1. D
\[ \log_{2018} \sin x + \log_{2018} \cos x + \log_{2018} \tan x = \log_{2018} (\sin x \cdot \cos x \cdot \tan x) \]
Plug in \( x = \pi \) and get \( \log_{2018} (0 \ast -1 \ast 0) \) which is undefined.

2. B
\[ ((2^3 + 7^0)^{\frac{3}{2}} + (27^1 - 36^2)^{\frac{1}{2}}) = ((8 + 1)^{\frac{3}{2}} + (3 - 6)^2)^{\frac{1}{2}} = (27 + 9)^{\frac{1}{2}} = 6 \]

3. A
Squaring both sides, we have \( (\log_a b)^2 + (\log_b a)^2 + 2 = 16 \). So, \( (\log_a b)^2 + (\log_b a)^2 = 14 \)

4. B
\[ \frac{1}{\log_2 7} \div \log_\frac{1}{3} 9 + \log_8 7 = \log_2 7 \div 2 + \frac{1}{3} \log_2 7 = \frac{5}{6} \log_2 7 \]

5. E
Take the base \( x \) logarithm of both sides of \( x^{\log_{16} x} = 8 \) to get \( \log_{16} x = \log_x 8 \). We can further simplify to \( \frac{1}{4} \log_2 x = 3 \log_2 x = \frac{3}{\log_2 x} \), so \( (\log_2 x)^2 = \frac{4}{3} \log_2 x = \frac{2}{\sqrt{3}} \) or \( -\frac{2}{\sqrt{3}} \).

The product of the solutions would thus be \( 2\sqrt{3} \ast 2^{-\sqrt{3}} = 1 \)

6. C
\( (\log_x 25)(\log_4 49)(\log_{27} x)(\log_{125} 64)(\log_x 81) = 2 \)
\( (\log_x 2)(\log_4 64)(\log_{27} 81)(\log_{125} 25)(\log_x 49) = 1 \ast 3 \ast \frac{4}{3} \ast \frac{2}{3} \ast \log_x 49 = 2 \)
\( \log_x 49 = 2 \log_x 7 = \frac{3}{4} \log_x 7 = \frac{3}{8} \), so \( \log_7 x = \frac{8}{3} \)

7. B
\[ \sqrt{2 + \sqrt{2^2 + \sqrt{2^4 + \sqrt{2^8 + \cdots}}} \text{ simplifies to } \sqrt{2(1 + \sqrt{1 + \sqrt{1 + \cdots}})}. \]
Now, solving for the inside part \( x = 1 + \sqrt{1 + \sqrt{1 + \cdots}} \), we get \( x - 1 = \sqrt{x} \). The quadratic formula gives \( x = \frac{3 + \sqrt{5}}{2} \) but the negative sign gives us an \( x \) value less than 1 where \( x \) is clearly greater than 1 so we have \( x = \frac{3 + \sqrt{5}}{2} \). Our final answer is \( \sqrt{2 \left( \frac{3 + \sqrt{5}}{2} \right)} = \frac{\sqrt{10 + \sqrt{2}}}{2} \)
\[ |10 - 2| = |2 - 10| = 8. \]

8. A
\( \ln(1 + \log_2 (3 + \log_4 (5 + x))) = 0 \)
\( 1 + \log_2 (3 + \log_4 (5 + x)) = 1 \)
\( 3 + \log_4 (5 + x)) = 1 \)
\( \log_4 (5 + x)) = -2 \)
\( x = 4^{-2} - 5 = -\frac{79}{16} \)

9. D
\( \log_3 r + \log_3 s + \log_3 t = \log_3 rst \). The product of the roots is \( -\frac{3}{9} = -\frac{1}{3} \), so \( \log_3 rst = \log_3 \frac{1}{3} = -1 \).

10. B
Note that \( -2 = \frac{5}{\sqrt{-32}} \), so \( x \) will be slightly less than -2.
11. A

\[ S = \frac{1}{1+r_1} + \frac{1}{1+r_2} + \frac{1}{1+r_3} + \ldots + \frac{1}{1+r_{20}} \]

We can use difference of squares to create two more manageable sums:

\[ S = \left( \frac{1}{r_1-l} + \frac{1}{r_2-l} + \frac{1}{r_3-l} + \ldots + \frac{1}{r_{20}-l} \right) - \left( \frac{1}{r_1+l} + \frac{1}{r_2+l} + \frac{1}{r_3+l} + \ldots + \frac{1}{r_{20}+l} \right) \]

The first sum can be calculated with a new polynomial \((x+i)^{20} - 7(x+i)^3 + 1\). Now we can use binomial theorem to get \(\frac{-21+20i}{2+7i} = \frac{98+187i}{53}\).

The second sum can be calculated with a new polynomial \((x-i)^{20} - 7(x-i)^3 + 1\). Now we can use binomial theorem to get \(\frac{-21-20i}{2-7i} = \frac{98-187i}{53}\).

