For all questions, answer choice (E) NOTA means that none of the above answers is correct. Choose the best answer for each question.

1. Use Euler's method to obtain the approximate $y(\frac{3}{2})$ for the solution of initial-value problem(**IVP**) $y' = x^2 + y^2, y(0) = -1$ with step size $h = \frac{1}{2}$.

(A) $-\frac{1}{2}$ (B) $-\frac{1}{4}$ (C) $\frac{9}{32}$ (D) $\frac{5}{8}$ (E) NOTA

2. What is the first slope for the Heun's method polygon for **IVP** $y' = 2x + y^2, y(0) = -1$ with step size $h = \frac{1}{2}$.

(A) $\frac{1}{8}$ (B) $\frac{3}{8}$ (C) $\frac{9}{8}$ (D) $\frac{5}{8}$ (E) NOTA

3. Suppose $y(x)$ is a solution of $y' = \frac{x^2}{1-3y^2}$ that passes through point $(\sqrt[3]{3}, -2)$. Then $y(x)$ also passes through which of the following points?

I) $(-\sqrt[3]{15}, 0)$ II) $(-\sqrt[3]{15}, 1)$ III) $(-\sqrt[3]{15}, -1)$ IV) $(0,1)$

(A) I (B) I and II (C) I, II and III (D) IV (E) NOTA

4. Suppose $y(x)$ is a solution of **IVP** $y' = \frac{\sin\sqrt{x}}{\sqrt{y}}, y(0) = 3^{\frac{2}{3}}$. What is the value of $y(\frac{\pi^2}{4})$?

(A) 0 (B) 1 (C) $3^{\frac{2}{3}}$ (D) $6^{\frac{2}{3}}$ (E) NOTA

5. Solve **IVP** $y' = \frac{2(1+y)}{1-x^2}, y(0) = 1$.

(A) $y = \frac{1-3x}{1-x}$ (B) $y = \frac{1+3x}{1-x}$ (C) $y = \frac{1+3x}{1+x}$ (D) $y = \frac{1-3x}{1+x}$ (E) NOTA

6. Let $y(x)$ be a solution of $y' - y\sin x = 2\sin x$ which passes through the point $(\frac{\pi}{2}, 1)$. Then $y(x)$ also passes through which of the following points?

I) $(-\frac{\pi}{2}, 1)$ II) $(-\pi, 1)$ III) $(0, -1)$ IV) $(0,1)$

(A) I (B) I and II (C) I, II and III (D) IV (E) NOTA

7. Find an integrating factor for $y' = 2y + 5$.

(A) x (B) x^2 (C) e^x (D) e^{-2x} (E) NOTA

8. Suppose $x(y)$ is a solution of $y' = \dfrac{y}{-x+2y}$ which passes through the point $(0,-1)$. What is the value of $x(1)$?

(A) 0 (B)1 (C) 2 (D) $3\ln x$ (E) NOTA

9. The sine integral function is defined by $\mathrm{Si}(x) = \int_0^x \frac{\sin t}{t} dt$, where the integrand is defined to be 1 at $t = 0$. Express the solution of the **IVP** $x^3 y' + 2x^2 y = 10\sin x, y(1) = 0$ in term of $\mathrm{Si}(x)$.

(A) $x^2 y = 10(\mathrm{Si}(x) - \mathrm{Si}(3))$ (B) $x^2 y = 10(\mathrm{Si}(x) - \mathrm{Si}(2))$

(C) $x^2 y = 10(\mathrm{Si}(x) - \mathrm{Si}(1))$ (D) $x^2 y = 10\mathrm{Si}(x) - \mathrm{Si}(10)$ (E) NOTA

10. The solution of **IVP** $(y + xy')\cos(xy) = x + 1, y(0) = \frac{\pi}{2}$ also passes through which of the following points?

I) $(0,1)$ II) $(1,0)$ III) $(-1,\frac{\pi}{6})$ IV) $(1,1)$

(A) I (B) I and II (C) I and III (D) I and IV (E) NOTA

11. Find the solution of Bernoulli Equations $xy' + y = xy^3$ that passes through the point $(1,1)$.

(A) $x(2-x)y^2 = 1$ (B) $x^2(2-x)y^2 = 1$

(C) $x(2-x)^2 y^2 = 1$ (D) $x^3(2-x)y^2 = 1$ (E) NOTA

12. Suppose $(x-y)y' = ay - 3x + \frac{y^2}{x}$ is Euler homogeneous. What is the value of a?

(A) 1 (B) 2 (C) 3 (D) any real number (E) NOTA

13. Let $y(x)$ be a solution of $y' = \dfrac{x(y+1) + (y+1)^2}{x^2}$ which passes through the point $(-1,0)$. What is the value of $y(1)$?

