Answers:

0. 3
1. 3
2. 3
3. $(-6,-2)$ (must be written in interval notation)
4. $48\sqrt{6}-32$
5. 80
6. 5
7. D, B, C, A (in this order)
8. 1
9. $2e$
10. 32
11. 256
12. $\frac{\pi}{4}$
13. π
14. $\frac{3\pi}{20}$

Solutions:

0. $A=\lim_{x\to 1}\left(\frac{2x^2-3x+3}{4x^2-2x-1}\right)=\frac{2\cdot 1^2-3\cdot 1+3}{4\cdot 1^2-2\cdot 1-1}=\frac{2}{1}=2$

$B=\lim_{x\to 2}\left(\frac{x^2+x-6}{2x^2-3x-2}\right)=\lim_{x\to 2}\frac{\cancel{(x-2)}(x+3)}{\cancel{(x-2)}(2x+1)}=\frac{2+3}{2\cdot 2+1}=\frac{5}{5}=1$

$A+B=2+1=3$

1. $A=\lim_{x\to 0}\left(\frac{\sin x}{\tan x}\right)=\lim_{x\to 0}\cos x=\cos 0=1$

$B=\lim_{x\to 1^-}\left(\frac{\arcsin x}{\arctan x}\right)=\frac{\arcsin 1}{\arctan 1}=\frac{\frac{\pi}{2}}{\frac{\pi}{4}}=2$

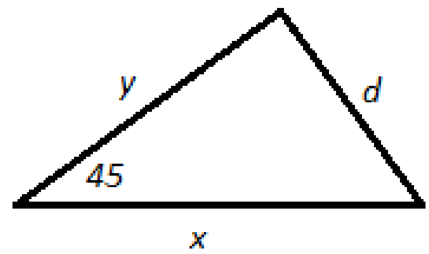
$A+B=1+2=3$

2. The slope of a tangent to this function is e^x (the derivative), so $e^a=a+1\Rightarrow a=0$ (based on the graphs of $y=e^x$ and $y=x+1$ intersecting only at $(0,1)$), which further implies that $b=1$. The tangent is therefore $y=x+1\Rightarrow c=-1$. Since this tangent has slope 1, $d=-1$. Therefore, $|a|+|b|+|c|+|d|=0+1+1+1=3$.

3. $f(x)=x^3+6x^2-36x+40=(x-2)^2(x+10)$, so *f* is positive on $(-10,2)\cup(2,\infty)$. $f'(x)=3x^2+12x-36=3(x-2)(x+6)$, so *f* is decreasing on $(-6,2)$. $f''(x)=6x+12$ $=6(x+2)$, so *f* is concave downward on $(-\infty,-2)$. The intersection of these three intervals is $(-6,-2)$.

4. Let x be positive so that it may count as a length. Since *x* runs from the *y*-axis to the outer edge of the rectangle, and since the rectangle is symmetric to the *y*-axis, the horizontal length of the rectangle is $2x$, and the vertical length of the rectangle is $36-2x^2$, making the area $R=2x(36-2x^2)=72x-4x^3$. Based on the side length restriction given in the problem, $1\le x\le 4$. $R'=72-12x^2\Rightarrow R'=0$ in this interval when $x=\sqrt{6}$. Sign analysis shows that this *x*-value gives a maximum area of $A=48\sqrt{6}$. Since this value is the only critical number in the interval, the minimum must occur at one of the two endpoints of the interval. $R(1)=68$ and $R(4)=32$, so $B=32\Rightarrow A-B=48\sqrt{6}-32$.

