$ E. NOTA

16. What is the volume of the region bounded by the function $y=|x|-1$ and the line $y=1$ and then rotated about the line $y=1$?

A. $\frac{8\pi}{3}$ B. $\frac{16\pi}{3}$ C. $\frac{\pi}{3}$ D. $\frac{2\pi}{3}$ E. NOTA

17. A ball's height, in feet above the ground follows the equation $y(t)=3t-t^2$. Once the ball reaches its maximum height it drops to the ground and starts bouncing back to a height $\frac{5}{14}$ as high as it had previously dropped. As it continues bouncing, what will the ball's total distance traveled, measured starting from $t=0$, approach?

A. $\frac{7}{2}ft$ B. $\frac{5}{4}ft$ C. $14\ ft$ D. $7\ ft$ E. NOTA

18. Find the shortest distance between the function $y=\sqrt{2x}$ and the point (3,0).

A. 6 B. 4 C. $\sqrt{6}$ D. $\sqrt{5}$ E. NOTA

19. The sum of the squares of seven consecutive positive integers is 476. What is the 5th largest of these seven consecutive integers?

A. 6 B. 8 C. 10 D. 12 E. NOTA

20. The population of a city increased from 3 million in 1973 to 6 million in 2003. Assuming the population grew exponentially, what would you expect the population to be in 2018?

A. $6\sqrt{2}$ million
B. 6 million
C. $4\sqrt{2}$ million
D. 4 million
E. NOTA

21. A radioactive isotope has a half-life of 5 days. How many days does it take for 87.5% of a given amount to decay?

A. 11 B. 13 C. 10 D. 8 E. NOTA

22. Find the determinant of $(AB - A^{-1})$ given that $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$.

A. -54 B. 61 C. -49 D. 12 E. NOTA

23. A bridge built as a semi-elliptical arch has a span of 10 yards (which serves as the ellipse's major axis). Its height 4 yards away from the center is 2.4 yards. What is the height of the arch at its peak?

A. 1.2
B. 5
C. $\frac{576}{25}$
D. 1.44
E. NOTA

24. Evaluate the infinite sum: $1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \frac{1}{1+2+3+4} + \cdots + \frac{1}{1+2+3+\cdots+n} + \cdots$

A. e
B. π
C. $\frac{1}{e}$
D. 2
E. NOTA

25. As a particle moves along the x-axis, its distance from the origin at any time t seconds can be described using the equation $x(t) = |10t - t^2|$. What is the greatest distance between the particle and the origin on the time interval $[-1,11]$ seconds?

A. 10 B. 11 C. 5 D. 25 E. NOTA

26. A car begins $(t = 0)$ at rest and starts accelerating at a constant rate of 1 meter per second and its position can be expressed as the function $= \frac{1}{2}t^2$. A second car starts ahead of the first, travels in the same direction and has a constant speed of 5 meters per second. Two seconds after the first car starts to accelerate, it is 45.5 meters behind the second car. After how many seconds from when then the first car started accelerating will it catch up to the second car?

A. 9.5 seconds B. 5 seconds C. 15 seconds D. 106 seconds E. NOTA

27. Hansel traveled to Gretel's house at a rate of 5 miles per hour. He stayed there for an hour and went back home at a rate of 10 miles per hour. The whole trip took two and a half hours. How far apart are the two houses?

A. 1 mile B. 1.5 miles C. 5 miles D. 15 miles E. NOTA

28. Two ladders of equal length both have rungs that are equally spaced from each other and from their ends. One of these ladders has 8 rungs. The other ladder has 11 rungs but are spaced 4 inches closer. How long, in feet, is each ladder? Assume the rungs have negligible height.

A. 12 feet B. 19 feet C. 4 feet D. $\frac{22}{3}$ feet E. NOTA

29. Jimmy and John invented a cryptocurrency that does transactions in number base x "dollars" rather than base 10. For example, if Jimmy buys a sandwich for 440 "dollars" and pays for it with a 1000 "dollar" bill he'll get 340 "dollars" back. What number base are they operating in?

A. 8 B. 7 C. 5 D. 2 E. NOTA

30. Each Earth day is a little bit longer than the preceding day. Today is one twenty-millionth of a second longer than yesterday. Assuming a constant increase in the length of a day, in approximately (rounded to the nearest hundred million) how many years will we have 26 hour long days? Assume today is exactly 24 hours in length.

A. 100 million
B. 200 million
C. 300 million
D. 400 million
E. NOTA