Answers and Solutions

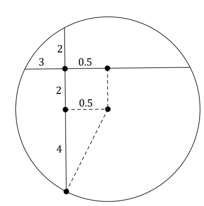
1. B
2. D
3. C
4. A
5. B
6. E $32i$
7. C
8. E 5
9. C
10. D
11. E 228
12. A
13. A
14. C
15. A
16. A
17. D
18. C
19. B
20. A
21. A
22. D
23. C
24. B
25. B
26. D
27. B
28. D
29. B
30. B

1. **B**. We need two vowels from the four available and two consonants from the seven that are available (b is already being used). Each letter can go in five places.
$$\binom{4}{2}\binom{7}{2}5!=(6)(21)(12)=15120$$
2. **D**. Let (x, y) be the point in question. We will write two equations equating the distances and then solve the system of equations.
$$\sqrt{(x+4)^2+(y-3)^2}=\sqrt{(x-5)^2+(y-6)^2}\rightarrow 3x+y=6$$
$$\sqrt{(x+4)^2+(y-3)^2}=\sqrt{(x-4)^2+(y+1)^2}\rightarrow 2x-y=-1$$
The intersection (a, b) is $(1, 3)$.
$(3-1)^2+(1-3)^3=4-8=-4.$
3. **C**. Let's first find out what is going on with the reciprocal function. Divide the numerator by the denominator to get $y=\frac{1}{x+1}+4,$ which is a left shift 1 and vertical shift 4. Now we can do the same to the parabola. $y=(x+1)^2-2(x+1)+3+4\rightarrow y=x^2+6.$
4. **A**. $4^{14}-1=2^{28}-1=\left(2^{14}+1\right)\left(2^{14}-1\right).$ We know that $2^{14}+1$ is divisible by 29, so let's use it to determine the remainder when $2^{14}-1$ is divided by 29. $2^{14}-1=\left[\left(2^{14}+1\right)-29\right]+27=29m+27.$ Since the expression in the brackets is a multiple of 29, the remainder must be 27.
5. **B**. The only primes will be the second term in each row. There are 15 primes below 50: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, and 47.
6. **E**. This can be done quickly by knowing that $(1+i)^2=2i$ and $(1-i)^2=-2i$. Rationalize first:
$$\left(\frac{1+i}{1-i}\square\frac{1+i}{1+i}-\frac{1-i}{1+i}\square\frac{1-i}{1-i}\right)^5\rightarrow\left(\frac{2i}{2}-\frac{(-2i)}{2}\right)^5\rightarrow(2i)^5=-32i.$$
7. **C**. Since the parallelogram has perpendicular diagonals, it is a rhombus, and the diagonals are bisected. This creates four congruent right triangles. Each right triangle has legs 5 and 10 and hypotenuse $5\sqrt{5}$. The distance r from the right angle to the hypotenuse is the radius of the inscribed circle. We can find r by equating two methods of finding the area of one of the triangles: $\text{VArea}=\frac{1}{2}(5)(10)=25=\frac{1}{2}\left(5\sqrt{5}\right)r\rightarrow r=2\sqrt{5}.$ The circle area will be $\left(2\sqrt{5}\right)^2\pi=20\pi.$
8. **E**. $(1.4x)(0.75y)=1.05xy\rightarrow 5\%.$

9. **C**. If $x=0$, only b and d remain. $f(g(0))=f(-2)=-7$.
10. **D**. Using a Vieta identity, the sum of the roots is −6, so the third root must be −9, and the product of the roots is $-c$. $(1)(2)(-9)=-c \rightarrow c=18$.
11. **E**. The y-intercept is (0, 212) and the value of c comes from the horizontal asymptote. Substituting some of the given information gives us $212=b\left(a^0\right)+32$, resulting in $b=180$. Using more of the given information, we get $112=180a^{-2}+32 \rightarrow \frac{80}{180}=a^{-2} \rightarrow a^2=\frac{9}{4} \rightarrow a=\frac{3}{2}$ only, as an exponential function cannot have a negative base. $ac+b=\left(\frac{3}{2}\right)(32)+180=228$.
12. **A**. For the sequence to be arithmetic, the differences between consecutive terms must be equal: $-4x-3x+2=-5x^2+4x \rightarrow 5x^2-11x+2=0 \rightarrow (5x-1)(x-2)=0$. The second terms of the resulting arithmetic sequences are $-\frac{4}{5}$ and −8. For the sequence to be geometric, the ratios of the consecutive terms must be equal: $\frac{-5x^2}{-4x}\left(=\frac{5x}{4}\right)=\frac{-4x}{3x-2} \rightarrow 15x^2-10x=-16x \rightarrow$ $3x(5x+2)=0 \rightarrow x=0, -\frac{2}{5}$. The second term of the geometric sequence is $\frac{8}{5}$. The sum of our three values is $-\frac{4}{5}+(-8)+\frac{8}{5}=-\frac{36}{5}$.