5. Using the diagram to the right, suppose the two people leave from the lower left vertex of the triangle. Using the Law of Cosines, we have that $d^2 = x^2 + y^2 - 2xy\cos 45^\circ = x^2 + y^2 - \sqrt{2}xy$. Differentiating implicitly with respect to time, $2d\frac{dd}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt} - \sqrt{2}\left(x\frac{dy}{dt} + y\frac{dx}{dt}\right)$. Since the two people walk for $\frac{1}{4}$ hour, $x = \frac{3}{4}$ mi and $y = \frac{1}{2}$ mi, and based on the relationship between d, x, and y, $d = \sqrt{\frac{9}{16} + \frac{1}{4} - \sqrt{2}\cdot\frac{3}{4}\cdot\frac{1}{2}} = \frac{\sqrt{13-6\sqrt{2}}}{4}$ mi. Therefore, plugging in these values are our given rates, $2\cdot\frac{\sqrt{13-6\sqrt{2}}}{4}\frac{dd}{dt} = 2\cdot\frac{3}{4}\cdot 3 + 2\cdot\frac{1}{2}\cdot 2 - \sqrt{2}\left(\frac{3}{4}\cdot 2 + \frac{1}{2}\cdot 3\right)$



$\Rightarrow \frac{\sqrt{13-6\sqrt{2}}}{2}\frac{dd}{dt} = \frac{13-6\sqrt{2}}{2} \Rightarrow \frac{dd}{dt} = \sqrt{13-6\sqrt{2}} \Rightarrow (A,B,C) = (13,6,2) \Rightarrow A\cdot B + C = 80$.

6. $A = \lim_{n\to\infty}\sum_{i=1}^{n}\left(\left(1+\frac{i}{n}\right)^2\cdot\frac{1}{n}\right) = \int_1^2 x^2dx = \frac{x^3}{3}\Big|_1^2 = \frac{2^3}{3} - \frac{1^3}{3} = \frac{7}{3}$ (between 2 and 3)

$B = \lim_{n\to\infty}\sum_{i=1}^{n}\left(\sin\left(\frac{\pi}{4} + \frac{\pi i}{4n}\right)\cdot\frac{\pi}{4n}\right) = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\sin x\,dx = -\cos x\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} = 0 + \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$ (between 0 and 1)

$C = \lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{2}{n}\cdot\ln\left(1+\frac{2i}{n}\right)\right) = \int_1^3 \ln x\,dx = (x\ln x - x)\Big|_1^3 = (3\ln 3 - 3) - (1\ln 1 - 1) = 3\ln 3 - 2$

(between 1 and 2)

Therefore, $\lfloor A\rfloor + \lceil B\rceil + \lceil C\rceil = 2+1+2 = 5$.

7. $A = \int_0^1 \frac{1}{x^2+1}dx = \arctan x\Big|_0^1 = \arctan 1 - \arctan 0 = \frac{\pi}{4} - 0 = \frac{\pi}{4} \approx 0.8$

$B = \int_0^1 \frac{x}{x^2+1}dx = \frac{1}{2}\ln\left(x^2+1\right)\Big|_0^1 = \frac{1}{2}\ln 2 - \frac{1}{2}\ln 1 = \frac{1}{2}\ln 2 \approx 0.35$

$C = \int_0^1 \frac{x}{x^4+1}dx = \frac{1}{2}\arctan\left(x^2\right)\Big|_0^1 = \frac{1}{2}\arctan 1 - \frac{1}{2}\arctan 0 = \frac{\pi}{8} - 0 = \frac{\pi}{8} \approx 0.38$

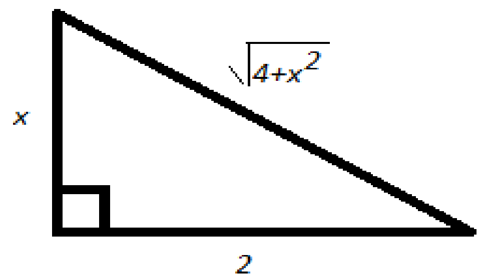
$D = \int_0^1 \frac{x^3}{x^4+1}dx = \frac{1}{4}\ln\left(x^4+1\right)\Big|_0^1 = \frac{1}{4}\ln 2 - \frac{1}{4}\ln 1 = \frac{1}{4}\ln 2 \approx 0.175$

Therefore, the values of these integrals, in increasing numerical order, is D, B, C, A.

