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| 1. | B | Since the perimeter of the square is 40, one side of the square is 10 & its area is 100. Since the diameter of the circle is 8 its radius is 4. One quarter of the circle is inside the square; its area is $A = \frac{1}{4}\pi(4^2) = \frac{1}{4}\pi(16) = 4\pi$. The area inside the square but outside the circle is $100 - 4\pi$. |
| 2. | C | Since the diameter of the sphere is 20 feet, its radius is 10 feet. $$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi 10^3 = \frac{4000\pi}{3}\ ; A = 4\pi r^2 = 4\pi 10^2 = 400\pi = \frac{1200\pi}{3};$$ $$A + V = \frac{1200\pi}{3} + \frac{4000\pi}{3} = \frac{5200\pi}{3}$$ |
| 3. | B | Since the diameter is 18 $\quad C = \pi d = 18\pi$ |
| 4. | A | $$V = {a^3}/{6\sqrt{2}} = {12^3}/{6\sqrt{2}} = {1728}/{6\sqrt{2}} = {288}/{\sqrt{2}} = {288\sqrt{2}}/{2} = 144\sqrt{2}$$ |
| 5. | D | Use the given circumference to find the radius of the circle. $C = 2\pi r,\ 40 = 2\pi r,\ 20 = \pi r,\ {20}/{\pi} = r$. If the inscribed angle is 45° its arc is 90° and has length 10, one quarter of the circumference. The chord of the segment is the hypotenuse of an isosceles right triangle whose legs are radii of the circle. $$h = leg\sqrt{2} = {20}/{\pi}\sqrt{2} = {20\sqrt{2}}/{\pi}$$ Perimeter of the segment is arc length plus chord: $10 + {20\sqrt{2}}/{\pi}$ |
| 6. | B | Since the diameter of the sphere is the space diagonal of the inscribed cube, we must first find the diameter of the sphere. $$V = \frac{4}{3}\pi r^3;\ \ \frac{125\pi\sqrt{3}}{2} = \frac{4}{3}\pi r^3;\ \ \frac{125\pi(3\sqrt{3})}{8\pi} = r^3;\ \ \frac{125\sqrt{27}}{8} = r^3;\ \ \frac{5\sqrt{3}}{2} = r$$ and diameter is $5\sqrt{3}$ Space diagonal of a cube is $a\sqrt{3} = 5\sqrt{3};\ \ a = 5$ and surface area is $6a^2 = 6 \cdot 5^2 = 150$ |
| 7. | D | Radius found using the circular cross-section area $A = \pi r^2;\ \ 9\pi = \pi r^2;\ \ r = 3;\ h = 3r == 9$ Volume is equal to volume of cylinder minus volume of hemisphere. $$V = \pi r^2 h - {2}/{3}\,\pi r^3 = 81\pi - {2}/{3} \cdot 27\pi = 81\pi - 18\pi = 63\pi$$ Surface area equal to cylinder lateral area plus one base area & surface area of hemisphere. $$A = 2\pi rh + \pi r^2 + 2\pi r^2 = 2\pi rh + 3\pi r^2 = 54\pi + 27\pi = 81\pi$$ Sum of volume and surface area: $Sum = 63\pi + 81\pi = 144\pi$ |
| 8. | D | Regular hexahedron is a cube with squares as faces. Since perimeter of a face is 16 an edge is 4. Volume is lwh or (4)(4)(4) = 64. |

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| 9. | C | Perimeter of segment equals diameter plus semicircle length. <br> $A = \pi r^2 = 36;\ \ r = {}^{6}/_{\sqrt{\pi}}\ \ and\ d = {}^{12}/_{\sqrt{\pi}};\quad arclength_{semi} = \pi r = {}^{6\pi}/_{\sqrt{\pi}}$ <br> Perimeter is ${}^{12}/_{\sqrt{\pi}} + {}^{6\pi}/_{\sqrt{\pi}} = {}^{(12+6\pi)}/_{\sqrt{\pi}} = {}^{(12+6\pi)\sqrt{\pi}}/_{\pi} = {}^{6\sqrt{\pi}(2+\pi)}/_{\pi}$ |
| 10. | A | Space Diagonal of cube: $D = a\sqrt{3};\ \ \sqrt{6} = a\sqrt{3};\ \ a = {}^{\sqrt{6}}/_{\sqrt{3}} = \sqrt{2}$ <br> Surface Area of cube: $A = 6a^2 = 6\left(\sqrt{2}^2\right) = 6 \cdot 2 = 12$ |
| 11. | B | Surface area of sphere: $4\pi r^2 = 256\pi;\ \ r^2 = 64;\ \ r = 8.$ <br> Volume of sphere: $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(512) = {}^{2048\pi}/_{3}$ |
| 12. | B | The greatest distance between two vertices of a regular hexagon is a diameter or 2a. 2a =10; a=5. <br> Area of regular hexagon is equal to area of six equilateral triangles with side 'a'. <br> $A = 6 \cdot \frac{a^2\sqrt{3}}{4} = 6 \cdot \frac{25\sqrt{3}}{4} = \frac{150\sqrt{3}}{4} = \frac{75\sqrt{3}}{2}$ |
