Nationals 2018 Alpha Ciphering Solutions

0. $\left(\cos^2 x+\sin^2 x\right)\left(\cos^2 x-\sin^2 x\right)=\left(\cos^2 x-\sin^2 x\right)=\cos 2x \to \frac{2\pi}{2}=\pi$

$$\cos x(\cos\frac{\pi}{3})-\sin x\left(\sin\frac{\pi}{3}\right)+\sin x\left(\cos\frac{\pi}{6}\right)+\cos x\left(\sin\frac{\pi}{6}\right)$$

1. $\frac{1}{2}\cos x-\frac{\sqrt{3}}{2}\sin x+\frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\cos x=\cos x$

$period = 2\pi \quad amplitude = 1 \quad 2\pi$

2. The number of combinations given at least one 1 occurs is $6^3-5^3=216-125=91$. The only ways to get a sum of 6 is 1,1, and 4 or 1,2, and 3. There are 3 ways for the former and 6 ways to get the latter, for a total of 9 ways. The answer is 9/91, so 9 + 91 =100

3. The critical numbers are -8 and 2, which creates 3 test zones. 3k-6+k+8=5 4k=3 k=3/4
6-3k+k+8=5 -2k=-9 k=9/2 6-3k-k-8=5 -4k=7 k=-7/4
All of these solutions are extraneous so there are 0 solutions!

4.
$$\sec x+1=\sec^2 x-1$$
$$\sec^2 x-\sec x-2=0$$
$$\left(\sec x-2\right)\left(\sec x+1\right)=0$$
$$\cos x=\frac{1}{2},-1\to\frac{\pi}{3}+\frac{5\pi}{3}+\pi=3\pi$$

L=3

5. $2\log_3 6-\log_3\frac{4}{27}=\log_3\left[\left(36\right)\left(\frac{27}{4}\right)\right]=\log_3 3^5=5$

6.

$$\frac{\begin{matrix} 4 & 1 & 8 & 4 & 1 \\ 3 & 2 & 5 & 3 & 2 \\ 0 & -3 & 9 & 0 & -3 \end{matrix}}{\begin{matrix} 4 & 1 & -2 & 4 & 1 \\ 3 & 2 & -1 & 3 & 2 \\ 0 & -3 & 1 & 0 & -3 \end{matrix}} = \frac{72-72+60-27}{8+18-12-3} = \frac{33}{11} = 3$$

7.

$$\sqrt{\left(13-\frac{1}{13}\right)^2} = 13-\frac{1}{13} = 12\frac{12}{13} \to 12+12+13 = 37$$

8.

$$\cos^2 x+\sin^2 x+2\sin x\cos x = \frac{1}{4}$$

$$1+\sin 2x = \frac{1}{4} \to \sin 2x = \frac{-3}{4}$$

$$\sin^2 2x = \frac{9}{16} \to \cos^2 2x = 1-\frac{9}{16} = \frac{7}{16}$$

9.

$$x^2-6x+9 = 12y+51+9$$

$$(x-3)^2 = 12(y+5)$$

$$Area = \frac{1}{2}(p)(4p) = 2p^2$$

$$4p = 12 \to p = 3 \to 2(3)^2 = 18$$

10. Several ways to solve this. I drew a picture and created some right triangles.

$$18^2 + r^2 = (12 + r)^2$$

$$324 + r^2 = 144 + 24r + r^2$$

$$24r = 180 \rightarrow r = \frac{15}{2} \rightarrow d = 15$$

11. $$\frac{a_5 - a_4}{a_2 - a_1} = \frac{a_1 r^4 - a_1 r^3}{a_1 r - a_1} = \frac{a_1 r^3 (r-1)}{a_1 (r-1)} = \frac{576}{9} = r^3$$

$$r = 4 \rightarrow a_1 = 3 \rightarrow 3 + 3(4) = 15$$

12. $$-5 < k - 2 < 5 \rightarrow -3 < k < 7$$

$$k - 2 > 1 \bigcup k - 2 < -1 \rightarrow k > 3 \bigcup k < 1$$

$$\bigcap = -2, -1, 0, 4, 5, 6 \rightarrow sum = 12$$

Answers:

1. 2π
2. 100
3. 0
4. 3
5. 5
6. 3
7. 37
8. 7/16
9. 18
10. 15
11. 15
12. 12

0. π