

1. **D.**

They intersect thrice.

2. **B.** The expression equals  $f(g(3))$ .  $g(3) = 48$ .  $f(48) = 4627$ 3. **C**

$$((3^{1/3})^{1/3})^{1/3} = 3^{1/27} = 27^{1/81}$$

4. **B**

$$(2^3)^2 = 2^6 = 64$$

5. **D**

$$2f(2) = 2 \cdot 4 = 8, 2g(8) = 512$$

6. **C**

For the positive side of the graph of the intersection we can easily find  $x=2,4$  to be solutions. We also know that  $g(x)$  always greater than  $f(x)$  on the interval  $(4, \infty)$  so there are only two positive solutions. At  $x=0$ ,  $g(x) > f(x)$ . We know that  $g(x)$  is decreasing and  $f(x)$  is increasing on the interval of  $(-\infty, 0)$  so there must be one point of intersection.

$$2+1=3$$

7. **C**

$$3x-1=5, \text{ so } x=2. 2^3+2^2-2-1=9$$

8. **C**

$P(x)$  can be defined as  $(x-2)(Q(x)) + R(x)$  such that  $Q(x)$  is the quotient of  $P(x)$  divided by  $x-2$  and  $R(x)$  is the remainder. Plugging in  $x = 2$  gives  $P(2) = R(2)$ , so we get 83.

9. **A**

The numerator factors into  $(x+1)(x-1)(x+2)(3x+4)$  and the denominator factors into  $(x+1)(x+2)(x+4)(3x+4)$ . There are 3 removable discontinuities, so there are  $4-3=1$  vertical asymptotes

10. **D**

I. True. Question 9 is an example of this.

II. False. This is the definition of a point discontinuity.

III. False. The function has 3 point discontinuities.

IV. True. They are the same names for each other

V. True. If one component defines the value for the hole, a piecewise function can make a continuous one.

11. **B**

$$\text{Plug in } x=4 \text{ and get } f(4) + 2f(1/4)=12$$

$$\text{Plug in } x=1/4 \text{ and get } f(1/4) + 2f(4) = 3/4$$

$$\text{Double the second equation and subtract to get } 3f(4)=-21/2, \text{ so } f(4)=-7/2.$$

12. **C**

First, one must recognize that these are the fibonacci numbers. The ratio of fibonacci numbers is the golden ratio:  $\frac{1+\sqrt{5}}{2}$

13. **A**

Plug in  $\text{root}(2/(3x))$ , to get  $g(x)$ .

$$[(4/9x^2)-4(2/(3x))]/[(16/9x^2)-(2/(3x))]=(12x-2)/(3x-8)$$

14. **B**

II and III are by definition the same

**15. E**

Only I is correct. If IV said about the y-axis, it would be correct as well.

**16. E**

An odd function of an even function results in an odd function. For example, if we let  $f(x)$  be an arbitrary odd function,  $x^3$  and  $g(x)$  be an even function,  $x^2$ .  $f(g(x)) = x^6$ , so it is even as well. For the same reason as question #15, only I is correct.

**17. B**

$\sin^4 x - \cos^4 x = (\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)$ . The 2nd part is an identity, so we're left with  $\sin^2 x - \cos^2 x$ . We can either use the identity or recognize this as  $-\cos 2x$ . Putting  $g(x)$  back in for  $\cos x$  yields  $1 - 2(g(x))^2$ .

**18. A**

Substitute  $a = x + y$

Factor  $(x+y)^4 - (x+y) = 0$

$a(a^3 - 1) = a(a-1)(a^2 + a + 1) = 0$

We have two real solutions of  $a=0$  and  $a-1=0$  while the solutions to  $a^2 + a + 1 = 0$  are imaginary

Substitute back  $a = x + y$  and we have two parallel lines  $x + y = 0$  and  $x + y - 1 = 0$ .

**19. C**

The vertex of a parabola is its largest value if the parabola faces down. With a vertex at  $\frac{-b}{2a} = 2$ , plugging in gets a value of 21.

**20. C**

Profit is Revenue - Cost =  $xP(x) - C(x) = (3x^2 + 9x) - (4x^2 - 7x + 3) = -x^2 + 16x - 3$  which describes a parabola that opens downwards. The vertex or maximum is at  $x = -\frac{16}{2(-1)} = 8$ .

**21. A**

Rearrange  $(y+1)^2$  to  $1 + y(y+2)$ .

Plug in  $y=2, 3, 4, 5 \dots$  and we get

$3^2 = 1 + 2(4)$ ,  $4^2 = 1 + 3(5)$ ,  $5^2 = 1 + 4(6) \dots$  and we can see a pattern that matches the infinite radical in the problem.