| 13. | B | If diagonal of square is 6, a side of square is ${}^{6}/_{\sqrt{2}} = 3\sqrt{2}$. Four of the sides of the octagon have one third the length of a side of the square, that is $\sqrt{2}$. Each of the other four sides is the hypotenuse of an isosceles right triangle with legs equal to $\sqrt{2}$. One of those sides is $\sqrt{2} \cdot \sqrt{2} = 2$. <br> Perimeter of octagon: $8 + 4\sqrt{2}$ |
| 14. | B | $V = \frac{4}{3}\pi r^3 = 288\pi;\ \ r^3 = 288 \cdot \frac{3}{4};\ \ r^3 = 216;\ \ r = 6$ <br> Surface area of the solid equals surface area of hemisphere plus circular cross-section area <br> $SA = 2\pi r^2 + \pi r^2 = 3\pi r^2 = 3\pi(6^2) = 108\pi$ |
| 15. | D | $V = Bh;\ 108\sqrt{3} = B \cdot 2;\ \ B = 54\sqrt{3};\qquad B = 6 \cdot \frac{a^2\sqrt{3}}{4};\ \ 54\sqrt{3} = \frac{3a^2\sqrt{3}}{2};\ \ a^2 = 36;\ \ a = 6;$ <br> $p = 6a = 36$ <br> $SA = 2B + ph;\ \ SA = 2(54\sqrt{3}) + 2(36) = 108\sqrt{3} + 72$ |

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| 16. | A | $C = 2\pi r = 14\pi;\ \ r = 7 \qquad V = {}^{4}/_{3}\,\pi r^3 = {}^{4}/_{3}\,\pi(7^3) = {}^{4}/_{3}\,\pi(343) = {}^{1372\pi}/_{3}$ |
| 17. | D | The cross-section forms a 1:3 similar smaller cone.<br>Use area of the larger base to find R: $\pi R^2 = 16\pi;\ \ R^2 16;\ \ R = 4$ because of 1:3 ratio $r = {}^{4}/_{3}$.<br>$C = 2\pi r = 2\pi({}^{4}/_{3}) = {}^{8\pi}/_{3}$ |
| 18. | D | $B = \pi r^2 = 9\pi;\ \ r^2 = 9;\ \ r = 3$ using the given $h = 9$ fill in sum of volume of cylinder and volume of hemisphere and add to sum of the surface area (minus one base) of the Cylinder and the surface area of the hemisphere.<br>$V_{cyl} + V_{hemi} + SA_{cyl} - B + SA_{hemi} = Bh + {}^{2\pi r^3}/_{3} + B + 2\pi rh + 2\pi r^2$<br>$= 9\pi(9) + {}^{2\pi(27)}/_{3} + 9\pi + 2\pi(3)(9) + 2\pi(9) = 81\pi + 18\pi + 9\pi + 54\pi + 18\pi = 180\pi$ |
| 19. | A | The surface of a regular octahedron is composed of 8 equilateral triangles, so the surface area equals 8 times the area of an equilateral triangle. $SA = 8\left({}^{a^2\sqrt{3}}/_{4}\right) = 2a^2\sqrt{3} = 200\sqrt{3}\,;\ \ a^2 = 100;$ *a=10.*<br>The volume formula for a regular octahedron given side *a*: $V = {}^{a^3\sqrt{2}}/_{3} = {}^{10^3\sqrt{2}}/_{3} = {}^{1000\sqrt{2}}/_{3}$ |
| 20. | B | The circumference of the circle is $10\pi$ so the 60°arc has length ${}^{10\pi}/_{6} = {}^{5\pi}/_{3}$. The chord of the 60° arc is a side of the equilateral triangle formed by two radii and the chord. Therefore, the chord has length *5* and will not be included in the perimeter of the bounded area. Therefore, the perimeter of the area inside the square and outside the circle is $32 - 5 + {}^{5\pi}/_{3} = 27 + {}^{5\pi}/_{3}$ |
| 21. | D | The slant height of the cone, the hypotenuse of a triangle with legs 5 and 7, has length $l = \sqrt{74}$<br>$LA_{cone} = \pi rl = 5\pi\sqrt{74} \qquad LA_{cyl} = 2\pi rh = 70\pi \qquad LA_{cone}\colon LA_{cyl} = 5\pi\sqrt{74}\colon 70\pi = \sqrt{74}\colon 14$ |
| 22. | D | Two congruent cones having only a base in common is the result of the rotation. The surface area of the solid is then two times the lateral area of either of the cones. Since the area of the square is 36, the side length is 6 and the length of a diagonal, the hypotenuse of 45-45-90 triangle with legs 6, is $6\sqrt{2}$. Half the length of a diagonal, $3\sqrt{2}$, is the length of the radius and also the height of a cone. The slant height of the cone is the hypotenuse of legs radius and height and is therefore, 6.<br>$SA_{cone} = \pi rl = \pi(3\sqrt{2})(6) = 18\pi\sqrt{2} \qquad SA_{solid} = 2(18\pi\sqrt{2}) = 36\pi\sqrt{2}$ |
