1) $\frac{b-a}{n}\left(f(x_0)+f(x_1)+...+f(x_n)\right)=\frac{12}{4}\left(\ln(1)+\ln(4)+\ln(7)+\ln(10)\right)=3\ln(280)$ **B**

2) $a(x)=\cos\left(x+\frac{\pi}{2}\right)=-\sin(x) \rightarrow a'(x)=-\cos(x) \rightarrow a''(x)=\sin(x)$ **A**

3) For a function to be differentiable at *c*, it must be defined at *c* and $\lim_{x\to c^-} f(x)=\lim_{x\to c^+} f(x)$ and $\lim_{x\to c^-} f'(x)=\lim_{x\to c^+} f'(x)$. These values must also be finite. This does not occur at $x=0$ for $\ln|x|$, but does for the others. Therefore, the answer is I, III, and IV only. **C**

4) For the first limit:

$$\lim_{x\to\infty}\left(\sqrt{x^2+x+1}-1\right)=\lim_{x\to\infty}\left(\sqrt{x^2+x+1}-x\left(\frac{\sqrt{x^2+x+1}+x}{\sqrt{x^2+x+1}+x}\right)\right)=\lim_{x\to\infty}\left(\frac{x+1}{\sqrt{x^2+x+1}+x}\right)$$

$$\lim_{x\to\infty}\left(\frac{x\left(1+\frac{1}{x}\right)}{|x|\left(\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+1\right)}\right)=\lim_{x\to\infty}\left(\frac{\left(1+\frac{1}{x}\right)}{\left(\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+1\right)}\right)=\frac{1}{2}$$

The second limit:

$$\lim_{x\to-\infty}\left(\frac{\sqrt{x^2-4}}{1-2x}\right)=\lim_{x\to-\infty}\left(\frac{|x|\sqrt{1-\frac{4}{x}}}{x\left(\frac{1}{x}-2\right)}\right)=\frac{1}{2}$$

For the third limit, this graph has an oblique asymptote that is a line with positive slope, so:

$\lim_{x\to\infty}\left(\frac{x^4-9x^2+3x}{x^3+x^2-x+1}\right)=\infty$ Therefore, two of the limits exist and are finite. **D**

5) After verifying that the conditions for the MVT for derivatives are met:

$$f'(c)=\frac{f(b)-f(a)}{b-a} \rightarrow \frac{2}{3}c-\frac{4}{3}=-\frac{2}{3} \rightarrow \frac{2}{3}c=\frac{2}{3} \rightarrow c=1$$ **B**

6) After verifying that the conditions for the MVT for integrals are met:

$$(b-a)f(c)=\int_{-2}^{4}\left(f(x)\right)dx$$

$$\left(4-(-2)\right)\left(\frac{1}{3}c^2-\frac{4}{3}c\right)=\left[\frac{1}{9}x^3-\frac{2}{3}x^2\right]_{-2}^{4} \rightarrow 2c^2-8c=0 \rightarrow c=0,\ c=4$$

The largest value of *c* that satisfies the MVT for integrals is 0 since it must be on the open interval (-2,4)

A

7) Solving for *y* yields: $y=\frac{4-|x|}{2|x|}$. However, the absolute value symbols makes the integration tougher. To integrate, simplify the integral as $\int_{-4}^{-1}\frac{4-|x|}{2|x|}dx=\int_{-4}^{-1}\left(\frac{2}{|x|}-\frac{1}{2}\right)dx$

Notice here that when $x<0$, $|x|=-x$ so

$$\int_{-4}^{-1}\left(\frac{2}{|x|}-\frac{1}{2}\right)dx=\int_{-4}^{-1}\left(-\frac{2}{x}-\frac{1}{2}\right)dx=\left[-2\ln|x|-\frac{1}{2}x\right]_{-4}^{-1}=\frac{1}{2}-\left(-\ln(16)+2\right)=\ln(16)-\frac{3}{2}$$ **B**

8) $2xy+e^y\cos(x)=e$ at the point $(0,1)$.

$$2\left(y+x\frac{dy}{dx}\right)+e^y\cos(x)\frac{dy}{dx}-e^y\sin(x)=0 \;\rightarrow\; 2x\frac{dy}{dx}+e^y\cos(x)\frac{dy}{dx}=e^y\sin(x)-2y$$