13. **A**. We need to find the values of k that make the determinant equal to 0. A consistent system has one or more solutions, so the only thing we don't want is a system with no solution.
$\begin{vmatrix} k & 2 & k \\ 3 & 14k & -5k \\ 2k & 5 & k \end{vmatrix}=(k)\left(14k^2+25k\right)-(2)\left(3k+10k^2\right)+(k)\left(15-28k^2\right)$. This simplifies to $14k^3-5k^2-9k=0$, which factors into $k(k-1)(14k+9)=0$. The sum of the solutions is $\frac{5}{14}$.

14. **C**. Perpendiculars from the center to the chords creates a right triangle with legs 0.5 and 4. This gives $r^2=0.5^2+4^2=\frac{65}{4}$, so the area is $\frac{65}{4}\pi$.



15. **A**. Let (x, y) be the point that has the shortest distance to the curve.
Distance $=\sqrt{\left(\frac{9}{2}-x\right)^2+\left(0-\sqrt{x-1}\right)^2}=\sqrt{\frac{81}{4}-9x+x^2=x-1}=\sqrt{x^2-8x+\frac{77}{4}}$.
Ignoring the square root, we have the equation for a parabola that has a minimum value. If we find the minimum value of the quadratic, then we can take the square root and have the minimum value of the square root. $x=-\frac{(-8)}{2(1)}=4,\ y=\sqrt{3} \rightarrow$
Distance $=\sqrt{16-32+\frac{77}{4}}=\sqrt{\frac{13}{4}}=\frac{\sqrt{13}}{2}$.

16. **A**. Let r represent the radius of the balls and the can. $V_{balls}=8\left(\frac{4}{3}\pi r^3\right)=\frac{32}{3}\pi r^3$ and $V_{cylinder}=\pi r^2 h=\pi r^2(16r)=16\pi r^3$. $\frac{\frac{32}{3}\pi r^3}{16\pi r^3}=\frac{2}{3}$, which is the value for any number of balls packed in this manner.

17. **D**. Take the equation and solve for *x*. $3x^2-2xy+(-y+4)=0\rightarrow x=\frac{2y\pm\sqrt{4y^2-12(-y+4)}}{6}\rightarrow$ $x=\frac{y\pm\sqrt{y^2+3y-12}}{3}$. The range will be the values for which the expression is defined, so now solve $y^2+3y-12\geq 0$. Due to the nature of the inequality, we know that the solution fits the description in the problem, so just solve with the quadratic formula to find the values of *a*, *b*, and *c*. $y=\frac{-3\pm\sqrt{9-(4)(1)(-12)}}{2}=\frac{-3\pm\sqrt{57}}{2}\rightarrow\frac{a+b}{c}=\frac{-3+57}{2}=27$.

18. **C**. By subtracting row 1 from rows 2 through 5, we get $\begin{vmatrix}1&1&1&1&1\\0&4&0&0&0\\0&0&2&0&0\\0&0&0&7&0\\0&0&0&0&9\end{vmatrix}$. You can now either (a) subtract column 1 from columns 2 through 5 to get a diagonal matrix whose determinant is (4)(2)(7)(9) or (b) notice that evaluating the determinant by expansion of minors using the first column, that you need only the product from option (a). The product is 504.

19. **B**. Since we want an element in the inverse, we go to row 3, column 2 of the original matrix and cover up that row and column. The determinant of that submatrix, the minor, is (3)(3) – (1)(4) = 5 and is in a "negative" position since the row number and column number have an odd sum. The determinant of the original matrix is –80. We now have $(-5)\left(-\frac{1}{80}\right)=\frac{1}{16}$.

20. **A**. The terms in a harmonic sequence are the reciprocals of the terms of an arithmetic sequence, so the term between 8 and 17 would be 12.5 and the common difference is 4.5. Subtracting, we get the first term of –1.