| 23. | C | The length of the semicircle, $\pi r = 6\pi$, is the circumference of the cone; since $6\pi = 2\pi r$, $r_{cone} = 3$. The radius of the semicircle is the slant height, $'l'$, of the cone and the hypotenuse of a triangle formed by the $r_{cone}, h_{cone}, and\ l_{cone}$. Therefore: $h_{cone} = \sqrt{6^2 - 3^2} = \sqrt{36 - 9} = \sqrt{27} = 3\sqrt{3}$.<br>$V = {}^{\pi r^2 h}/_{3} = {}^{\pi(3^2)(3\sqrt{3})}/_{3} = 9\pi\sqrt{3}$ |

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| 24. | E | $V_{cyl} = \pi r^2 h = \pi 2^2(6) = 24\pi$ which is the volume of the new cone.<br>$V_{cone} = \pi r_{cone}^2 h / 3$; $24\pi = 9\pi(h)/3$; $24\pi = 3\pi h$; $h_{cone} = 8$.<br>Slant height is hypotenuse for legs 8 & 3, therefore, $l = \sqrt{64+9} = \sqrt{73}$.<br>Surface Area of Cone: $SA = \pi r^2 + \pi r l = 9\pi + 3\pi\sqrt{73}$ |
| 25. | A | The height and diameter of the cylinder are equal to 6, the side length of the cube.<br>$V_{cube} - V_{cyl} = a^3 - \pi r^2 h = 6^3 - \pi(3^2)6 = 216 - 54\pi$ |
| 26. | B | The slant height is the hypotenuse to legs apothem and height of the pyramid. The apothem of the square base (and the radius of the cone) is 3.<br>Therefore, height of pyramid and cone: $h = \sqrt{9^2 - 3^2} = \sqrt{72} = 6\sqrt{2}$.<br>$V_{cone} = \pi r^2 h / 3 = \pi(3^2)(6\sqrt{2}) / 3 = 18\pi\sqrt{2}$ |
| 27. | D | The first information needed is the area a segment of a circular cross-section, perpendicular to the axis, that represents the liquid in the tank. Since the diameter is 8, the radius is 4 and the sector of the circle that contains the needed segment is composed of an isosceles triangle with legs 4 and height 2. The height divides the triangle into two 30-60-90 triangles with the height (2) as the short leg and half the chord of the segment as the long leg $(2\sqrt{3})$; across from the long leg with vertex at the center of the circle is the $60°$ angle and since there are two 30-60-90 triangles, the central angle of the sector is $120°$.<br>Area of segment = area of sector – area of triangle<br>$A = \pi r^2/3 - bh/2 = 16\pi/3 - (4\sqrt{3})(2)/2 = 16\pi/3 - 4\sqrt{3}$.<br>All that is needed now to find the volume of the liquid is to multiply the segment area times the length of the tank. $V = (16\pi/3 - 4\sqrt{3})20 = 320\pi/3 - 80\sqrt{3} = (320\pi - 240\sqrt{3})/3$ |
| 28. | A | $LA_{cyl} = 2\pi r h$; $2\pi r^2 = 2\pi r h$; $r = h = 2$; $d = 4$ Hypotenuse of right triangle is $\sqrt{4^2 + 2^2} = 2\sqrt{5}$<br>Perimeter of right triangle: $diameter + height + hypotenuse = 4 + 2 + 2\sqrt{5} = 6 + 2\sqrt{5}$ |
| 29. | E | Regular hexahedron is a cube. The diagonal $(3\sqrt{2})$ of the cube face is the diameter of the cylinder, the radius is then $3\sqrt{2}/2$ or $3/\sqrt{2}$. The height of the cylinder is equal to an edge (3) of the cube.<br>Surface area of the cylinder:<br>$SA_{cyl} = 2\pi r^2 + 2\pi r h = 2\pi\left(3/\sqrt{2}\right)^2 + 2\pi\left(3\sqrt{2}/2\right)(3) = 2\pi(9/2) + 9\pi\sqrt{2} = 9\pi + 9\pi\sqrt{2}$<br>$SA_{cube} = 6a^2 = 6(9) = 54$ Since both surface areas include the two shared bases of the cube they must be subtract from each area before adding to answer the question.<br>$SA_{cyl} - 2a^2 + SA_{cube} - 2a^2 = 9\pi + 9\pi\sqrt{2} - 2(9) + 54 - 2(9) = 9\pi + 9\pi\sqrt{2} - 18$ |
| 30. | B | The solid formed consists of two congruent cones that only have their base in common. The radius of each cone is the height of the original equilateral triangle. Since the height of the triangle is the long leg of a 30-60-90 triangle and a side (4) of the equilateral triangle is its hypotenuse, the height of the triangle has length $2\sqrt{3}$. The height of each cone is (2), half the length of a side of the triangle.<br>$V_{solid} = 2\left(\pi r^2 h/3\right) = 2\left(\pi(2\sqrt{3})^2(2)/3\right) = 2(8\pi) = 16\pi$ |