1. **A.**
   Given the conditions defined by the problem, Rolle’s theorem states that there exists at least one value c in between a and b, exclusive such that \( f'(c) = 0 \).

2. **C.**
   Chain rule for \( h(x) = f(g(x)) \) is \( f'(g(x))g'(x) \). So we have \( (2(3x^2)+3)(6x) \) for \( x=2 \) and we get 324.

3. **B.**
   Using \( u \)-substitution for \( u = x+1 \), we get the integral of \( u^2 + u \) from 18 to 20, which we can integrate to get \( \frac{2282}{3} \).

4. **E.**
   The \( x^7 \) is hidden behind the other terms and since the power of the numerator is greater than the power of the denominator, the limit DNE.

5. **B.**
   The cross sections have area \( y^2 \), so the new integral is the integral from 0 to 5 of \( (y)^2 \).
   Since \( y = x^2 \), we want the integral from 0 to 5 of \( x^4 \) which is 625.

6. **B.**
   \( L(x) = f(a) + f'(a)(x - a) \). In this case, \( f'(a) = \frac{1}{2\sqrt{a}} \). Therefore, we have \( L(45) = f(49) + \frac{1}{2\sqrt{49}(45-49)} = 7 - \frac{2}{7} = \frac{47}{7} \).

7. **C.**
   Looking at the problem we notice that the expansion of the entire product yields a numerator and denominator of equal powers and a starting coefficient of 1. The \( 2018^n \) cancels with the \( 2018^{n+1} \) to make a single 2018. So the final expansion looks something along the lines of \( \frac{2018n^{4036} + \ldots}{n^{4036} + \ldots} \). This converges to 2018.

8. **D.**
   Using tabular, we get the expression \( x^2 \cos x + 2x \sin x - 2 \cos x \). Evaluating from 0 to \( \frac{\pi}{2} \) yields \( \pi - 2 \).

9. **C.**
   The 2nd fundamental theorem of calculus states that for a function \( g(x) \) such that
   \[ g(x) = \int_{u(x)}^{v(x)} f(t) \, dt, \quad g'(x) = v'(x)f(v(x)) - u'(x)f(u(x)), \]
   Therefore, with \( v(x) = 4x^3 \) and \( u(x) = 3x^2 \), our answer is \( 12x^2 \sin 16x^6 - 6x \sin(9x^4) \).

10. **A.**
    I converges as a direct comparison to \( \sum_{n=0}^{\infty} \left( \frac{3}{4} \right)^n \).
    II diverges by \( n^{th} \) term test and L'Hôpital's as you initially get \( \frac{\infty}{\infty} \).
    III diverges by Alternating Series Theorem as \( \sec 3(x) \) is not monotonically decreasing.
    IV diverges by \( n^{th} \) term test as \( \frac{(2x)!}{x^x} \) approaches infinity as the terms \( 2x, 2x-1, 2x-2, \ldots 2x-x \) contain \( x \) amount of terms each which is greater than \( x \). Therefore, their product is greater than \( x^x \) already.
11. A
Use L'Hopitals: \((1 - 6/5*a^{1/5})/(1/4*a^{1/5} - 10/9*a^{1/9})\)
Plug in a=1 and get \(36/155\). \(36 + 155 = 191\)

12. D
Use implicit to find that \(3x^2 - \frac{x}{y} \frac{dy}{dx} - \ln y = e^x \frac{dy}{dx} + ye^x\), isolating \(dy/dx\) and then dividing by \(x/y + e^x\) we get \(\frac{3x^2y - y\ln y - y^2e^x}{x + ye^x}\)

13. A
Taking the natural log of both sides gives \(\ln y = \sqrt{x}\ln x\). Taking the derivative of both sides gives \(\frac{1}{y} \frac{dy}{dx} = \frac{1}{2\sqrt{x}} \ln x + \frac{\sqrt{x}}{x}\). Multiplying by \(y\) gives \(\frac{dy}{dx} = \frac{1}{2}x\sqrt{x} - \frac{1}{2}\ln x + x\sqrt{x} - \frac{1}{2}\). Factoring out \(x\sqrt{x} - \frac{1}{2}\) gives \(x\sqrt{x} - \frac{1}{2}(\frac{1}{2}\ln x + 1)\). Plugging in 3 gives a=3, b=½, c=3, d=1.
\(2a+4b+c-d = 10\).