(A) 1 (B) -2 (C) -1 (D) 2 (E) NOTA

14. If $(x^2 + 2y)dy + (axy + bsinx)dx = 0$ is an exact equation, what is the value of a ?

(A) 1 (B) 2 (C) -2 (D) 3 (E) NOTA

15. Find the solution of $(x^2e^y + cosx - 1)dy + (2xe^y - ysinx)dx = 0$.

(A) $x^2e^y + y(\cos x + 1) = c$ (B) $x^2e^y + y(\cos x - 1) = c$

(C) $x^2e^y + y(\sin x + 1) = c$ (D) $x^2e^y + y(\sin x - 1) = c$ (E) NOTA

16. Equation $(x^2 + xy)dy + (3xy + y^2)dx = 0$ is not exact. Which one of the following can be an integrating factor?

(A) $\frac{1}{x}$ (B) x (C) x^2 (D) $\ln x$ (E) NOTA

17. Solve $xydx + (2x^2 + 3y^2 - 20)dy = 0$.

(A) $x^2y^4 - y^6 - 10y^4 = c$ (B) $x^2y^4 + y^6 + 10y^4 = c$

(C) $x^2y^4 + y^6 - 10y^4 = c$ (D) $x^3y^4 + y^6 - 10y^4 = c$ (E) NOTA

18. Suppose $y_1 = x^2$ is a solution of $x^2y'' - 3xy' + 4y = 0$. Use y_1 to find a second solution.

(A) $\frac{1}{x}$ (B) x (C) x^3 (D) $x^2 \ln x$ (E) NOTA

19. Find the solution of **IVP** $y'' + 4y' - 5y = 0, y(0) = 4, y'(0) = 2$.

(A) $y = 3e^{-x} + e^{-5x}$ (B) $y = 3e^{x} + e^{-5x}$

(C) $y = 3e^{-x} + e^{5x}$ (D) $y = -3e^{-x} + e^{5x}$ (E) NOTA

20. Suppose $y(x)$ is a solution of $y'' + 4y = 0$ that passes through the point $(\frac{\pi}{4}, 2)$. What is the value of $y'(0)$?

(A)-1 (B) 1 (C) 2 (D) 4 (E) NOTA

21. Which of the following functions can be a solution to $(y')^2 + y^2 - 1 = 0$?

I) $y = 1$ II) $y = -1$ III) $x + c = \sin^{-1} y$ IV) $x + c = 2\cos^{-1} y$

(A) I and II (B) I, II and IV (C) I, II and III (D) ALL (E) NOTA

22. Find the critical points (equilibrium solution) of $y' = (y+1)(y-2)(4-y)$.

I) $y = -1$ II) $y = 2$ III) $y = 4$ IV) NONE

(A) I (B) I and II (C) I, II and III (D) IV (E) NOTA

23. Suppose $y(x)$ is a solution of **IVP** $y'' = x + y - y^2, y(0) = -1, y'(0) = 1$. We further assume $y(x)$ possesses a Taylor series expression center at 0: $y(x) = y(0) + y'(0)x + \frac{y''(0)}{2!}x^2 + \frac{y'''(0)}{3!}x^3 + \cdots$

What is the value of $\frac{y'''(0)}{3!}$?

(A)-1 (B)1 (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) NOTA

24. Suppose $y(x)$ is a solution to $y'y'' = x, y(0) = 1, y'(1) = \sqrt{2}$. Find $y'''(0)$.

(A) 1 (B) -1 (C) ± 1 (D) $\frac{2}{3}$ (E) NOTA

25. Suppose $y(x)$ is a solution to $y'' = 1 - (y')^2, y(0) = 1, y'(1) = \sqrt{2}$. Find $y^{(4)}(0)$.

(A) 6 (B) -6 (C) 10 (D) -10 (E) NOTA

26. Suppose $y = 1 + x + x^2 + x^3 + \cdots, 0 < |x| < 1$. Then y satisfies

I) $y' = y^2$ II) $y'' = 2y^3$ III) $y''' = 3y^4$ IV) $y^{(4)} = 4y^5$

(A) I (B) I and II (C) I, II and III (D) IV (E) NOTA

27. Find a particular solution of $y'' - 2y' + y = e^x$.

(A) $y_p = \frac{1}{2}e^x$ (B) $y_p = \frac{1}{2}x^2e^x$ (C) $y_p = \frac{1}{2}xe^x$ (D) $y_p = x^2e^x$ (E) NOTA

28. Suppose $y(x)$ is a particular solution of $y=(x+1)y'+(y')^2$, $y(1)=-1$. Then $y(x)$ passes through which of the following points?

I) $(0,0)$ II) $(0,\frac{1}{4})$ III) $(-1,0)$ IV) $(2,-2)$

(A) I (B) I and II (C) I, II and III (D) I and IV (E) NOTA

29. Find the form of the particular solution of $y''' + y'' = e^x(\cos x - 2\sin x)$.

(A) $y_p = (\frac{2}{5}\sin x + \frac{3}{10}\cos x)e^x$ (B) $y_p = (\frac{3}{10}\sin x + \frac{2}{5}\cos x)e^x$

(C) $y_p = (\frac{2}{5}\sin x - \frac{3}{10}\cos x)e^x$ (D) $y_p = (\frac{3}{10}\sin x - \frac{2}{5}\cos x)e^x$ (E) NOTA

30. Tom tries to solve **IVP** $y' + y\cot x = 2\cos x,\ y(\frac{\pi}{2}) = 5$. He got general solution $y_T(x)$ by mistake, using initial value $y(\frac{\pi}{2}) = 3$. Suppose the real solution of given problem is $y(x)$. What is value of $(y(x) - y_T(x))|_{x=\frac{\pi}{6}}$?

(A) 0 (B) 1 (C) 2 (D) 4 (E) NOTA