

For all questions, answer choice “E) NOTA” means that none of the above answers is correct.

1. Evaluate the following definite integral.

$$\int_{-1}^3 |2x - 1| dx$$

- A) 4 B) 4.5 C) 6 D) 8.5 E) NOTA

2. Evaluate the following definite integral.

$$\int_0^{\pi/4} \frac{\sin(x) - \cos(x)}{\cos(x)} dx$$

- A) $\frac{1}{4}(\ln 4 - \pi - 4)$ B) $\frac{1}{4}(\ln 4 - \pi)$ C) $\frac{1}{4}(\ln 4 - \pi + 4)$ D) diverges E) NOTA

3. Evaluate the following definite integral.

$$\int_{-5}^{-1} \frac{dx}{x^2 + 6x + 13}$$

- A) 0 B) $\frac{\pi}{12}$ C) $\frac{\pi}{6}$ D) $\frac{\pi}{2}$ E) NOTA

4. Evaluate the following definite integral.

$$\int_1^{\pi/2} (\ln(x) + 4\sin^2(x)) dx$$

- A) $\frac{\pi}{2} \left(\ln \frac{\pi}{2} + 1 \right) - \sin(2) - 1$
B) $\frac{\pi}{2} \left(\ln \frac{\pi}{2} + 1 \right) + \sin(2) - 1$
C) $\frac{\pi}{2} \left(\ln \frac{\pi}{2} - 1 \right) + \frac{4}{3} \cos^3(1)$
D) $\frac{\pi}{2} \left(\ln \frac{\pi}{2} - 1 \right) + \frac{4}{3} (\sin^3(1) - 1)$
E) NOTA

5. The region between $f(x) = 3x \ln(x)$ and the x -axis over the interval $[1, e]$ is revolved about the y -axis to form a solid. What is the volume of this solid?

- A) $\frac{3\pi}{4}(e^2 + 1)$ B) $\frac{2\pi}{3}(e^3 + e - 1)$ C) $\frac{2\pi}{3}(2e^3 + 1)$ D) $\frac{\pi}{3}(5e^3 - 2)$ E) NOTA

6. A particle moves along the x -axis so that its velocity is given by $v(t) = 12t^3 - 8t + 4t^{-5}$ for $t > 0$. Compute the displacement of the particle from time $t = 3^{-1/2}$ to $t = 1$.

- A) $-\frac{8}{9}$ B) 8 C) $\frac{74}{9}$ D) $\frac{52}{3}$ E) NOTA

7. The length of the polar curve $r = e^\theta$ from $\theta = 0$ to $\theta = 2\pi$ is

- A) less than e^π D) between $e^{2\pi}$ and $e^{5\pi/2}$
 B) between e^π and $e^{3\pi/2}$ E) NOTA
 C) between $e^{3\pi/2}$ and $e^{2\pi}$

8. Suppose that $\frac{d^2y}{dx^2} = 4x - 5$, that $y'(0) = 3$, and that $y(0) = 4$. Compute $y(6)$.

- A) 64 B) 70 C) 78 D) 84 E) NOTA

9. Suppose $f(x)$ is a twice-differentiable function for all real numbers. Values of $f(x)$ and $f'(x)$ are shown in the table below.

x	1	2	3	4
$f(x)$	1	2	4	-2
$f'(x)$	0	-1	3	-2

Evaluate the following definite integral.

$$\int_2^4 \frac{f'(x) dx}{4 + (f(x))^2}$$

- A) $-\frac{\pi}{8}$ B) $-\frac{\pi}{4}$ C) $\frac{\pi}{4}$ D) $\frac{\pi}{8}$ E) NOTA

10. Suppose $f(x)$ is a twice-differentiable function for all real numbers. Values of $f(x)$ and $f'(x)$ are shown in the table below.

x	1	2	3	4
$f(x)$	1	2	4	-2
$f'(x)$	0	-1	3	-2

Evaluate the following definite integral.

$$\int_2^4 \frac{f(x)f'(x) dx}{4 + (f(x))^2}$$

- A) 0 B) $\frac{\pi}{4}$ C) $\frac{3}{2} \ln(2)$ D) $3 \ln(2)$ E) NOTA

11. Compute the area of the region in the plane bounded by the graphs of $y = x^3 + x^2 + x + 1$ and $y = x^4 + x^3 - x^2 + x + 2$.

- A) $\frac{13}{15}$ B) $\frac{26}{15}$ C) $\frac{12}{5}$ D) $\frac{46}{15}$ E) NOTA

12. Suppose $f(x)$ is a thrice-differentiable function for all real numbers. Values of $f(x)$ and $f'(x)$ are shown in the table below.

x	1	2	3	4
$f(x)$	1	2	3	-2
$f'(x)$	0	-1	-2	3
$f''(x)$	-1	0	1	-1

Evaluate the following definite integral.

$$\int_1^3 xf''(x) dx$$

- A) -8 B) $\frac{3}{2}$ C) 27 D) undefined E) NOTA

13. Compute the following, if it exists.

$$\lim_{k \rightarrow \infty} \sum_{n=1}^k \frac{n^{2017}}{k^{2018}}$$

- A) $\frac{1}{2018}$ B) $\frac{1}{2017}$ C) 1 D) ∞ E) NOTA

14. Determine the x -coordinate of the maximum of the function

$$f(x) = \int_{-50}^{24x^2 - 8x^3 - 6x^4} e^{-t^2} dt.$$

- A) -2 B) -1 C) 0 D) 1 E) NOTA

15. Suppose $f(x)$ is a differentiable function for all real numbers that satisfies

$$\int_2^5 f(x) dx = 7 \quad \text{and} \quad \int_2^9 f(x) dx = -2.$$

Evaluate the following definite integral.

$$\int_5^9 (3x^2 + 4f(x)) dx$$

- A) 568 B) 595 C) 624 D) 640 E) NOTA

16. Find the value of a such that the line $y = a$ divides the region bounded by the graphs of $y = 7$ and $y = 5x^2$ into two regions of equal area.