14. D
Setting the two functions equal to each other, we find that they intersect at the points \((-1/3, 11/9), (0,0),\) and \((1,3)\).
\(\left|\int_{-1/3}^{0} (3x^3 - x + 1) - (2x^2 + 1)\right| = \frac{7}{324}\)
\(\left|\int_{0}^{1} (3x^3 - x + 1) - (2x^2 + 1)\right| = \frac{5}{12}\)
Sum the two areas and we get \(\frac{71}{162}\)

15. C
The expression in the integral can be expressed as the partial sum of \(1/(x+1)+1/x+1/(x^2+1)\). Integrating, we get \(\ln(x^2+x)+\tan^{-1}(x)\) and then we plug in \(x=1\) to \(x=\sqrt{3}/3\) to get \([\ln(\sqrt{3}/3 + 1/3) + \pi/4] - [\ln(2) + \pi/3] = \ln(\sqrt{3}/6 + \frac{1}{3}) - \pi/12\)

16. B
Taking the derivative of \(f(x)\) gives \(12x^3 + 132x^2 - 24x - 1440\). This factors into \(12(x+10)(x+4)(x-3)\), so on the interval \([0,1]\), \(f(x)\) is decreasing. By definition, a decreasing function is over approximated by a left hand riemann sum. (You can draw a decreasing curve and see that the riemann rectangles are taller than the curve). Therefore, II and III are correct.

17. E
Using the ratio test
\[
\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \frac{\frac{4x^{n+1}}{(n+5)(n+4)!}}{\frac{4x^n}{(n+4)!}}
\]
This equals \(\frac{x}{n+5}\). Since \(n\) is infinitely large, no set constant for \(x\) will result in a divergent series, so the bounds are \((-\infty, \infty)\).
18. D

Using $f(x) = (x^2+4)^{1/2}$, we find that $f(0) = 2$, $f'(0) = 0$, $f''(0) = \frac{1}{2}$.

Thus the second degree polynomial is $2 + \frac{1}{4}x^2$. Taking the integral, we get $2x + \frac{1}{12}x^3$.

Plugging in $x = 2$, we get $40/3$.

19. C

The cost function is $C(x)=16(24-x)+20(x^2+400)^{1/2}$, we set $C'(x)=16+10(x^2+400)^{-\frac{1}{2}}(2x) = 0$ and get $x=\frac{80}{3}$. We plug back into $\sqrt{x^2+400}$ and get $\frac{100}{3}$

20. D

One can use the cosine double angle identity to make $\frac{1+\cos^2(x)}{2\cos^2(x)}$ which simplifies to

$$\frac{1}{2} \int \sec^2 x + \int \frac{1}{2}$$ which equals $\frac{1}{2} \tan(x) + \frac{1}{2}x + C$ after integrating.

21. A

quotient rule and dividing by $\frac{dy}{dt}\begin{align*}
\frac{d^2 y}{dx^2} &= \frac{d}{dt} \left( \frac{dy}{dx} \right) \\
\frac{dy}{dx} &= \frac{1}{1+t^2} \\
\frac{dx}{dt} &= t \ln 10 \\
\frac{dy}{dx} &= \frac{t \ln 10}{1+t^2} \\
\frac{d^2 y}{dx^2} &= \frac{t(1-t^2)(\ln 10)^2}{(1+t^2)^2}
\end{align*}$ using

and when $t = 2\sqrt{2}$ it is $-14\sqrt{2}(\ln 10)^2 \div 81$

22. A

Using the shell method, the volume about the y-axis would be $\int_1^5 x(\ln(x) + 5)\,dx$ if it were bounded by the x axis, but since the solid has a bottom boundary of $y=2$, our new integral is $\int_1^5 x(\ln(x) + 5 - 2)\,dx$, so the volume is $60\pi + 25\pi \ln(5)$. 
23. A

Using the Taylor series for $e^x$, we find that

$$e^{-x^2} = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} x^{2k}$$

Taking the integral from $x=0$ to $x=3$, we get

$$4 \sum_{k=0}^{\infty} \left[ \frac{(-1)^k}{k!(2k+1)} x^{2k+1} \right]_0^3$$

Which is the same as

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{k!(2k+1)} 4 \cdot 3^{2k+1}$$

Factoring out a 3 from the exponent of $3^{2k+1}$ and then simplify $3^{2k}$ to $9^k$ which is answer choice A.

24. B

Since $f(x)$ is odd, $\int_{-2}^{0} f(x)\,dx = \int_{0}^{2} f(x)\,dx = 4$. We’re given $\int_{0}^{8} f(x)\,dx = 21$, $\int_{2}^{8} f(x)\,dx = 17$, so $\int_{2}^{8} f(x)\,dx = 17$. Next, we’re given $\int_{0}^{5} f(x)\,dx = 3$. Since we are moving right to left and $f(x)$ is odd, $\int_{0}^{5} f(x)\,dx = \int_{0}^{5} f(x)\,dx = 3$. Using that, we get $\int_{5}^{8} f(x)\,dx = 18 \cdot \int_{2}^{5} f(x)\,dx = \int_{2}^{8} f(x)\,dx - \int_{5}^{8} f(x)\,dx$ Therefore, $\int_{2}^{5} f(x)\,dx = 17 - 18 = -1$

