For all questions, answer choice "E) NOTA" means that none of the above answers is correct.

1. The solution set of the inequality below can be written as $(p, q]$. Compute $p + q$.

$$\frac{3x-5}{x+2} \leq 2$$

A) 7 B) 9 C) 11 D) 13 E) NOTA

2. Which of the following is a fourth root of $4 - 4i\sqrt{3}$?

A) $\sqrt[4]{8}(\cos(65°) + i\sin(65°))$

B) $\sqrt[4]{8}(\cos(165°) + i\sin(165°))$

C) $\sqrt[4]{8}(\cos(245°) + i\sin(245°))$

D) $\sqrt[4]{8}(\cos(265°) + i\sin(265°))$

E) NOTA

3. The vector $\vec{w}$ is the cross product of $\langle 1, 2, 2\rangle$ and $\langle 7, 0, 24\rangle$. Compute $\|\vec{w}\|$.

A) $\sqrt{634}$ B) $\sqrt{2353}$ C) $10\sqrt{26}$ D) $2\sqrt{986}$ E) NOTA

4. Which of the following ellipses does **not** have a focus at $(0, 4)$?

A) $x^2 + 10y^2 - 6x - 80y + 159 = 0$

B) $3x^2 + 7y^2 - 42x + 56y + 238 = 0$

C) $x^2 + 3y^2 - 4x - 24y + 46 = 0$

D) $9x^2 + 10y^2 - 18x - 80y + 79 = 0$

E) NOTA

5. Let A be a 2×3 matrix and let B be a 3×2 matrix. How many of the following are defined?

I. $A + B$ II. AB III. BA IV. $AB - BA$

A) 0 B) 1 C) 2 D) 3 E) NOTA

6. If point R has coordinates $(5, 3, 4)$, point H has coordinates $(3, 0, 2)$, and O is the origin, compute the area enclosed by triangle RHO.

A) 9 B) $\sqrt{131}$ C) $\frac{5\sqrt{26}}{2}$ D) $\frac{5\sqrt{34}}{2}$ E) NOTA

7. If $x^2 + 2xy - 3y = 3$, then the value of the slope of the tangent line at $x = 2$ is

A) -2 B) $-\frac{1}{2}$ C) 1 D) 2 E) NOTA

8. A random real number k is chosen from the interval $[-5/7, 2/3]$. Compute the probability that

$$kx^2 - \frac{x\sqrt{k}}{2} + \frac{k}{4}$$

has two real roots.

A) $\frac{21}{116}$ B) $\frac{42}{145}$ C) $\frac{12}{29}$ D) $\frac{16}{21}$ E) NOTA

9. When the mean, median, and mode of the set $\{10, 2, 5, 2, 4, 2, n\}$ are arranged in increasing order, they form a non-constant arithmetic progression. Compute the sum of all possible real values of n.

A) 6 B) 9 C) 17 D) 20 E) NOTA

10. Let P be the product of the numbers 4112018, 6142018, 7192018, and 7232018. Compute the value of $\log P$ to the closest integer.

A) 24 B) 25 C) 26 D) 27 E) NOTA

11. Suppose A, B, and C are 3×3 invertible matrices such that $ABC = I$, where I is the 3×3 identity matrix. What is BCA?

A) A^{-1} B) A^2 C) BC D) A E) NOTA

12. Compute the limit below.

$$\lim_{x \to 0} \frac{3^x - 1}{2^x - 1}$$

A) 0 B) $\ln\left(\frac{3}{2}\right)$ C) $\log_3 2$ D) $\log_2 3$ E) NOTA

13. Let N be the positive integer such that

$$\frac{N + 314}{N + 123}$$

is an integer. Compute the sum of the digits of N.

A) 5 B) 9 C) 11 D) 14 E) NOTA

14. Compute the area enclosed by a triangle with medians of lengths 60, 60, and 96.

A) 576 B) 624 C) 2304 D) 2496 E) NOTA

15. A barrel contains a selection of colored cubes, each of which is yellow, blue, or green. The number of green cubes is at least half of the number of blue cubes, and at most one-third of the number of yellow cubes. The cubes that are blue or green number at least 34. Compute the minimum number of yellow cubes.

A) 27 B) 36 C) 44 D) 52 E) NOTA

16. Let a, b, and c satisfy the equation

$$a(x-1)(x+1) + b(x-1)(x-3) + c(x+1)(x-3) = 2x - 20.$$

Compute $a + b + c$.

A) -3 B) 0 C) 1 D) 3 E) NOTA

17. Let g be the greatest common divisor of 4897 and 1357. Compute the sum of the digits of g.

A) 6 B) 8 C) 10 D) 12 E) NOTA

18. If you write the product $(2^{51}+1)(2^{50}+1)$ in binary, how many 0s will you need?

A) 98 B) 99 C) 100 D) 101 E) NOTA

19. Let $\lfloor x \rfloor$ represent the greatest integer less than or equal to the real number x. Compute

$$\left\lfloor \frac{3^{31}+2^{31}}{3^{29}+2^{29}} \right\rfloor.$$

A) 1 B) 3 C) 4 D) 8 E) NOTA

20. Suppose you roll three standard, fair six-sided dice. What is the probability that the smallest number you rolled was 1, given that the sum of the three numbers is prime?