So we have \(S = \frac{98+187i}{53} - \frac{98-187i}{53} = \frac{187}{53} \cdot 187-53=134\).

12. D

\((\log_2a 4^x)(1 + \log_2 a) = (\log_2a 4^x)(\log_2 2a) = (\log_2a 2a)(\log_2 4^x) = 2x\)

13. C

\(3^{x-4} = 4^{x-3}\)

\((x - 4) \ln 3 = (x - 3) \ln 4\)

\(x \ln 3 - 4 \ln 3 = x \ln 4 - 3 \ln 4\)

\(x (\ln 3 - \ln 4) = \ln 81 - \ln 64\)

\(x = \frac{\ln 81 - \ln 64}{\ln 3 - \ln 4} = \frac{\ln 64 - \ln 81}{\ln 4 - \ln 3}\)

14. D

First, we must multiply \(F(x)\) by 8 to get \(2^{x+3} + 56\). Then, we add 8 to get \(2^{x+3} + 64\).

15. C

\(\frac{\log n}{2 \log m} + \frac{\log m}{2 \log n} = 1\). Substituting \(x = \log_m n\), we have \(x + \frac{1}{x} = 2\) with the only solution of \(x = 1\). Thus, \(n = m\).

16. D

\(a \log_{1440} 5 + b \log_{1440} 2 + c \log_{1440} 3 = d\)

\(\log_{1440} 5^a + \log_{1440} 2^b + \log_{1440} 3^c = d\)

\(5^a 2^b 3^c = 1440^d = 5^d 2^{5d} 3^{2d}\)

Thus, \(a = 1, b = 5, c = 2, d = 1\). \(1 \times 5 + 2 \times 1 = 7\)

17. E

\[ \sqrt{6 + (1 + \sqrt{3 + (1 + \sqrt{3 + \sqrt{8}})^2})^2} = \sqrt{6 + (1 + \sqrt{3 + (1 + (1 + \sqrt{2}))^2})^2} \]

\[ \sqrt{6 + \left(1 + \sqrt{3 + (2 + \sqrt{2})^2}\right)^2} = \sqrt{6 + \left(1 + \sqrt{9 + 4\sqrt{2}}\right)^2} = \sqrt{6 + (2 + 2\sqrt{2})^2} = \]
4 + \sqrt{2}. 4+2=6.

18. B
\log_x y + \frac{1}{\log_x y} = \frac{10}{3}. We can use substitution a quadratic to solve this or recognize that the
reciprocals \frac{1}{3} + 3 = \frac{10}{3}. So, \log_x y = \frac{1}{3} and x = y^3. Now we have, y^4 = 400 and
y = 2\sqrt{5}. Thus, x = \frac{400}{2\sqrt{5}} = 40\sqrt{5}. 40\sqrt{5} - 2\sqrt{5} = 38\sqrt{5}.

19. D
Simplify \log_4(\log_{64} x) = \log_{64}(\log_4 x) to \frac{1}{2}\log_2(\frac{1}{6}\log_2 x) = \frac{1}{6}\log_2(\frac{1}{2}\log_2 x). Substitute
a = \log_2 x and get \frac{1}{2}\log_2(\frac{1}{6}a) = \frac{1}{6}\log_2(\frac{1}{2}a) or \log_2((\frac{1}{6}a)^3) = \log_2(\frac{1}{2}a) or \frac{1}{216}a^3 = \frac{1}{2}a.
Solving, we get a^2 = 108.

20. C
Using approximations of \log_{10} 2 and \log_{10} 3 where only up to 3 decimal places are
needed, take the base 10 logarithm of 18^{50} = 3^{100}2^{50} to get 100(0.477) + 50(0.301) =
62.75. 12^{50} = 10^{62.75}, thus the number of digits is 62.75 rounded up or 63.

21. A
(3 + \sqrt{7})^6 + (3 - \sqrt{7})^6 = 2(3^6 + 15 * 3^4 * 7 + 15 * 3^2 * 7^2 + 7^3) = 32384.
We know that (3 - \sqrt{7})^6 < 1, so 32384 - 1 < (3 + \sqrt{7})^6 < 32384. Thus the greatest
integer less than (3 + \sqrt{7})^6 would be 32384 - 1 = 32383. 3+2+3+8+3=19

22. C
Using basic matrix identities, we have \left(-\frac{e^5}{e^9}\right) + \left(-\frac{e^5}{e^9}\right) = -2e^{-4}.

23. B
The units digit of 3^n repeats after every fourth number with 3, 9, 7, 1. In mod4, 5^{79} is
1^{79} = 1, so the units digit of will be the first number in the cycle, so 3.