Now, by plugging in the values of *x* and *y*:

$$\frac{dy}{dx}=\frac{e^y\sin(x)-2y}{e^y\cos(x)+2x}=\frac{e\sin(0)-2(1)}{e\cos(0)+2(0)}=\frac{-2}{e}$$ **D**

9) $g(x)=\sqrt{x^2+7} \;\rightarrow\; g'(x)=\frac{x}{\sqrt{x^2+7}} \;\rightarrow\; g'(3)=\frac{3}{4}$

Because the slope of the tangent line is $\frac{3}{4}$, the slope of the perpendicular line is $-\frac{4}{3}$. **A**

10) This revolution forms an Ellipsoid. **A**

11) $\lim_{x\to 0}\left(\frac{x}{\sqrt{x+1}-1}\right)=\lim_{x\to 0}\left(\frac{x}{\sqrt{x+1}-1}\left(\frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}\right)\right)=\lim_{x\to 0}\left(\sqrt{x+1}+1\right)=2$

Note for this problem l'hopital's rule is another method of solving. **C**

12) Choice A is only true if this graph is odd with respect to the origin, and the problem does not specify if it is.
<u>Choice B</u> is always true because $f'(x)<0$ and $f''(x)<0$ for all *x*.
Choice C is not true. The inverse is a function because the original graph passes the horizontal line test.
Choice D is not applicable to this situation because there is no value of *x* where the second derivative is 0. **B**

13) $\int_{-\infty}^{0}e^{2x}dx+\int_{0}^{\infty}e^{-x}dx=\left[\frac{1}{2}e^{2x}\right]_{-\infty}^{0}-\left[e^{-x}\right]_0^{\infty}=\frac{3}{2}$ **B**

14) Using differentials, Newton's method is used to approximate zeros for various functions:

$$0=f(x_n)+f'(x_n)(x_{n+1}-x_n) \;\rightarrow\; x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)} \;\rightarrow\; x_{n+1}=x_n-\frac{(x_n)^2-x_n-5}{2(x_n)-1}$$

Now, using the initial guess:

$$x_1=4-\frac{7}{7}=3 \;\therefore\; x_2=3-\frac{1}{5}=\frac{14}{5} \;\therefore\; x_3=\frac{14}{5}-\frac{\frac{196-70-125}{25}}{\frac{28-5}{5}}=\frac{321}{115}$$ **B**

15) $Area=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\left(\cos^2(x)-\left(-\sin^2(x)\right)\right)dx=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}(1)\,dx=\frac{\pi}{12}$ **D**

16) $\int_0^{\frac{\pi}{4}}\sin^2(x)\cos^2(x)\tan(x)\,dx=\int_0^{\frac{\pi}{4}}\sin^3(x)\cos(x)\,dx$. This integral can be computed using *u*-substitution with

$u=\sin(x)$: $\int_0^{\frac{\sqrt{2}}{2}}u^3du=\left[\frac{u^4}{4}\right]_0^{\frac{\sqrt{2}}{2}}=\frac{1}{16}$ **C**

17) First, work with the cube and find the rate of change of the main diagonal D. Let the length of the side of the cube be s.

$$V = s^3 \rightarrow \frac{dV}{dt} = 3s^2\frac{ds}{dt} \rightarrow 15 = 3(5)^2\frac{ds}{dt} \quad \therefore \quad \frac{ds}{dt} = \frac{1}{5}$$

Using the Pythagorean theorem, the main diagonal of a cube is $D = \sqrt{3}s$ so $\frac{dD}{dt} = \sqrt{3}\frac{ds}{dt} = \frac{\sqrt{3}}{5}$

However, D is the diameter of the circumscribed sphere, so the rate of change of the radius is $\frac{\sqrt{3}}{10}$. **A**

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18) To find the interval of convergence, use the ratio test:

$$\lim_{n\to\infty}\left|\frac{(n+1)(1-x)^{\frac{n}{3}}(1-x)^{\frac{1}{3}}}{3^n}\left(\frac{3^n}{3n(1-x)^{\frac{n}{3}}}\right)\right| = \lim_{n\to\infty}\left|\frac{(n+1)(1-x)^{\frac{1}{3}}}{3n}\right| = \frac{\left|(1-x)^{\frac{1}{3}}\right|}{3} < 1$$

Now create a preliminary interval to eventually isolate x:

$$-1 < \frac{(1-x)^{\frac{1}{3}}}{3} < 1 \rightarrow -3 < (1-x)^{\frac{1}{3}} < 3 \rightarrow -27 < 1-x < 27 \rightarrow -26 < x < 28$$