A) $\frac{8}{31}$ B) $\frac{73}{216}$ C) $\frac{3}{8}$ D) $\frac{31}{73}$ E) NOTA

21. Compute the limit below, if it exists.

$$\lim_{x\to\infty}\left(\sqrt{x^2+3x+9}-\sqrt{x^2-5x+16}\right)$$

A) -2 B) 0 C) 2 D) 4 E) NOTA

22. Given a $3\times 3\times 3$ coordinate lattice with opposite corners at the origin and $(3,3,3)$, how many ways are there to move along the lattice (unit lengths only, parallel to the axes) from the origin to the point $(3,3,3)$ without passing through the point $(2,2,2)$? Assume movement is never away from $(3,3,3)$.

A) 540 B) 720 C) 1140 D) 1252 E) NOTA

23. Let x, y, and z be real numbers which satisfy the equations below.

$$\log(3xy)=\log(x)\log(y)$$
$$\log(yz)=\log(y)\log(z)$$
$$\log(3xz)=\log(x)\log(z)$$

Compute $x+y+z$.

A) 300 B) 500 C) 600 D) 1053 E) NOTA

24. Let $ABCD$ be a convex quadrilateral such that $AB=5$, $BC=17$, $CD=7$, and $DA=25$. Given that $\measuredangle ABC+\measuredangle BCD=270°$, compute the area of $ABCD$.

A) 60 B) 90 C) 120 D) 150 E) NOTA

25. Given that the sum of all positive integers with exactly two proper factors, each factor of which is less than 30, is 2397, compute the sum of all positive integers with exactly three proper factors, each factor of which is less than 30. (Note: a proper divisor of n is a positive integer that divides but is not equal to n.)

A) 4794 B) 5040 C) 6241 D) 7122 E) NOTA

26. Halmos is thinking of a function, $f(x)$. He reveals to Polya that the function is a polynomial of the form $f(x) = ax^7 + bx^5 + cx^3 + dx^2 + e$, where a, b, c, d, and e are real number coefficients. Polya wishes to determine the value of d. For any real number x that Polya asks about, Halmos will tell him the value of $f(x)$. At least how many values of x must Polya ask about in order to definitively determine the value of d?

A) 2 B) 3 C) 5 D) 7 E) NOTA

27. Let $f(x)$ equal the number of zeroes to the right of the rightmost non-zero digit in the decimal form of $x!$ and let $n = \frac{1}{4}(5^{2018} - 1)$. Given that $f(n)$ can be written as $\frac{1}{a}(b^c - d)$, where a and b are relatively prime positive integers, c is a positive integer, and d is a positive integer less than 10^4, compute $a + b + c + d$.

A) 10,107 B) 10,108 C) 10,111 D) 10,112 E) NOTA

28. A 30°-60° right triangle has an incircle of radius r and a circumcircle of radius R. Compute r/R.

A) $\frac{\sqrt{3}-1}{2}$ B) $\frac{\sqrt{5}-1}{2}$ C) $\frac{\sqrt{3}+1}{2}$ D) $\frac{\sqrt{5}+1}{2}$ E) NOTA

29. Let $P(x) = x^4 + ax^3 + bx^2 + cx + d$, where a, b, c, and d are real numbers. Suppose $P(1) = 827$, $P(2) = 1654$, and $P(3) = 2481$. Compute the value of $\frac{1}{4}[P(5) + P(-1)]$.

A) 827 B) 863 C) 3308 D) 4135 E) NOTA

30. A *perfect out-shuffle* on a deck of n cards is defined as follows: the deck is cut exactly in half between the $(n/2)$th card and the $(n/2 + 1)$th card, forming two piles A and B, the top card of A being the top card of the original deck. The cards of A and B are perfectly interwoven into one pile so that the top card of A is now the top card of the deck, the top card of B is now the second card in the deck, the second card of A is now the third card of the deck, etc. (Note that the top card of A ends up being the top card of the deck after the shuffle, and the bottom card of B ends up being the bottom card of the deck after the shuffle; this means that the top card stays on top and the bottom card stays on bottom.) Define $f(n)$ as the least positive number of perfect out-shuffles on a deck of n cards that will return the deck to its original order. (For example, $f(4) = 2$ because it takes 2 shuffles to return a deck of 4 cards to its original order.) Let $N = f(2) + f(4) + f(6) + f(8) + f(10)$. Compute the sum of the digits of N.

A) 5 B) 7 C) 9 D) 11 E) NOTA