1. **A**.
   Given the conditions defined by the problem, Rolle's theorem states that there exists at least one value c in between a and b, exclusive such that f '(c) = 0
2. **C**.
   Chain rule for h(x) = f(g(x)) is f'(g(x))g'(x). So we have $(2(3x^2)+3)(6x)$ for x=2 and we get 324.
3. **B**.
   Using u-substitution for u = x+1, we get the integral of $u^2 + u$ from 18 to 20, which we can integrate to get $\frac{2282}{3}$.
4. **E**.
   The x^7 is hidden behind the other terms and since the power of the numerator is greater than the power of the denominator, the limit DNE.
5. **B**.
   The cross sections have area y^2, so the new integral is the integral from 0 to 5 of $(y)^2$. Since $y = x^2$, we want the integral from 0 to 5 of x^4 which is 625.
6. **B**.
   L(x) = f(a) + f'(a)(x-a). In this case, $f'(a) = \frac{1}{2\sqrt{a}}$. Therefore, we have L(45) = f(49) + $\frac{1}{2\sqrt{49}}(45-49) = 7 - \frac{2}{7} = \frac{47}{7}$
7. **C**.
   Looking at the problem we notice that the expansion of the entire product yields a numerator and denominator of equal powers and a starting coefficient of 1. The 2018^n cancels with the 2018^{n+1} to make a single 2018. So the final expansion looks something along the lines of $\frac{2018n^{4036} + \dots}{n^{4036} + \dots}$. This converges to 2018.
8. **D**.
   Using tabular, we get the expression $x^2 \cos x + 2x \sin x - 2\cos x$. Evaluating from 0 to $\frac{\pi}{2}$ yields $\pi - 2$
9. **C**.
   The 2nd fundamental theorem of calculus states that for a function g(x) such that $g(x) = \int_{u(x)}^{v(x)} f(t)dt$, $g'(x) = v'(x)f(v(x)) - u'(x)f(u(x))$. Therefore, with v(x) = $4x^3$ and u(x) = $3x^2$, our answer is $12x^2 \sin 16x^6 - 6x\sin(9x^4)$
10. **A**
    I converges as a direct comparison to $\sum_{n=0}^{\infty} (\frac{3}{4})^n$
    II diverges by n^{th} term test and L'Hôpital's as you initially get $\frac{\infty}{\infty}$
    III diverges by Alternating Series Theorem as sec3(x) is not monotonically decreasing.
    IV diverges by n^{th} term test as $\frac{(2x)!}{x^x}$ approaches infinity as the terms 2x, 2x-1, 2x-2... 2x-x contain x amount of terms each which is greater than x. Therefore, their product is greater than x^x already.

11. **A**

Use L'Hopitals: (1-6/5*$a^{1/5}$)/(1/4*$a^{-3/4}$-10/9*$a^{1/9}$)

Plug in a=1 and get 36/155. 36+155= 191

12. **D**

Use implicit to find that $3x^2 - \frac{x}{y}\frac{dy}{dx} - lny = e^x\frac{dy}{dx} + ye^x$, isolating dy/dx and then diving by x/y+e^x we get $\frac{3x^2y - ylny - y^2e^x}{x + ye^x}$

13. **A**

Taking the natural log of both sides gives lny=$\sqrt{x}$lnx. Taking the derivative of both sides gives $\frac{1}{y}\frac{dy}{dx} = \frac{1}{2\sqrt{x}}lnx + \frac{\sqrt{x}}{x}$. Multiplying by y gives $\frac{dy}{dx} = \frac{1}{2}x\sqrt{x} - \frac{1}{2}$lnx+x$\sqrt{x} - \frac{1}{2}$. Factoring out x$\sqrt{x} - \frac{1}{2}$ gives x$\sqrt{x} - \frac{1}{2}(\frac{1}{2}lnx + 1)$. Plugging in 3 gives a=3, b=½, c=3, d=1. 2a+4b+c-d = 10.

14. **D**

Setting the two functions equal to each other, we find that they intersect at the points (-1/3/ 11/9), (0,0), and (1,3).