25. E

The volume of a frustum is defined by $V = \frac{\pi h}{3} (R^2 + Rr + r^2)$, where $R$ is the big radius, $r$ is the small radius, and $h$ is the height. Since the smaller radius is on the bottom, $r$ always equals 10. Now we’re only dealing with 2 variables. We can find $h$ in terms of $R$ because their ratio will be proportional to the ratio of the bowl. Therefore, we know that $h=\frac{3}{5} R$. Our new volume formula is $V = \frac{\pi}{5} (R^3 + 10R^2 + 100R)$. Taking the derivative of both sides gets $\frac{dV}{dt} = \frac{\pi}{5} (3R^2 \frac{dR}{dt} + 20R \frac{dR}{dt} + 100 \frac{dR}{dt})$. Since we know that the height is 9 and $h = \frac{3}{5} R$, we know that $R=15$ cm. Plugging in with the fact that the volume changes at a rate of 12, we get $\frac{dR}{dt} = \frac{12}{175\pi}$.

26. B
This is simply the integral of \(f(x) - g(x) = x^2 + x + 3 + 1/x + 1/x^2\) on the interval from -8 to -2. We have \(1/3x^3 + 1/2x^2 + 3x + \ln(|x|) - 1/x\) and can plug in the values of \(x= -2\) and \(x= -8\) and subtract.

27. C

\[
AL = \int_0^\frac{\pi}{3} \sqrt{\theta^2 + 1} \, d\theta \text{ using trig substitution of } \theta = \tan(x) \text{ we get } AL = \\
\int_0^\frac{\pi}{3} \sqrt{\tan^2(x) + 1} \, \sec^2(x) \, dx = \int_0^\frac{\pi}{3} \sec^3(x) \, dx \text{ which can be rearranged by integration by parts and the identity } \tan^2(x) + 1 = \sec^2(x) : \\
AL = \sec(x)\tan(x)\Big|_0^{\frac{\pi}{3}} \text{ and } 0 - \int_0^{\frac{\pi}{3}} \sec(x)\tan^2(x) \, dx \\
= 2\sqrt{3} - \int_0^{\frac{\pi}{3}} \sec(x)(\sec^2(x) - 1) \, dx = 2\sqrt{3} - AL + \int_0^{\frac{\pi}{3}} \sec(x) \, dx \\
\sec(x) \text{ can be integrated into } \ln(\sec(x) + \tan(x)) \text{ so the final steps are } 2AL = \sqrt{3} + \ln(\sec(x) + \tan(x)) | \Big|_0^{\frac{\pi}{3}} \text{ and } 0 = 2\sqrt{3} + \ln(2 + \sqrt{3}) \\
\text{thus } AL = \frac{1}{2}(2\sqrt{3} + \ln(2 + \sqrt{3}))
\]

28. C

The given integral is the same as \(\int_0^1 \frac{1}{x} \sum_{n=1}^{\infty} \frac{(-1)^n+1}{n} \frac{x^n}{(2n+1)^2} = \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} - \sum_{n=1}^{\infty} \frac{1}{(2n)^2} \)
using the taylor series for \(\ln(x+1)\).

\[
\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6} \text{ which can be found with sine inverse maclaurin series.} \\
\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} = \sum_{n=1}^{\infty} \frac{1}{n^2} - \frac{1}{4} \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{8} \\
\sum_{n=1}^{\infty} \frac{1}{(2n)^2} = \frac{1}{4} \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{24} \\
\frac{\pi^2}{8} - \frac{\pi^2}{24} = \frac{\pi^2}{12}
\]

29.B

First we have \(ds/dt=4s\). Integrating and simplifying we get \(s=Ce^{4t}\). He’s initially 3 ft\(^3\) so \(C=3\) and we have \(s=3e^{4t}\) for first 5 seconds. Plugging in \(t=5\), we get his size as \(3e^{20}\). The next five minutes require a new growth function. \(ds/dt=4s-8s^2\). Integrating, we get \(\ln[(s)/(2s-1)]=4t+C\). Isolating \(s\), we get \(s=(Ce^{4t})/(2Ce^{4t}-1)\). The initial \(s\) is \(3e^{20}\), which gives us \(C=(3e^{20})/(6e^{20}-1)\). Over the next five seconds his size will be \([(3e^{20})/(6e^{20}-1)\]*\(e^{16}\)\(^5\)/\([2(3e^{20})/(6e^{20}-1)\]*\(e^{16}\)\(^5\)-1\]] = \(\frac{3e^{100}}{6e^{100}-6e^{20}+1}\).

30. D

Use partial fractions to get \(1/(x+1)+2/(x+2)\) and integrate to get \(\ln[(x+1)(x+2)^2]\). Plug in \(x=1\) to get \(\ln(18)\) and \(x=0\) to get \(\ln(4)\). Subtract to get \(\ln(9/2)\).