After testing the endpoints for divergence (or convergence) by plugging them into the series, the interval remains open ended. **A**

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19) In polar coordinates, it is difficult to represent straight lines with equations. To mediate this, tangent lines are written using cartesian coordinates. This requires conversions.

$$y = r\sin(\theta) \qquad x = r\cos(\theta)$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{dr}{d\theta}\sin(\theta) + r\cos(\theta)}{\frac{dr}{d\theta}\cos(\theta) - r\sin(\theta)}$$

$r(\theta) = 3\cos(2\theta) \rightarrow r'(\theta) = -6\sin(2\theta)$ at $\theta = \frac{\pi}{6}$. By plugging in all the respective values:

$$\frac{dy}{dx} = \frac{-3\sqrt{3}\left(\frac{1}{2}\right) + \frac{3}{2}\left(\frac{\sqrt{3}}{2}\right)}{-3\sqrt{3}\left(\frac{\sqrt{3}}{2}\right) - \frac{3}{2}\left(\frac{1}{2}\right)} = \frac{\sqrt{3}}{7}$$

D

---

20) Use the MVT for derivatives in such a way that $f'(x) = L'(x) = \frac{f(b) - f(a)}{b-a}$

$$\frac{3}{2}x^2 = \frac{259-7}{8-2} = 42 \quad \therefore \quad x = 2\sqrt{7}$$

The point of tangency is therefore $\left(2\sqrt{7},\ 28\sqrt{7}+3\right)$. Use slope intercept form with information acquired to find the equation of $L(x)$ and compute its x-intercept.

$$y - 28\sqrt{7} - 3 = 42\left(x - 2\sqrt{7}\right) \rightarrow y = 42x - 84\sqrt{7} + 28\sqrt{7} + 3 \rightarrow y = 42x - 56\sqrt{7} + 3$$

x-intercept: $\left(\frac{56\sqrt{7}-3}{42},\ 0\right)$ **A**

21) Use trig identities. Once it is simplified, use u-substitution.

$$\int\sqrt{36x^2\sec^5\left(2x^2\right)-36x^2\sec^3\left(2x^2\right)}\,dx$$
$$=\int 6x\sqrt{\sec^3\left(2x^2\right)\left(\sec^2\left(2x^2\right)-1\right)}\,dx$$
$$=\int 6x\sec\left(2x^2\right)\sqrt{\sec\left(2x^2\right)\tan^2\left(2x^2\right)}\,dx$$
$$=\int 6x\sec\left(2x^2\right)\tan\left(2x^2\right)\sqrt{\sec\left(2x^2\right)}\,dx$$
$$=\int 6x\sec\left(2x^2\right)\tan\left(2x^2\right)\sqrt{\sec\left(2x^2\right)}\,dx \rightarrow \left\{\begin{array}{c} u=\sec\left(2x^2\right) \\ dx=\dfrac{du}{4x\sec\left(2x^2\right)\tan\left(2x^2\right)}\end{array}\right\} \rightarrow \frac{3}{2}\int\sqrt{u}\,du$$

Integrate normally and undo the substitution. *Never* forget the constant of integration on indefinite integrals.

$u^{\frac{3}{2}}+C \rightarrow \sec^{\frac{3}{2}}\left(2x^2\right)+C$ **C**

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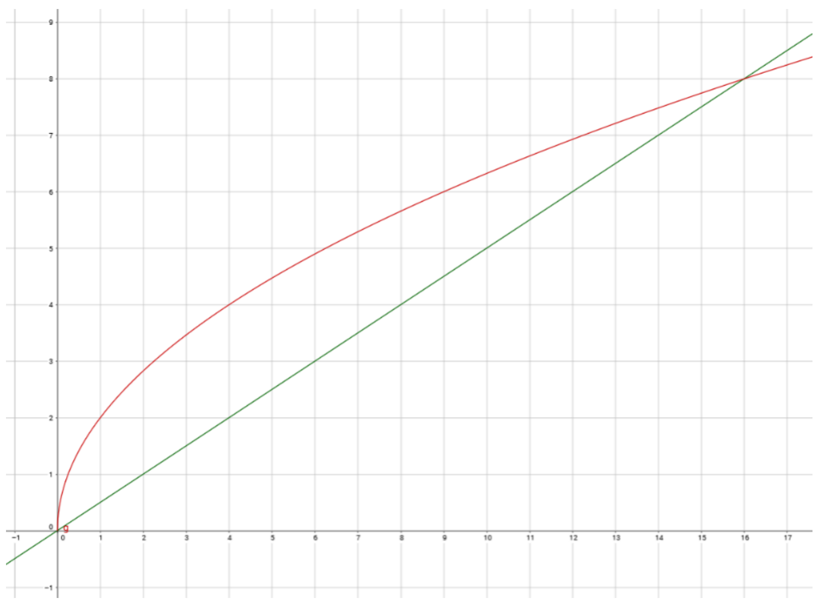
22) When changing the order of integration, the goal is to rewrite the bounds so that $f(x,y)$ is easier to integrate. Do not change the integral itself, as that would lead to a different answer, so B is out.