$\left|\int_{-1/3}^{0}(3x^3 - x + 1) - (2x^2 + 1)\right|$ =7/324

$\left|\int_{0}^{1}(3x^3 - x + 1) - (2x^2 + 1)\right|$ =5/12

Sum the two areas and we get 71/162

15. **C**

The expression in the integral can be expressed as the partial sum of 1/(x+1)+1/x+1/(x^2+1). Integrating, we get ln(x^2+x)+$\tan^{-1}$(x) and then we plug in x=1 to x=√3/3 to get [ln(√3/3 +1/3)+π/4]-[ln(2)+π/3] = ln(√3/6+⅙)-π/12

16. **B**

Taking the derivative of f(x) gives $12x^3+132x^2-24x-1440$. This factors into 12(x+10)(x+4)(x-3), so on the interval [0,1], f(x) is decreasing. By definition, a decreasing function is over approximated by a left hand riemann sum. (You can draw a decreasing curve and see that the riemann rectangles are taller than the curve). Therefore, II and III are correct.

17. E

Using the ratio test

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \frac{\frac{4x^{n+1}}{(n+5)(n+4)!)}}{\frac{4x^n}{(n+4)!}}.$$

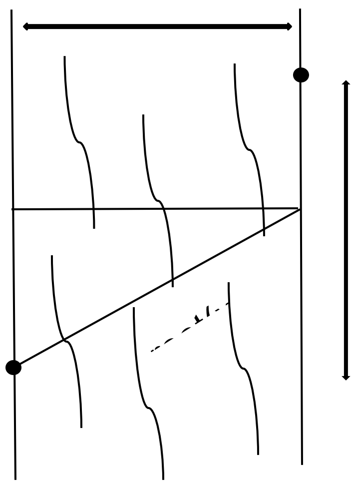
This equals $\frac{x}{n+5}$. Since n is infinitely large, no set constant for x will result in a divergent series, so the bounds are $(-\infty, \infty)$.

18. D

Using $f(x) = (x^2+4)^{1/2}$, we find that $f(0) = 2, \quad f'(0) = 0, \quad f''(0) = \frac{1}{2}.$

Thus the second degree polynomial is $2 + \frac{1}{4}x^2$. Taking the integral, we get $2x + \frac{1}{12}x^3$. Plugging in x = 2, we get 40/3.

19. C



The cost function is $C(x)=16(24-x)+20(x^2+400)^{1/2}$, we set $C'(x)=-16+10(x^2+400)^{-1/2}(2x) = 0$ and get x=80/3. We plug back into $\sqrt{(x^2+400)}$ and get 100/3

20. **D**

One can use the cosine double angle identity to make $\frac{1+cos^2(x)}{2cos^2(x)}$ which simplifies to $\frac{1}{2}\int \ sec^2x + \int \ \frac{1}{2}$ which equals $\frac{1}{2}tan(x) \ + \ \frac{1}{2}x + C$ after integrating

21. **A**

$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right)}{\frac{dx}{dt}}$ $\quad \frac{dy}{dt} = \frac{1}{1+t^2}, \frac{dx}{dt} = \frac{1}{tln10}$ $\quad \frac{dy}{dx} = \frac{tln10}{1+t^2}$ using quotient rule and dividing by $\frac{dx}{dt}, \frac{d \ \ ^2y}{d \ \ ^2x} = \frac{t(1-t^2)(ln10)^2}{(1+t^2)^2}$ and when t= $2\sqrt{2}$ it is $\frac{-14\sqrt{2}(ln10)^2}{81}$

22. **A**

Using the shell method, the volume about the y-axis would be $2\pi \int_1^5 x(\ln(x) + 5)dx$ if it were bounded by the x axis, but since the solid has a bottom boundary of y=2, our new integral is $2\pi \int_1^5 x(\ln(x) + 5 - 2)dx$, so the volume is $60\pi + 25\pi \ln 5$.

23. A

Using the taylor series for e^x, we find that

$$e^{-x^2} = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} x^{2k}$$

Taking the integral from x=0 to x=3, we get

$$4\sum_{k=0}^{\infty} \left[\frac{(-1)^k}{k!(2k+1)} x^{2k+1}\right]_0^3$$

Which is the same as

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{k!(2k+1)} 4 \cdot 3^{2k+1}$$

Factoring out a 3 from the exponent of 3^{2k+1} and then simplify 3^{2k} to 9^k which is answer choice A.