8. Using the triangle for first quadrant angles,

$\sin\left(\arctan\frac{x}{2}\right)=\frac{x}{\sqrt{4+x^2}}$, so the integral

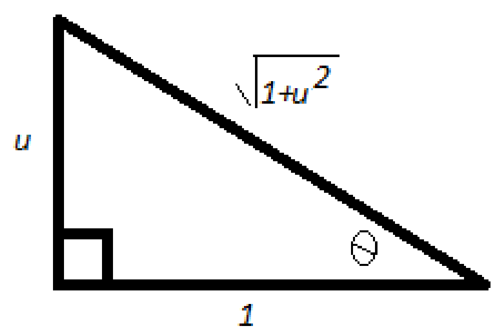
becomes $\int_0^{4\sqrt{6}}\frac{x}{\sqrt{4+x^2}}dx$. Now, using the



substitution $u=4+x^2$, $\frac{1}{2}du=xdx$, this

integral becomes $\frac{1}{2}\int_4^{100}u^{-\frac{1}{2}}du=\sqrt{u}\Big|_4^{100}=10-2=8$. Therefore, $A=8$.

For the second integral, we will work the integral as $\int_t^4\sqrt{1+\frac{1}{2x}}\,dx$ and take the limit as $t\to 0^+$ once the antiderivative is found. Make the substitution $u^2=2x,\ udu=dx$ to get $\int_{\sqrt{2t}}^{2\sqrt{2}}\sqrt{\frac{u^2+1}{u^2}}\cdot udu=\int_{\sqrt{2t}}^{2\sqrt{2}}\sqrt{u^2+1}du$. At this point, use the triangle to make the substitution $u=\tan\theta,\ du=\sec^2\theta d\theta$ to get $\int_{\arctan\sqrt{2t}}^{\arctan(2\sqrt{2})}\sec^3\theta d\theta$.



Now, to find this integral, we must use integration by parts to evaluate this integral by using $w=\sec\theta,\ dw=\sec\theta\tan\theta d\theta,\ v=\tan\theta,$ $dv=\sec^2\theta d\theta$, and this integral becomes

$$\int_{\arctan\sqrt{2t}}^{\arctan(2\sqrt{2})}\sec^3\theta d\theta=\sec\theta\tan\theta\Big|_{\arctan\sqrt{2t}}^{\arctan(2\sqrt{2})}-\int_{\arctan\sqrt{2t}}^{\arctan(2\sqrt{2})}\sec\theta\tan^2\theta d\theta=\sec\theta\tan\theta\Big|_{\arctan\sqrt{2t}}^{\arctan(2\sqrt{2})}$$

$$-\int_{\arctan\sqrt{2t}}^{\arctan(2\sqrt{2})}\left(\sec^3\theta-\sec\theta\right)d\theta=\sec\theta\tan\theta\Big|_{\arctan\sqrt{2t}}^{\arctan(2\sqrt{2})}-\int_{\arctan\sqrt{2t}}^{\arctan(2\sqrt{2})}\sec^3\theta d\theta+\int_{\arctan\sqrt{2t}}^{\arctan(2\sqrt{2})}\sec\theta d\theta$$

$$=\sec\theta\tan\theta\Big|_{\arctan\sqrt{2t}}^{\arctan(2\sqrt{2})}-\int_{\arctan\sqrt{2t}}^{\arctan(2\sqrt{2})}\sec^3\theta d\theta+\ln\left|\sec\theta+\tan\theta\right|\Big|_{\arctan\sqrt{2t}}^{\arctan(2\sqrt{2})}\Rightarrow 2\int_{\arctan\sqrt{2t}}^{\arctan(2\sqrt{2})}\sec^3\theta d\theta$$