As written, the bounds are

$\frac{x}{2}\le y\le\sqrt{4x}$ and $0\le x\le 16$

The area of integration is shown. Instead of representing the graphs in terms of x, put them in terms of y to get the horizontal bounds, and integrate vertically from 0 to 8.



Rewrite the bounds as

$0\le y\le 8$ and $\frac{y^2}{4}\le x\le 2y$.

Finally, to keep everything consistent, integrate with respect to x first, and y next.

Final answer: $\displaystyle\int_0^{16}\int_{\frac{x}{2}}^{\sqrt{4x}}\left(\frac{1+x^2}{y^2}\right)dy\,dx=\int_0^{8}\int_{\frac{y^2}{4}}^{2y}\left(\frac{1+x^2}{y^2}\right)dx\,dy$ **D**

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23) Euler's method uses a differential equation to approximate y values if the antiderivative of a function is tough to compute, but can be applied to easy functions as well.

n	$x_{n+1}=x_n+h$	$y_{n+1}=y_n+h\left(y'(x_n)\right)$	(x_n,y_n)
0	Given = 1	Given = 0	(1, 0)
1	$x_1=1+\frac{1}{2}$	$y_1=0+\frac{1}{2}(0+2-0)$	$\left(\frac{3}{2},1\right)$
2	$x_2=\frac{3}{2}+\frac{1}{2}$	$y_2=1+\frac{1}{2}\left(\frac{3}{2}(1)+2\left(\frac{3}{2}\right)-2(1)\right)$	$\left(2,\frac{9}{4}\right)$

B

24) Let x be the amount of rope that is pulled up. At the bottom of the hole, $x=0$, and at the top of the hole $x=80$. As the rope is pulled, there is $80-x$ feet of rope remaining. Now, using *Hooke's Law*:

$$W=\int_0^{80} F(x)\,dx$$

Force in this case is the weight of the system (rope and briefcase) as the rope is pulled. Since the rope has weight, the weight of the system changes, and this gives us a integrable function with respect to x.

$$F(x)=\frac{3}{2}(80-x)+50$$

$$W=\int_0^{80}\left(\frac{3}{2}(80-x)+50\right)dx=\int_0^{80}\left(170-\frac{3}{2}x\right)dx=8,800 \text{ foot-pounds}$$ **D**

25) Choice A is always true. If the product of 1 + a_n is written as a sum of logs, use the limit comparison test to prove that it converges. This works because a_n approaches 0 as n approaches infinity.

$\prod_{n=1}^{\infty}(1+a_n)=\sum_{n=1}^{\infty}\ln(1+a_n)$ Make the comparison: $\lim_{x\to 0}\frac{\ln(1+x)}{x}=1 \;\therefore\; \lim_{\substack{n\to\infty \\ a_n\to 0}}\frac{\ln(1+a_n)}{a_n}=1$

Choice B is only true if $0<a_n<b_n$, which is not stated.

Choice C is not true. If the ratio test yields a value of 1, the outcome is either divergent, convergent, or conditionally convergent, but we do not know which unless another test is used.

Choice D is false because the harmonic series is divergent but the alternating harmonic series is convergent. **A**

26) $$\frac{\partial x}{\partial y}=\frac{\frac{\partial f}{\partial y}}{\frac{\partial f}{\partial x}}=\frac{3\left(2x+4xy^3\right)^2\left(12xy^2\right)}{3\left(2x+4xy^3\right)^2\left(2+4y^3\right)}=\frac{12xy^2}{2+4y^3}$$ **B**

27) This problem isn't tough per-se, but it is tricky to the eye. Make sure you correctly represent the factorials and the exponent on the e term.