- A) $\sqrt[3]{3}$ B) $\frac{7}{2}\sqrt[3]{2}$ C) $7\sqrt[3]{4}$ D) $\sqrt[3]{7}$ E) NOTA

17. Suppose $f(x)$ and $g(x)$ are twice-differentiable functions for all real numbers such that $f(x) \neq 0$. Values of $f(x)$, $g(x)$, $f'(x)$, and $g'(x)$ are shown in the table below.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
0	16	1	-3	2
1	4	3	1	-2
2	9	0	2	1

Evaluate the following definite integral.

$$\int_0^2 \frac{(1 + g'(x))(f'(x + g(x)))}{\sqrt{f(x + g(x))}} dx.$$

- A) -2 B) -1 C) 1 D) 2 E) NOTA

18. Evaluate the following definite integral.

$$\int_0^2 x\sqrt{2x - x^2} dx$$

- A) $\frac{\pi}{2}$ B) π C) 1 D) 2 E) NOTA

19. Suppose $f(x) = x^5 - x^3 + 4x$. Evaluate the following definite integral.

$$\int_0^4 f^{-1}(x) dx$$

- A) $\frac{3}{1952}$ B) $\frac{23}{24}$ C) $\frac{24}{23}$ D) $\frac{25}{12}$ E) NOTA

20. Using the first two nonzero terms of the Maclaurin series for $f(x) = \ln(1 + \sin(x))$, approximate the value of the following definite integral.

$$\int_0^{1/4} \ln(1 + \sin(x)) dx$$

- A) $\frac{11}{384}$ B) $\frac{13}{384}$ C) $\frac{5}{192}$ D) $\frac{7}{192}$ E) NOTA

21. The exact value of the definite integral

$$\int_0^1 \frac{dx}{1 + x^{50}}$$

is

- A) less than 0.91 D) between 0.97 and 1
 B) between 0.91 and 0.94 E) NOTA
 C) between 0.94 and 0.97

22. Which of the following expressions is an antiderivative of the function below?

$$g(x) = \frac{12x^5 - 8x^3 + 2}{x^3}$$

- A) $\frac{8(x^5 - x + 1)}{x^3}$ B) $\frac{6(4x^5 - 4x^3 + 4x^2 - 1)}{x^4}$ C) $4x^3 - 8x - 1$ D) $\frac{4x^5 - 8x^3 + 5x^2 - 1}{x^2}$ E) NOTA

23. Evaluate the following definite integral.

$$\int_0^1 \frac{x^4 + 4x^3 + 6x^2 + 4x}{(x + 1)^4} dx$$

- A) $\frac{17}{24}$ B) $\frac{49}{64}$ C) $\frac{129}{160}$ D) $\frac{7}{8}$ E) NOTA

24. Suppose the function $f(x)$ is defined as follows.

$$f(x) = 16x^3 - 15x^2 + 2x \int_1^2 f(t) dt - 21$$

Compute $f(2)$.

- A) -33 B) $-\frac{47}{3}$ C) 39 D) 55 E) NOTA

25. Compute the following, if it exists.

$$\lim_{n \rightarrow \infty} \frac{1 + \sqrt[n]{e} + \sqrt[n]{e^2} + \sqrt[n]{e^3} + \dots + \sqrt[n]{e^{n-1}}}{n}$$

- A) e^{-1} B) $e - 1$ C) e D) $e + 1$ E) NOTA

26. Evaluate the following definite integral.

$$\int_3^7 \sqrt{10x - 21 - x^2} dx$$

- A) 0 B) 4 C) $\frac{32}{3}$ D) $\sin^{-1} 4$ E) NOTA

27. Evaluate the following definite integral.

$$\int_0^4 \sqrt{\frac{4-x}{4+x}} dx$$

- A) $2(\pi - 2)$ B) $2(\pi - 1)$ C) $4(\pi - 2)$ D) $4(\pi - 1)$ E) NOTA

28. Evaluate the following definite integral.

$$\int_2^4 \frac{\sqrt{x^2 - 4}}{x} dx$$

- A) $\sqrt{3} - \frac{\pi}{3}$ B) $2\sqrt{3} - \frac{2\pi}{3}$ C) $4\sqrt{3} - \frac{4\pi}{3}$ D) $4\sqrt{3} - \frac{2\pi}{3}$ E) NOTA

29. Evaluate the following definite integral.

$$\int_{-2}^{-1} \frac{-3x^2 + 6x - 2}{3x^2 - 2x} dx$$

- A) $\ln\left(\frac{5}{16}\right) - 1$ B) $\ln\left(\frac{\sqrt[3]{5}}{4}\right) - 1$ C) $\ln\left(\frac{5}{16}\right) + 1$ D) $\ln\left(\frac{\sqrt[3]{5}}{4}\right) + 1$ E) NOTA

30. Evaluate the following definite integral.

$$\int_{\pi/6}^{\pi/3} \frac{dx}{\sin(x)(1 + \sin(x))}$$

- A) $\ln(\sqrt{3} + 2) + \frac{1}{3}\sqrt{3} - 2$ D) $\ln(2\sqrt{3} + 3) + \frac{4}{3}\sqrt{3} - 2$
B) $\ln\left(\frac{2}{3}\sqrt{3} + 1\right) - \frac{4}{3}\sqrt{3} + 2$ E) NOTA
C) $\ln\left(\frac{4}{3}\sqrt{3} + 1\right) - \frac{2}{3}\sqrt{3} + 2$