24. B

Since f(x) is odd, $-\int_{-2}^{0} f(x)dx = \int_{0}^{2} f(x)dx = 4$. We're given $\int_{0}^{8} f(x)dx = 21$, so $\int_{2}^{8} f(x)dx = 17$. Next, we're given $\int_{0}^{-5} f(x)dx = 3$. Since we are moving right to left and f(x) is odd $\int_{0}^{-5} f(x)dx = \int_{0}^{5} f(x)dx = 3$. Using that, we get $\int_{5}^{8} f(x)dx = 18$. $\int_{2}^{5} f(x)dx = \int_{2}^{8} f(x)dx - \int_{5}^{8} f(x)dx$ Therefore,

$$\int_{2}^{5} f(x)dx = 17 - 18 = -1$$

25. E

The volume of a frustum is defined by $\frac{\pi h}{3}(R^2 + Rr + r^2)$, where R is the big radius, r is the small radius, and h is the height. Since the smaller radius is on the bottom, r always equals 10. Now we're only dealing with 2 variables. We can find h in terms of R because their ratio will be proportional to the ratio of the bowl. Therefore, we know that h=⅗ R. Our new volume formula is $V = \frac{\pi}{5}(R^3 + 10R^2 + 100R)$. Taking the derivative of both sides gets $\frac{dV}{dt} = \frac{\pi}{5}(3R^2\frac{dR}{dt} + 20R\frac{dR}{dt} + 100\frac{dR}{dt})$. Since we know that the height is 9 and h = ⅗ R, we know that R=15 cm. Plugging in with the fact that the volume changes at a rate of 12, we get $\frac{dR}{dt} = \frac{12}{175\pi}$.

26. B

This is simply the integral of f(x)-g(x) = $x^2+x+3+1/x+1/x^2$ on the interval from -8 to -2. We have $1/3x^3+1/2x^2+3x+\ln(|x|)-1/x$ and can plug in the values of x= -2 and x= -8 and subtract.

27. **C**

$AL = \int_0^{\sqrt{3}} \sqrt{\theta^2+1}\, d\theta$ using trig substitution of $\theta = tan(x)$ we get $AL = \int_0^{\frac{\pi}{3}} \sqrt{tan^2(x)\ +1}\ \ sec^2(x)dx\ =\ \int_0^{\frac{\pi}{3}} sec^3(x)\, dx$which can be rearranged by integration by parts and the identity $tan^2x + 1 = sec^2x$:

$$AL = sec(x)tan(x)|\frac{\pi}{3}\ and\ 0\ - \int_0^{\frac{\pi}{3}} sec(x)tan^2(x)\, dx$$

$$= 2\sqrt{3} - \int_0^{\frac{\pi}{3}} sec(x)(sec^2(x)-1)\, dx\ =\ 2\sqrt{3}\ -\ AL + \int_0^{\frac{\pi}{3}} sec(x)\, dx$$

sec(x) can be integrated into ln|sec(x) + tan(x)| so the final steps are

2AL= $\sqrt{3} + ln|sec(x)\ +\ tan(x)|\ |\frac{\pi}{3}\ and\ 0\ =\ 2\sqrt{3}\ +\ ln(2\ +\ \sqrt{3})$

thus AL = $\frac{1}{2}(2\sqrt{3} + ln(2+\sqrt{3}))$

28. C

The given integral is the same as $\int_0^1 \frac{1}{x}\sum_{n=1}^{\infty} \frac{(-1)^{n+1}x^n}{n} = \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} - \sum_{n=1}^{\infty} \frac{1}{(2n)^2}$ using the taylor series for ln(x+1).

$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ which can be found with sine inverse maclaurin series.

$\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} = \sum_{n=1}^{\infty} \frac{1}{n^2} - \frac{1}{4}\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{8}$

$\sum_{n=1}^{\infty} \frac{1}{(2n)^2} = \frac{1}{4}\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{24}$

$$\frac{\pi^2}{8} - \frac{\pi^2}{24} = \frac{\pi^2}{12}$$

29.B

First we have ds/dt=4s. Integrating and simplifying we get $s=Ce^{4t}$. He's initially 3 ft^3 so C=3 and we have $s=3e^{4t}$ for first 5 seconds. Plugging in t=5, we get his size as $3e^{20}$. The next five minutes require a new growth function. $ds/dt=4s-8s^2$. Integrating, we get ln[(s)/(2s-1)]=4t+C. Isolating s, we get $s=(Ce^{4t})/(2Ce^{4t}-1)$. The initial s is $3e^{20}$, which gives us $C=(3e^{20})/(6e^{20}-1)$. Over the next five seconds his size will be $[(3e^{20})/(6e^{20}-1)*e^{16*5}]/[\ 2(3e^{20})/(6e^{20}-1)*e^{16*5}-1] = \frac{3e^{100}}{6e^{100}-6e^{20}+1}$.

30. D

Use partial fractions to get 1/(x+1)+2/(x+2) and integrate to get $\ln[(x+1)(x+2)^2]$. Plug in x=1 to get ln(18) and x=0 to get ln(4). Subtract to get ln(9/2).