$$=\sec\theta\tan\theta+\ln\left|\sec\theta+\tan\theta\right|\Big|_{\arctan\sqrt{2t}}^{\arctan(2\sqrt{2})}.$$ Therefore, we want

$$=\frac{\sec\theta\tan\theta+\ln\left|\sec\theta+\tan\theta\right|}{2}\Bigg|_{\arctan\sqrt{2t}}^{\arctan(2\sqrt{2})}=\frac{\sec\left(\arctan 2\sqrt{2}\right)\tan\left(\arctan 2\sqrt{2}\right)}{2}$$

$$+\frac{\ln\left(\sec\left(\arctan 2\sqrt{2}\right)+\tan\left(\arctan 2\sqrt{2}\right)\right)}{2}-\frac{\sec\left(\arctan\sqrt{2t}\right)\tan\left(\arctan\sqrt{2t}\right)}{2}$$

$$-\frac{\ln\left(\sec\left(\arctan\sqrt{2t}\right)+\tan\left(\arctan\sqrt{2t}\right)\right)}{2}=\frac{3\cdot 2\sqrt{2}}{2}+\frac{\ln\left(3+2\sqrt{2}\right)}{2}$$

$-\dfrac{\sec\left(\arctan\sqrt{2t}\right)\tan\left(\arctan\sqrt{2t}\right)}{2}-\dfrac{\ln\left(\sec\left(\arctan\sqrt{2t}\right)+\tan\left(\arctan\sqrt{2t}\right)\right)}{2}$. Now, since $\left(1+\sqrt{2}\right)^2=3+2\sqrt{2}$ and we want the limit as $t\to 0^+$, this equals $3\sqrt{2}+\ln\left(1+\sqrt{2}\right)$ $-\dfrac{1\cdot 0}{2}-\dfrac{\ln(1+0)}{2}=3\sqrt{2}+\ln\left(1+\sqrt{2}\right)$. Therefore, $B=3$, $C=2$, $D=1$, and $E=2$.

$\dfrac{B\cdot C+D\cdot E}{A}=\dfrac{3\cdot 2+1\cdot 2}{8}=1$.

9. $A=e$, as this is a definition of Euler's number. Further, since *f* is an increasing function for $x>0$ and has a horizontal asymptote, $B=0$ (if this doesn't convince you, use logarithmic differentiation to get $f'(x)=\left(1+\dfrac{1}{x}\right)^x\left(-\dfrac{1}{x+1}+\ln\left(1+\dfrac{1}{x}\right)\right)$, and therefore, $\lim\limits_{x\to\infty} f'(x)=e\cdot 0=0$).
Since $g(x)=(\log 5)10^{\log_5 x}=(\log 5)x^{\log_5 10}$, use the Power Rule to get $g'(x)=x^{\log_5 10-1}=x^{\log_5 2}=2^{\log_5 x}$, so $C=2$.
Thus, $C\cdot(A+B)=2\cdot(e+0)=2e$.

10. Since $a_n=3a_{n-1}-2$, $a_{n+1}=3a_n-2$. Subtracting the first equation from the second yields $a_{n+1}-a_n=3a_n-3a_{n-1}\Rightarrow a_{n+1}=4a_n-3a_{n-1}$. Using a linear recurrence relation, $a_n=a\cdot 3^n+b\cdot 1^n$ for some real numbers *a* and *b*. Using the first term is 5 and the second term is 13, the explicit formula for the sequence is $a_n=\dfrac{4}{3}\cdot 3^n+1$, so $A=-1$ and $B=3$.
Since $b_n=3b_{n-1}-2b_{n-2}$, using a linear recurrence relation, $b_n=c\cdot 2^n+d\cdot 1^n$ for some real numbers *c* and *d*. Using the first term is 5 and the second term is 13, the explicit formula for the sequence is $b_n=4\cdot 2^n-3$, so $C=3$ and $D=2$.
$(A+B)^{C+D}=(-1+3)^{3+2}=32$.