21. **A**. Notice that the sum of the expressions in the first two sets of parentheses is the expression on the right hand side of the equation. Let's simplify this equation to $a^3+b^3=(a+b)^3\rightarrow$ $a^3+b^3=a^3+b^3+3ab(a+b)\rightarrow ab(a+b)=0$. Now we have three equations to solve:

$5^x-7=0$	$25^x+1=0$	$25^x+5^x-6=0$
$5^x=7$	$25^x=-1$	$5^{2x}+5^x-6=0$
$x=\log_5 7$	$\varnothing$	$(5^x+3)(5^x-2)=0$
		$\varnothing$ $\quad x=\log_5 2=\log_5\frac{10}{5}=\log_5 10-1$

22. **D**. We need to match up the appropriate factors from each expression:

$$51x^{50}+50x^{49}+49x^{48}+\ldots+26x^{25}$$
$$1\quad +x\quad +x^2\quad +\ldots+x^{25}$$

We can see that the coefficients are consecutive integers from 26 to 51 inclusive, so the resulting coefficient will be $\frac{26(26+51)}{2}=13(77)=1001$.

23. **C**. The basic form of the entire ellipse, centered at the origin, will be $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$. Substituting values we know, we get $\frac{15^2}{20^2}+\frac{30^2}{a^2}=1 \to \frac{900}{a^2}=1-\frac{9}{16}=\frac{7}{16} \to 7b^2=14400 \to b^2=\frac{14400}{7}$.
The volume of the tunnel is the area of the semiellipse multiplied by the depth of the tunnel.
$\frac{1}{2}ab\pi gD=\frac{1}{2}(20)\left(\frac{120}{\sqrt{7}}\right)\left(40\sqrt{7}\right)\pi=48000\pi.$

24. **B**. If each child must get one banana, then there are 3 remaining to be distributed. This is now a "stars and bars" problem.
Bananas: $\binom{3+4-1}{3}=20$ Oranges: $\binom{6+4-1}{6}=84$ $(20)(84)=1680$

25. **B**. The graph is the right side of the hyperbola $4x^2-9y^2=1$. By definition of a hyperbola, the value in question is the value of $2a$, the distance from the center to the vertex. This is a horizontal hyperbola and the equation for the entire hyperbola is $\frac{x^2}{\frac{1}{4}}-\frac{y^2}{\frac{1}{9}}=1$. The value of a^2 is $\frac{1}{4}$, so $a=\frac{1}{2}$. $2a=1$.

26. **D**. $\left(r+r^{-1}\right)^3=r^3+3r^2r^{-1}+3rr^{-2}+r^{-3}=r^3+r^{-3}+3\left(r+r^{-1}\right)$. Substituting, we get $\left(\sqrt{5}\right)^3=r^3+r^{-3}+3\sqrt{5} \to 5\sqrt{5}-3\sqrt{5}=2\sqrt{5}=r^3+r^{-3}$.

27. **B**. This is a "quadratic type" equation that we will need to rewrite first as $2x^2-5x-3+3\sqrt{2x^2-5x-3}-4=0$. Let $a=\sqrt{2x^2-5x-3} \to a^2+3a-4=0 \to (a+4)(a-1)=0$.
We can ignore the $a=-4$ root since the positive square root can't be negative. We now must solve $\sqrt{2x^2-5x-3}=1$. $2x^2-5x-3=1 \to 2x^2-5x-4=0 \to x=\frac{5\pm\sqrt{25-4(2)(-4)}}{4}=\frac{5\pm\sqrt{57}}{4}$.
So, $a+b+c=5+57+4=66$.

28. **D**. A boy must be first. $\left(\frac{4}{7}\right)\left(\frac{3}{6}\right)\left(\frac{3}{5}\right)\left(\frac{2}{4}\right)\left(\frac{2}{3}\right)\left(\frac{1}{2}\right)\left(\frac{1}{1}\right)=\frac{1}{35}$.

29. **B**. Let the smallest angle be $n°$. To find the maximum value of n, the difference between the angle measures need to be as small as possible. The smallest possible difference is 1°.
$n+(n+1)+(n+2)+...+(n+8)=9n+36=360 \to 9n=324 \to n=36$.

30. **B**. $\begin{cases} f(1)=a+b \\ f(a+b)=a+b(a+b)=a+ab+b^2 \\ f\left(a+ab+b^2\right)=a+b\left(a+ab+b^2\right)=a+ab+ab^2+b^3=29 \end{cases}$ $\begin{cases} f(0)=a \\ f(a)=a+ab \\ f(a+ab)=a+ab+ab^2=2 \end{cases}$

$2+b^3=29 \to b^3=27 \to b=3$ $a+3a+9a=2 \to 13a=2 \to a=\frac{2}{13}$ $a+b=\frac{41}{13}$