THETA CIPHERING ANSWERS

0. 54

1. \( \frac{152\pi}{15} \)

2. (-1.5, 0)

3. \( \frac{169}{24} \) or \( 7 \frac{1}{24} \)

4. 4

5. \[
\begin{bmatrix}
3 & 0 \\
-8 & -7
\end{bmatrix}
\]

6. \( 72\sqrt{3} - 36\pi \)

7. 1

8. -5, -1, 4

9. \( \frac{1 + 3\sqrt{5}}{2} \)

10. \( \left( \frac{2}{3}, 1 \right) \cup (1, 2) \cup (3, \infty) \)

11. \( \frac{x^2 + 1}{x^3 + 2x} \)

12. 128

13. 4

14. \( \frac{7}{2} \)
0. 54  

3 \cdot 6 \cdot 3 = 3 \text{ choices for hundreds place, } 6 \text{ choices for tens place, } 3 \text{ even digits for ones place.}

1. \frac{152\pi}{15}  

Arc length is a fraction (38/60) of the circumference (16\pi)

2. (-1.5, 0)  

Slope of perpendicular line will be $-\frac{4}{3}$ and the midpoint of the segment is (-3, 2). Using point-slope form $y - 2 = -\frac{4}{3}(x + 3)$ leads to $4x + 3y = -6$ and letting $y = 0$, gives $x = -1.5$.

3. \frac{169}{24} \text{ or } 7 \frac{1}{24}  

\(\triangle ABC\) is a right triangle by the Pythagorean Theorem. \(\triangle AMP \cong \triangle BMP\) by SAS, so AP=PB. Let AP = x. Using the Pythagorean Theorem on \(\triangle APC\), $25 + (12-x)^2 = x^2$ which gives $25 + 144 - 24x + x^2 = x^2$ and so $x = 169/24$.

4. 4  

The center (h, k) is (0, -4) and c = 4. Since a vertex is (0, 1), $a = 5$. $b^2 = a^2 - c^2$ resulting in $b = 3$. So, $a + b + h + k = 5 + 3 + 0 + (-4) = 4$.

5. \[
\begin{bmatrix}
3 & 0 \\
-8 & -7
\end{bmatrix}
\]

$AB = \begin{bmatrix}
6 & 0 \\
-12 & -9
\end{bmatrix}$ and $B^{-1}A = \frac{1}{3} \begin{bmatrix}
2 & 1 \\
-3 & 0
\end{bmatrix} \begin{bmatrix}
4 & 2 \\
1 & -4
\end{bmatrix} = \begin{bmatrix}
3 & 0 \\
-4 & -2
\end{bmatrix}$

Subtract the two results and you get \[
\begin{bmatrix}
3 & 0 \\
-8 & -7
\end{bmatrix}
\]

6. $72\sqrt{3} - 36\pi$  

The radius of the circle is 6 so its area is $36\pi$. Using the 30-60-90 triangle formed by drawing a radius of the circle to the point of tangency to the hexagon (an apothem) and a segment from the center to a vertex of the hexagon, the short leg of the triangle is $2\sqrt{3}$. This makes the side of the hexagon $4\sqrt{3}$. The formula for area = \(\frac{1}{2}\) (apothem)(perimeter) so $A = \frac{1}{2} \cdot 6 \cdot 24\sqrt{3} = 72\sqrt{3}$. The area between them is $72\sqrt{3} - 36\pi$.

7. 1  

$4(-1) + 2 \left(\frac{3}{2}\right) + 6 \left(\frac{1}{3}\right) = -4 + 3 + 2 = 1$

8. -5, -1, 4  

Try the test for -1 as a root of f(-x). It works! Synthetically divide by -1 to get the quotient $x^2 + x - 20 = 0$ which factors to $(x + 5)(x - 4) = 0$ and gives zeros of -5 and 4.
9. Let \( x = \frac{1 + 3\sqrt{5}}{2} \) Squaring both sides gives \( x^2 = 11 + x \) which can be solved by the quadratic formula as \( x = \frac{1 + 3\sqrt{5}}{2} \). Only the positive solution works.

10. \( \left( \frac{2}{3}, 1 \right) \cup (1, 2) \cup (3, \infty) \) The argument of the logarithm must be positive which gives the resulting domain of \((-3, 2) \cup (3, \infty)\). The base of the logarithm, \( 3x - 2 \), must also be positive and not equal to 1. This means that \( x \) must be greater than \( \frac{2}{3} \) but not equal to 1.

11. \( \frac{x^2 + 1}{x^3 + 2x} \) \((x + \frac{1}{x})^{-1}\) becomes \( \frac{x}{x^2 + 1} \). \([x + \frac{x}{x^2 + 1}]^{-1} = \left(\frac{x^3 + 2x}{x^2 + 1}\right)^{-1} = \frac{x^2 + 1}{x^3 + 2x} \)

12. \( 2^7 \) or 128 The prime factorization of 2016 is \( 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 7 \). The smallest prime factor is 2 and the largest prime factor is 7.

13. \( 4 \) Solving the first inequality gives \(-10 \leq 3x - 4 \leq 10 \) resulting in \(-2 \leq x \leq \frac{14}{3} \). Solving the second gives \( 3x + 2 > 4 \) or \( 3x + 2 < -4 \) leading to the solution of \( x > \frac{2}{3} \) or \( x < -2 \). The only integers that satisfy both are 1, 2, 3, and 4.

14. \( \frac{7}{2} \) Let \( m = 2^{2x} \). The equation becomes \( 3(2^{2x} \cdot 2^3) - (2^{2x})^2 = 128 \) and after substituting we get \( 24(m) - m^2 = 128 \), or \( m^2 - 24m + 128 = 0 \). This factors into \((m - 16)(m - 8) = 0 \) so \( m = 8 \) or \( m = 16 \). Solving for \( x \), we get \( 2^{2x} = 8 \) which yields \( x = 3/2 \) and \( 2^{2x} = 16 \) which yields \( x = 2 \). The sum is \( 3/2 + 2 = 7/2 \).