For example, square root the third equation to get  $5 = \sqrt{1 + 4(6)}$  and plug that into the second equation and square root it to get  $4 = \sqrt{1 + 3(\sqrt{1 + 4(6)})}$  and plug that into the first equation and square root it to get  $3 = \sqrt{1 + 2\sqrt{1 + 3(\sqrt{1 + 4(6)})}}$ . The pattern continues infinitely so our answer is 3.

**22. C**

If Stewie misread the product of the roots, he has the correct sum of the roots and if Kenny misread the sum of the roots, he has the correct product of the roots. This makes the function  $y = x^2 - 14x + 40$ , for which  $x=4$  and  $x=10$  are roots.  $4^2 + 10^2 = 116$

**23. B**

Squaring and rearranging the equation, we can get the equation of an ellipse:

$(x-1)^2/4 + y^2/36 = 1$ . The area of the ellipse is  $\sqrt{4} * \sqrt{36} * \pi = 12\pi$ . However, the square root restricts the original equation to only positive values of  $f(x)$  which also has a line of symmetry at  $y=0$ , so we half  $12\pi$  to get our answer,  $6\pi$ .

**24. D**

We need to find an  $n$  such that  $n!$  has at most 6 powers of 5. We can either go through very large factorials or recognize that  $25!$  has 6 powers of five because  $25/5=5$  and  $25/25=1$  and  $5+1=6$ . However, the question asks for the largest value of  $n$  that satisfies the condition, so we want the largest factorial with 6 powers of 5. If we were to go up 5 to  $30!$ , the resulting expansion would have  $30/5 + 30/5$  (rounded down) = 7 trailing zeros, so the value of  $n$  is 29. The sum of the first  $n$  even numbers is  $n(n+1)$ , so  $29(30) = 870$

**25. E**

$$(2-5)^7 = -2197$$

**26. A**

$$G(x) = x^2 - 6x + 1 = 28 \text{ so } x = -3 \text{ or } 9$$

$$F^{-1}(x) = 9^{\frac{x}{3}} = 9 \text{ [cannot be } -3], \text{ so } x = 3$$

**27. C**

$$F(x) = \sum_{n=0}^6 x^n (x - 1)$$

We can combine and factor this function into  $(x - 1)(1 + x + x^2 + x^3 + x^4 + x^5 + x^6) = x^7 - 1$ . Since  $x-1$  is a constant, we can factor that out. Then we recognize  $F(x)$  to be

$$(x - 1)(1 + x + x^2 + x^3 + x^4 + x^5 + x^6) = x^7 - 1. \quad 3^7 - 1 = 2186.$$

$$2+1+8+6 = 17$$

**28. D**

To find the coefficients of  $G(x)$ , we can use repeated long division.

$$\begin{array}{r}
 -3 \overline{) \phantom{1} 1 \phantom{-4} -7 \phantom{26} 2 \phantom{-91} -5} \\
 \phantom{-3} \underline{\phantom{1} \phantom{-4} -3 \phantom{26} -15 \phantom{-91} 39} \\
 \phantom{-3} \phantom{1} \phantom{-4} \phantom{-3} \phantom{26} -13 \phantom{-91} 34 \\
 \phantom{-3} \phantom{1} \phantom{-4} \phantom{-3} \phantom{26} \phantom{-91} -78 \\
 \phantom{-3} \phantom{1} \phantom{-4} \phantom{-3} \phantom{26} \phantom{-91} \phantom{34} -91 \\
 \phantom{-3} \phantom{1} \phantom{-4} \phantom{-3} \phantom{26} \phantom{-91} \phantom{34} \phantom{-91} 56 \\
 \phantom{-3} \phantom{1} \phantom{-4} \phantom{-3} \phantom{26} \phantom{-91} \phantom{34} \phantom{-91} \phantom{56} -13 \\
 \phantom{-3} \phantom{1} \phantom{-4} \phantom{-3} \phantom{26} \phantom{-91} \phantom{34} \phantom{-91} \phantom{56} \phantom{-13} 1
 \end{array}$$

The  $x^2$  term would be the middle number, 56.

29. **A**

Q(x) is  $x = 2(y + 3)^2 + 5(y + 3) - 4$  and so plugging in 21 for x gives us that  $y = 2$  or  $\frac{11}{2}$  and the shortest distance to the origin is from (21, 2), whose distance is  $\sqrt{445}$

30. **E. 0**

Graphing each equation and using the horizontal line test conclusively finds that none of the roman numerals are one to one.