$f^{(2018)}(-2018)$=-[$(2019!)2018-2018^{2018}e^{(2018)^2}$]

E

28) There are a couple different ways to prove that the area of polygons with a given circumradius (c) approaches the area of a circle with radius r. The method of exhaustion states that $\lim_{n\to\infty}\left(\frac{1}{2}nc\sin\left(\frac{2\pi}{n}\right)\right)=\pi c^2$. Taking into account that $c=6$: $\lim_{n\to\infty}\left(\frac{1}{2}n(n-1)\sin\left(\frac{2\pi}{n}\right)\right)=\pi(6)^2=36\pi.$ **A**

29) $y=\sinh^{-1}(x)$

$$x=\sinh(y)=\frac{e^y-e^{-y}}{2}\left(\frac{e^y}{e^y}\right)=\frac{e^{2y}-1}{2e^y} \quad\to\quad e^{2y}-2xe^y-1=0$$

Use the quadratic formula to solve for e^y:

$e^y=x+\sqrt{x^2+1} \quad\to\quad y=\ln\left(x+\sqrt{x^2+1}\right)=\sinh^{-1}(x)$. Now find the derivative of the natural log:

$\frac{dy}{dx}=\frac{1}{\sqrt{x^2+1}}$ *Continued on the next page*

Using this information, use integration by parts to compute the integral:

$\int x^2 \sinh^{-1}(x)\,dx = \left(\frac{x^3 \sinh^{-1}(x)}{3}\right) - \int \frac{x^3}{3}\left(\frac{1}{\sqrt{x^2+1}}\right)dx$. Then via *u*-substitution:

$$\int x^2 \sinh^{-1}(x)\,dx = \left(\frac{x^3 \sinh^{-1}(x)}{3}\right) - \int \frac{x^3}{3}\left(\frac{1}{\sqrt{x^2+1}}\right)dx \;\rightarrow\; \left\{\begin{matrix} u = x^2+1 \\ x^2 = u-1 \\ dx = \frac{du}{2x} \end{matrix}\right\} \rightarrow \left(\frac{x^3 \sinh^{-1}(x)}{3}\right) - \int \frac{x}{3}\left(\frac{(u-1)}{\sqrt{u}}\right)\frac{du}{2x}$$

By simplifying the integral:

$$\int x^2 \sinh^{-1}(x)\,dx = \left(\frac{x^3 \sinh^{-1}(x)}{3}\right) - \frac{1}{6}\int\left(\sqrt{u} - \frac{1}{\sqrt{u}}\right)du$$

$$= \left(\frac{x^3 \sinh^{-1}(x)}{3}\right) - \frac{1}{6}\left[\frac{2}{3}u^{\frac{3}{2}} - 2\sqrt{u}\right]$$

$$= \left(\frac{x^3 \sinh^{-1}(x)}{3}\right) - \frac{1}{6}\left(\frac{2u^{\frac{3}{2}} - 6\sqrt{u}}{3}\right)$$

$$= \left(\frac{x^3 \sinh^{-1}(x)}{3}\right) - \frac{1}{6}\left(\frac{\left(2\sqrt{u}(u-3)\right)}{3}\right)$$

Then, by resubstituting $u = x^2+1$:

$$\int x^2 \sinh^{-1}(x)\,dx = \left(\frac{x^3 \sinh^{-1}(x)}{3}\right) - \frac{\sqrt{x^2+1}\left(x^2+1-3\right)}{9}$$

Finally:

$$\boxed{\int x^2 \sinh^{-1}(x)\,dx = \frac{3x^3 \sinh^{-1}(x) - \sqrt{x^2+1}\left(x^2-2\right)}{9} + C}$$ **C**

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30) The imaginary number *i* acts as a constant, and the derivative is taken with respect to *x*:

$$f(x) = \ln\left(\frac{i(x!)}{(x+1)!}\right) = \ln\left(\frac{i}{x+1}\right) \;\rightarrow\; f'(x) = \frac{x+1}{i}\left(-\frac{i}{(x+1)^2}\right) = -\frac{1}{x+1} \;\rightarrow\; f'(3) = -\frac{1}{4}$$ **B**

Answer Key:

1) B	11) C	21) C
2) A	12) B	22) D
3) C	13) B	23) B
4) D	14) B	24) D
5) B	15) D	25) A
6) D	16) C	26) B
7) B	17) A	27) A
8) D	18) A	28) A
9) A	19) D	29) C
10) A	20) A	30) B