11. $f(x)=x^4+4x^3-48x^2+Ax+B\Rightarrow f'(x)=4x^3+12x^2-96x+A\Rightarrow f''(x)=12x^2+24x$ $-96=12(x+4)(x-2)$, and f'' changes signs at both –4 and 2, so these are the values of *C* and *D* (based on the expression of the sought value, it does not matter which is which). Further, $f(-4)=f(2)\Rightarrow -768-4A+B=-144+2A+B\Rightarrow A=-104$. Using this value of *A*, we have $E=f(2)=-352+B$, and while this gives neither the value of *B* nor

E, we do have that $B-E=352$. Therefore,
$A+B-C\cdot D-E=A+(B-E)-C\cdot D=-104+352-(4\cdot -2)=256$.

12. $\tan\left(\arctan\left(\frac{1}{n^2+n+1}\right)\right)=\tan\left(\arctan\left(\frac{\frac{1}{n(n+1)}}{\frac{n^2+n+1}{n(n+1)}}\right)\right)=\tan\left(\arctan\left(\frac{\frac{1}{n}-\frac{1}{n+1}}{1+\frac{1}{n}\cdot\frac{1}{n+1}}\right)\right)$

$=\frac{\frac{1}{n}-\frac{1}{n+1}}{1+\frac{1}{n}\cdot\frac{1}{n+1}}=\tan\left(\arctan\frac{1}{n}-\arctan\frac{1}{n+1}\right)$, so because the argument of the tangent function is positive for all positive integers n, $\arctan\left(\frac{1}{n^2+n+1}\right)=\arctan\frac{1}{n}-\arctan\frac{1}{n+1}$, so $\sum_{n=1}^{\infty}\left(\arctan\left(\frac{1}{n^2+n+1}\right)\right)=\sum_{n=1}^{\infty}\left(\arctan\frac{1}{n}-\arctan\frac{1}{n+1}\right)$, and the nth partial sum of this series is $s_n=\arctan 1-\arctan\frac{1}{n+1}$. Therefore, $\sum_{n=1}^{\infty}\left(\arctan\left(\frac{1}{n^2+n+1}\right)\right)$
$=\lim_{n\to\infty}\left(\arctan 1-\arctan\frac{1}{n+1}\right)=\arctan 1-\arctan 0=\frac{\pi}{4}-0=\frac{\pi}{4}$.

13. Multiply both sides of the equation by r to get
$r^2+163=16r\cos\theta+20r\sin\theta\Rightarrow x^2+y^2+163=16x+20y\Rightarrow(x-8)^2+(y-10)^2=1$, so the graph is a circle with radius 1, thus enclosing an area of π.

14. $A=\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}(\csc x-x\csc x\cot x)dx=\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\left(\frac{\sin x-x\cos x}{\sin^2 x}\right)dx=\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\left(\frac{\sin x-x\cos x}{\sin^2 x}\right)dx$

$=\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\left(\frac{x}{\sin x}\right)'dx=\left.\frac{x}{\sin x}\right|_{\frac{\pi}{6}}^{\frac{\pi}{2}}=\frac{\pi}{2}-\frac{\pi}{3}=\frac{\pi}{6}$

Since the fourth-degree Maclaurin polynomial for $\cos x$ is $1-\frac{x^2}{2!}+\frac{x^4}{4!}$, the second-degree Maclaurin polynomial for $\cos\left(\sqrt{x}\right)$ is $1-\frac{\sqrt{x}^2}{2!}+\frac{\sqrt{x}^4}{4!}=1-\frac{x}{2}+\frac{x^2}{24}$. Therefore,
$B=\int_0^2\left(1-\frac{x}{2}+\frac{x^2}{24}\right)dx=\left.x-\frac{x^2}{4}+\frac{x^3}{72}\right|_0^2=2-1+\frac{1}{9}=\frac{10}{9}$.

$\frac{A}{B}=\frac{\frac{\pi}{6}}{\frac{10}{9}}=\frac{3\pi}{20}$