24. B
We can ignore the tens, hundreds, and thousands digits of all number in both
summations. We have
\left(1^{2017} + 2^{2017} + 3^{2017} + \ldots + 7^{2017}\right) + \left(1^{2018} + 2^{2018} + 3^{2018} + \ldots + 8^{2018}\right)
2017 is 1mod4 and 2018 is 2mod4. Thus, we can simplify because the units digits repeat
every 4th power.
\left(1^1 + 2^1 + 3^1 + \ldots + 7^1\right) + \left(1^2 + 2^2 + 3^2 + \ldots + 8^2\right).
The first part can be grouped up by 10 numbers (summing 0 through 9 to get 45 which
has a units digit of 5). 2017 divided by 10 is 201 remainder 7 which gives us 201 * 5 +
1 + 2 + 3 + 4 + 5 + 6 + 7 = 1033, so the first sum has a units digit of 3.
The second part can still be grouped up by 10 numbers with some simplification. We
have 0 + 1 + 4 + 9 + 16 + 25 + 36 + 49 + 64 + 81 = 285 which also has a remainder
of 5. Thus, 201 * 5 + 0 + 1 + 4 + 9 + 16 + 25 + 36 + 49 + 64 = 1209, so the second
sum has a units digit of 9.
3+9=12, so our answer is 2.

25. A
Take mod 5 of F(n) and get 1 + (-1)^n + 2^n + (-2)^n so whenever n is odd, then we get
0. There are 1009 odd integers.
Now, if \( n \) is even, we can plug in \( n=2x \) to get \( 1 + 1^x + (-1)^x + (-1)^x \) so only odd values of \( x \) work so we have 505 odd values of \( x \) from 1 to 1009. \( 1009+505=1514 \).

26. C

After 12 minutes, he’s left with \( 6.4kg \times \left(\frac{1}{2}\right)^{\frac{720 \text{ seconds}}{144 \text{ seconds}}} = 0.2kg \) of Zn-7, which means he has 175g of Zn-71. \( 2800g \times \left(\frac{1}{2}\right)^x = 175g \) gives us \( x = 4 \). \( \frac{720 \text{ seconds}}{4} = 180 \text{ seconds} \).

27. E

\[
\cos(\pi \sqrt{x^2 + 7}) - 1 \leq 0, \quad \cos(\pi \sqrt{x^2 + 7}) \geq 1.
\]

The maximum value of a cosine function is 1, so \( \cos(\pi \sqrt{x^2 + 7}) = 1 \) with the restriction that \( \sqrt{x^2 + 7} \) is an even integer. Thus, \( \log_2(-x^2 + 7x - 10) = 1, \) \( x^2 - 7x + 12 = 0 \) so \( x = 3 \) or 4. Only \( x = 3 \) satisfies the \( \sqrt{x^2 + 7} \) restriction.

28. D

The vertical asymptote is where \( 1 - 4x = 0 \) or \( x = \frac{1}{4} \).

The x-intercept is where \( f(x) = 2 - \log_3(1 - 4x) = 0 \). \( x = -2 \), so the shortest distance between (-2,0) and the line \( x = \frac{1}{4} \) would be along the x axis or simply \( |-2| + \left|\frac{1}{4}\right| = \frac{9}{4} \).

29. C

We have a telescoping series with

\[
\sqrt[3]{9} \cdot \sqrt[5]{81} \cdot \sqrt[7]{729} \cdot \sqrt[9]{6561} \ldots = 9^{\frac{3}{3+5+7}} \cdot 9^{\frac{5}{3+5+7}} \cdot 9^{\frac{7}{3+5+7}} \ldots
\]

\[9^{\frac{3}{3+5+7}} \ldots \] The telescoping series \( \frac{1}{3} + \frac{2}{3+5} + \frac{3}{3+5+7} + \frac{4}{3+5+7+9} \ldots \) can be expressed as \( \sum_{n=1}^{\infty} \frac{n}{(2n+1)!!} \) which converges to \( \frac{1}{2} \) so \( 9^{\frac{1}{2}} = 3 \).

30. A

We essentially such a series:

\[ S = 1 \cdot 2 + 2 \cdot 2^2 + 3 \cdot 2^3 + \cdots + 100 \cdot 2^{100} \]

Now, we can manipulate this series by multiplying by 2 to get

\[ 2S = 2 \cdot 2^2 + 2 \cdot 2^3 + 3 \cdot 2^4 + \cdots + 100 \cdot 2^{101} \]

Subtracting, we have

\[ -S = 2 + 2^2 + 2^3 + \cdots + 2^{100} - 100 \cdot 2^{101} \]

Or

\[ -S = 2^{101} - 100 \cdot 2^{101}, \] so \( S = 99 \cdot 2^{101} \)

Taking the base 2 log of \( S \), we get \( 101 + \log_2 99 \) while the restriction prevents the expressing from being manipulated. \( 101 + 2 + 99 = 202 \).